THE

YOUNG GEOMETRICIAN'S COMPANION;

BEING

A New and Comprehensive COURSE of

PRACTICAL GEOMETRY;

CONTAINING,

I. An easy Introduction to *Decimal Arithmetic*, with the Extractions of the *Square*, *Cube*, *Biquadrate*, and *other Roots*.

II. Such *Definitions*, *Axioms*, *Problems*, *Theorems*, and *Characters*, as necessarily lead to the Knowledge of this Science.

III. *Planometry*, or the Mensuration of *Superficies*; as *Squares*, *Parallelograms*, *Triangles*, *Circles*, *Segments*, &c.

IV. *Stereometry*, or the Mensuration of *Solids*; as *Cubes*, *Parallelopipedons*, *Prisms*, *Cones*, *Pyramids*, *Cylinders*, *Spheres*, *Frustums*, &c.

V. The *Sections of a Cone*; as *Ellipses*, *Parabolas*, *Hyperbolas*, *Spheroids*, *Conoids*, *Spindles*, &c.

VI. The *Platonic Bodies*; as *Tetraedrons*, *Hexaëdrons*, *Octaëdrons*, *Dodecaëdrons*, and *Icosaëdrons*.

TO WHICH IS ADDED

A Collection of curious and interesting Problems, shewing that *Lines* and *Angles*, (and consequently the *least Particle of Matter*) may be divided *in infinitum*; that *Superficies* and *Solids* may be so cut as to appear considerably augmented; and, that the famous Problem of *Archimedes*, *of moving the Earth*, is capable of an easy and accurate *Demonstration*.

Calculated for the Use of SCHOOLS and ACADEMIES.

And is necessary to be gone through by the Scholar before he proceeds to the higher and more abstruse Branches of the *Mathematics*, *Indivisibles*, *Infinites*, *Algebra*, and *Fluxions*.

By the REVEREND R. TURNER, LL.D.

Rector of COMBERTON, and Vicar of ELMLY; Author of a View of the EARTH, or a Short System of MODERN GEOGRAPHY—View of the HEAVENS—HEAVENS SURVEYED—PLAIN TRIGONOMETRY made Easy—And a New Introduction to BOOK-KEEPING.

Καθαρμοὶ Ψυχῆς λογικῆς εισιν αἱ μαθηματικαὶ ἐπιστῆμαι.

HIEROCLES.

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TO

YOUNG GENTLEMEN

ENTERING UPON

MATHEMATICAL STUDIES.

GENTLEMEN,

YOU are here presented with an easy *Introduction* to the leading Branches of the MATHEMATICS. In the Pursuit of which, you will do well to make yourselves perfect in one Definition and Problem, before you proceed to another. * And though the Demonstration does not immediately accom-

* MATHO happening to look into two or three last Pages of a new Book of *Geometry*, was so frighted with the complicated Diagrams he found there, respecting the *Frustums of Cones*, *Pyramids*, and *Conic Sections*, that he shut the Book immediately in Despair, thinking none but a *Newton* was capable of reading it. But his Tutor happily persuading him to begin the first Pages about *Lines* and *Angles*, he found such surprizing Pleasure in a Week or two's Time in the Advances he daily made, that at last he became one of the greatest Geometricians of his Age.

pany every Rule, let not that be any Difcouragement; for the Reafon of many of the Operations will naturally arife as you go along, from the Connection of the Problems themfelves. But if a Youth be unwilling to proceed without a Demonftration of every Thing, he may as well refufe to *move* or *eat*, becaufe he has not by him a full Explication of the Caufe of *Mufcular Motion*, and the Nature of *Maftication* and *Digeftion* of his Food. Befides, many of the Rules arife only from *Algebraical* and *Fluxionary* Proceffes, which are impoffible for him to underftand at prefent. The *Young Geometer* muft therefore defer them till he has, by the following, or fome other Introduction, initiated himfelf into a Knowledge of the firft Principles of this moft excellent and ufeful Science; which, we flatter ourfelves, he may eafily, and in a fhort Time, accomplifh, by a careful Perufal of the following Work.

And, as moſt Solutions in *Geometry* require ſome Knowledge of *Decimal Arithmetic* and the *Extraction of Roots*, we have therefore judg'd it proper to prefix them, as neceſſary to be gone over by the Learner, before he enters fully upon his *Geometrical* Studies.

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To

To

To

☞ PLATO, a celebrated Greek Philosopher, who flourished about 350 Years before *Christ*, was used, in his Lectures, to illustrate and demonstrate to his Pupils the Truth of his Propositions by *Geometry*; and EUCLID, who lived about fourscore Years after him, being educated in PLATO's School, is said to have compiled his whole System of *Geometrical Elements* only in Reference to Applications of that Kind. But now, the Utility of *Geometry* extends to every Art and Science in Human Life.

ERRATUM.

Page 73, line 7, after the Period, read, "With the same Extent, and one Foot in *b*, make a Mark at *c*."

THE YOUNG Geometrician's Companion.

DECIMAL ARITHMETIC.

THIS is a particular Kind of *Arithmetic*, which enables us to treat *Fractions* as whole Numbers; and it is of the greatest Use in all Parts of Mathematical Learning. It receives its Name from *Decem* (Latin for Ten), because it always supposes the *Unit* or *Integer*, let it be what it will, whether 1 *Pound*, 1 *Mile*, 1 *Gallon*, to be divided into ten equal Parts, and each of those into 10 more, and so on, as far as we please.

Definitions.

A *Fraction* is a Number expressing some *Part* or Parts of an Unit or Integer: So the *Half*, a *Third*, or *Tenth* Part of any Thing are *Fractions*.

Every Fraction consists of two Numbers, the *Numerator*, and the *Denominator*. The *Denominator* shews into how many Parts the Unit or Integer is divided; and the *Numerator* is the Number expressing how many of those Parts are intended by the Fraction.

All Fractions are either *Vulgar* or *Decimal.*

In *Vulgar Fractions* the Denominator may be any Number whatſoever, and is always ſet under the Numerator with a Line between; ſo two Thirds is thus expreſſed $\frac{2}{3}$; three Fourths, thus $\frac{3}{4}$; and eleven Fifteenths, thus $\frac{11}{15}$.

But in *Decimal Fractions* the *Numerator* only is expreſſed, or wrote down; the *Denominator* being underſtood by Places, and is always 10, 100, 1000, &c. being an *Unit* with as many Cyphers annexed to it, as there are Figures in the *Numerator.* So the Denominator of .5 is 10; the Denominator of .47 is 100; and of .358 is 1000, &c.

A *Decimal Fraction* is known from a *whole Number* by a ſmall *crooked Daſh* before it, called a parting Line: Sometimes a *Point* or *Dot* is uſed inſtead of it. Thus ‚58 or .397 are Decimals.

If a Number conſiſts of a whole Number and a Decimal, it is called a *mixt Number.* If a Decimal ends at a certain Number of Places, it is ſaid to be *Finite:* But if it runs on *without terminating*, it is ſaid to be *Infinite.* When *one* of the Figures in the Decimal is repeated, as .666, &c. it is called a ſingle *circulating* or *recurring* Decimal. When *two* or more are repeated, as .602.602, &c. it is called a *compound circulating* or *recurring Decimal.* In Numbers where the ſame Figure continually *circulates*, make a Daſh acroſs the firſt, thus $.\cancel{6}66$; but where every *two* or *three* Figures repeat, make a Daſh acroſs the *firſt* and *laſt*, thus $.\cancel{6}0\cancel{2}$; the reſt may at preſent be omitted.

Nota-

Notation *of* Decimals.

AS in *Common Arithmetic* whole Numbers increaſe towards the *Left* Hand in a tenfold Proportion; ſo on the contrary, in this *Kind* of Arithmetic, Decimals decreaſe towards the *Right* Hand in the ſame Proportion, as appears in the following Table.

Hundreds of Thouſands.	Tens of Thouſands.	Thouſands.	Hundreds.	Tens.	Units Place.	Tenth Parts of Unity.	Hundredth Parts.	Thouſandth Parts.	Ten Thouſandths.	Hundred Thouſandths.
6	5	4	3	2	1	2	3	4	5	6
Whole Numbers.						Decimal Parts.				

From this Table it is evident, that the *firſt* Figure in the Decimals is ſo many *tenth* Parts of the Unit or whole Number, as the Figure itſelf denotes: The *ſecond* is ſo many hundredth Parts: The *third* ſo many thouſandth Parts. Thus .2 is $\frac{2}{10}$, or two tenth Parts; .23 is $\frac{23}{100}$, or twenty-three hundredth Parts; .234 is $\frac{234}{1000}$, or two hundred and thirty-four thouſandth Parts.

This Table is the very Foundation of *Decimals*, and on that Account, ought to be attentively conſidered.

Cyphers annexed or *added* on the Right Hand of any Decimal Fraction neither increase nor diminish its Value. Thus $\frac{250}{1000}$ is equivalent to $\frac{25}{100}$, or to .25 hundredth Parts. —But Cyphers, if placed *before* the Decimal, *decrease* its Value in a tenfold Proportion: As .3 tenths having a Cypher placed before it becomes $\frac{03}{100}$; if two Cyphers are prefixed it is $\frac{003}{1000}$; that is, only three thousandth Parts; and so of any others.

Hence it follows, that when you are to write down a Decimal Fraction, whose *Denominator* has more Cyphers in it than there are Figures in the *Numerator*, they must be supplied by prefixing so many Cyphers before the Figures of the Numerator. As suppose $\frac{16}{1000}$ was to be written down without its Denominator; in this Case, because there are *three* Cyphers in the Denominator, and but *two* Figures in the Numerator, we must therefore prefix a Cypher before the 16, and set it down thus, .016.

A *mixt Number* is composed of a *whole* Number and a Fraction, and is thus written 5.7, *viz.* five and seven Tenths; and 25.47, which is twenty-five, and forty-seven hundredth Parts of another.

The more Places a Decimal (which does not terminate) consists of, the nearer it expresses the Truth; but in Practice we seldom use more than three Places; and in high and accurate Calculations not above 4, or 5, or 6 at the most; and when the Decimal consists of several *Nines*, we reject them, and make the next Figure on the Left Hand one more: Thus, for 5.199 we write 5.2; and for 9.99 we write 10.

Addi-

Addition of Decimals.

IN *Addition of Decimals*, carefully place the Numbers under each other according to their reſpective Places; that is, Units (in whole Numbers) under Units; and Tenths (in Decimals) under Tenths, &c. Then add as if they were all whole Numbers, cutting off as many Figures from the Sum towards the Right Hand for Decimal Parts as there are Decimals found in any of the Numbers to be added.

The following Examples will make all plain.

Ex. 1.	Ex. 2.	Ex. 3.	
Decimal Parts of a Foot.	£.	Yards.	
.21	.002	5.16	
.32	.361	11.66	
.59	.1682	9.2	
.65	.28	3.721	
.82	.86	102.1	
2.59	1.6712	131.841	the Sum Total.

The chief Care to be taken here, is to keep the Dots, or ſeparating Points, exactly under one another; and to cut off in the whole Sum as many Figures towards the Right Hand, as there are Decimals in the greater Number. Thus, in the *Second Example*, becauſe there are 4 Decimals in the third Line, four Figures are ſeparated or cut off towards the Right Hand in the total Sum.

Subtraction *of* Decimals.

IN this Rule, we muſt alſo carefully ſet the Units under Units, and the Tenths under Tenths, &c. and ſubtract as in whole Numbers, always remembering to cut off as many Figures in the Remainder as there are Decimal Places in either of the other Numbers;

As in theſe Examples.

	Feet.	Yards.	Gallons.
From	915.315	201.125.	30.5...
Take	79.172	5.5785	7.2597
Remains	836.143	195.5465	23.2403

Note. If the Number of Places in the Decimals be more in that which is to be ſubtracted, than in that which we ſubtract from; we muſt ſuppoſe Cyphers to make up the Number of void Places, as in the laſt Example above, where three Cyphers are ſuppoſed to be added, and the Subtraction made accordingly.

Multiplication of Decimals.

THIS Rule is performed exactly the same as in whole Numbers; only we must observe to cut off as many Places of Decimals in the Product, as there are Decimals in the Multiplicand and Multiplier added together.

But if the Product hath not so many Figures as there should be Places cut off, the Deficiency must be supplied by prefixing a Cypher or Cyphers on the Left Hand, and then cut them off. A few Examples will sufficiently explain this Rule.

Example 1.

```
Multiplicand  .4267
Multiplier     .584
             ------
              17068
             34136
            21335
           --------
           .2491928 Product.
```

Here are *four* Figures cut off in the Multiplicand, and *three* in the Multiplier; therefore I cut off *seven* in the Product.

Example 2.

Multiplicand	.4
Multiplier	.2
	.08

The Number of Places cut off in the Multiplicand and Multiplier are *two*; but as the Product consists only of one Figure, the Defect is made up by prefixing the Cypher, and then cutting them *both* off.

Example 3.

```
  3.65
  1.35
  ----
  1825
 1095
 365
------
4.9275
```

Example 4.

```
  41.376
    .248
--------
  331008
 165504
 82752
---------
10.261248
```

The Rule compared with these Examples will explain each other

To multiply Decimals by 10, 100, 1000, &c. only remove the Point as many Places further towards the Right Hand as there are Cyphers in the Multiplier.

So .943 multiplied by 10 is - -	9.43
If multiplied by 100 it is - - -	94.3
And if multiplied by 1000 it is - -	943

Divi-

Division *of* Decimals.

HERE the Operation is the very ſame as in whole Numbers; only when the Quotient is found, we muſt ſubtract the Number of Places cut off in the Diviſor, out of the Number cut off in the Dividend, and the Remainder ſhews how many Places muſt be cut off in the Quotient.

But if the Quotient hath not ſo many Figures as there ſhould be Decimal Places in it, we muſt prefix as many Cyphers as will make up the Number of Places, and then cut them off, as in the following Examples.

Example 1.

Divide 4.6732 by .23

```
.23) 4.6736 (20.32 Anſwer.
     46
     ——
      73
      69
      ——
       46
       46
       ——
       . .
```

Here the Decimal Parts in the Dividend exceed thoſe in the Diviſor by two; we therefore cut off *two* Places for Decimals in the Quotient. By this Means we find that the Diviſor is contained in the Dividend 20 Times, and 32 hundredth Parts of another.

Example 2.

Divide .6474 by 73.

```
73) .6474 (.0088 the Answer.
     584
    -----
     634
     584
    -----
    (50)
```

Here the Quotient is 88; but as there are four Places of Decimals in the Dividend, and none in the Divisor, *four* Places must be cut off in the Quotient. We therefore prefix two Cyphers to the Figures in the Quotient, and it becomes .0088, the Answer, with a small Remainder.

Example 3.

Divide 295.75 by 8.45.

```
8.45) 295.75 (35
      2535
      -----
       4225
       4225
      -----
       ....
```

Here the Number of Decimal Places in the Dividend and Divisor being equal, the Quotient will be a whole Number; that is, the Divisor is contained in the Dividend just *thirty-five* Times.

Exam-

Example 4.

Divide 192.1 by 7.684.

```
7.684 ) 192.100 ( 25 the Anſwer required.
         15368
        ------
          38420
          38420
        -------
          ....
```

Becauſe here are not ſo many Places of Decimals in the Dividend as they are in the Diviſor; we annex Cyphers to the Dividend to make them equal, and the Quotient is in this Caſe a whole Number.

Example 5.

Divide 500*l.* among 26 Men, and give each Man's Share.

```
26 ) 500 ( 19.23l. for Anſwer.
     26
    ----
     240
     234
    -----
       60.
       52
      ----
        80.
        78
       ----
        (2)
```

In Queſtions of this Kind, where a *whole Number* is divided by a *whole Number*, we may find the Value of the Remainder to what Exactneſs we pleaſe, by adding a Cypher at a Time to it, remembering, that for every Cypher we add, there muſt be a Decimal in the Quotient. Here are *two* Cyphers added, therefore we have two Decimal Places in the Quotient.

To divide *Decimals* by 10, 100, 1000, &c. only remove the Point as many Places further towards the Left Hand as there are Cyphers in the Divisor.

Thus, 943 divided by 10 is	-	94.3
If divided by 100 is	-	9.43
If divided by 1000 is	-	.943
And, divided by 10000 is		.0943

Reduc-

Reduction *of* Decimals.

THIS Rule teaches us (1st) to reduce or change a Vulgar Fraction into a Decimal of the same Value.—(2dly) To find the Value of a Vulgar or Decimal Fraction.—And (3dly) To reduce any Part of *Coin*, *Weight*, or *Measure*, to a Decimal. We will begin with the first, which is,

To reduce a Vulgar Fraction into a Decimal of the same Value.

Rule.

As the Denominator of the Vulgar Fraction is to its Numerator, so is 10, 100, 1000, &c. (or any other Number, which is intended for a Denominator) to the Numerator of a Decimal Fraction.

Example 1.

Let $\frac{3}{4}$ of any Thing be given, to be reduced to a Decimal of two Places; viz. to have 100 for its Denominator.

$$\begin{array}{rl} \text{As } 4 : 3 :: & 100 \\ & \underline{\quad 3} \\ 4\,) & 300\ (\ .75 \\ & \underline{28\ \ } \\ & \ \ 20 \\ & \underline{\ \ 20} \\ & \ \ \cdot\ \cdot \end{array}$$

Here are cut off two Places in the Quotient, because there are 2 Cyphers in the third Number: So that we find, that *three Parts* in four of any Thing is equal to *seventy-five* Parts in a hundred of the same Thing. In the same Manner $\frac{1}{4}$ will be found equal to .25.

Exam-

Example 2.

What Decimal Fraction is equal to $\frac{15}{94}$ of any Thing?

$$\begin{array}{r}
\text{As } 94 : 15 :: 1000 \\
15 \\
\hline
5000 \\
1000 \\
\hline
\end{array}$$

$$\begin{array}{l}
94\,)\,15.000\,(\,.159 \text{ the Answer.} \\
\quad\;\; 94 \\
\hline
\quad\;\; 560 \\
\quad\;\; 470 \\
\hline
\quad\;\;\; 900 \\
\quad\;\;\; 846 \\
\hline
\quad\;\;\; (54)
\end{array}$$

This Decimal of .159 is not exactly equal to the given Vulgar Fraction; but by continuing the Division it may be carried to what Exactness we please; though in some Cases it never can be exact. But in this Example, the Remainder does not amount to more than the Half of one ten thousandth Part of the Whole, which is near enough in common Cases.

(2dly.)

(2dly.) To find the Value of a Vulgar or Decimal Fraction.

Rule.

As the Denominator of the Fraction is to its Numerator, so is the Number of Parts in the Integer (or whole Number) to the Value required.

Example 1.

What is the Value of $\frac{7}{16}$ of an Hundred Weight?

As 16 : 7 : : 112 Pounds in an Hundred Weight.

```
        7
    ------
16 ) 784 ( 49 Pounds.
     64
    ----
     144
     144
    ----
     ...
```

Example 2.

What is the Value of .384 of an Hogshead?

1000 : 384 : : 63 Gallons in an Hogshead.

```
          63
        -----
        1152
       2304
      ------
1000 ) 24192 ( 24 Gallons.
       2000
      ------
        4192
        4000
       -----
         192
           8 Pints in a Gallon.
        -----
1000 )  1536 ( 1 Pint.
        1000
       -----
        (536)
```

Answer 24 Gallons, 1 Pint, and $\frac{536}{1000}$ Parts of a Pint, which is a little more than half one. Note.

Note. The Value of a Decimal Fraction may be found more elegantly, by only multiplying the Decimal by the Number of Parts in the Integer; remembering to cut off as many Figures in the Product, as there are Places cut off in the Decimal given. Then multiply those Figures cut off, by the Number of Parts in the next inferior Denomination; and cut off as many Places as before; and so proceed till you have brought it down to the lowest Denomination required.

Examples.

What is the Value of .384 of an Hogshead?
63 Gallons in an Hogshead.

1152
2304

Gallons 24.192
8 Pints in a Gallon.

Pints 1.536

Here the Answer is the same as on the other Side; but is discovered in less than Half the Quantity of Figures.

What is the Value of .5789 of a Pound Sterling.
20 Shillings in a Pound.

Shillings 11.5780
12 Pence in a Shilling.

11560
5780

Pence 6.9360
4 Farthings in 1*d.*

Farthings 3.7440

Answer 11 Shillings and 6 Pence 3 Farthings, and something more.

(3dly.)

(3dly.) Some Part of an Integer being given, to find (nearly) what Decimal it is of the Integer:—or, to reduce any Parts of Money, Weights, and Measures, to a Decimal equal to it.

Rule.

First, make it a Vulgar Fraction by writing the Number of the same Parts contained in the Integer under it; then reduce that Fraction to a Decimal as before.

Examples.

What Decimal of a Foot is 3 Inches?

In a Foot are 12 Inches, consequently the Vulgar Fraction is $\frac{3}{12}$.

$$\begin{array}{rl} \text{Then, as } 12 : 3 :: & 100 \\ & \underline{\quad 3} \\ 12\,) & 300\,(.25 \\ & \underline{24} \\ & 60 \\ & \underline{60} \\ & \cdot\,\cdot \end{array}$$

Let it be required to reduce 12*s.* 2*d.* to the Decimal of a Pound Sterling.

In 12*s.* 2*d.* are 146 Pence; and in a Pound are 240 Pence; therefore the Vulgar Fraction is $\frac{146}{240}$.

$$\begin{array}{rl} \text{Then, as } 240 : 146 :: 1000 & \\ & \underline{1000} \\ 240\,) & 146000\,(\,.608 \text{ Answer.} \\ & \underline{1440} \\ & 2000 \\ & \underline{1920} \\ & (80) \end{array}$$

What

What Decimal of an Hundred Weight is 71 lb.? In an Hundred Weight are 112 lb. therefore the Vulgar Fraction is $\frac{71}{112}$.

Then, as 112 : 71 : : 1.000

$$\begin{array}{r} 71 \\ \hline 1000 \\ 7000 \\ \hline \end{array}$$

$$\begin{array}{l} 112\ )\ 71000\ (\ .633 \text{ Anſwer.} \\ \quad\ \ 672 \\ \quad\ \ \overline{\ \ 380} \\ \quad\ \ \ \ 336 \\ \quad\ \ \overline{\ \ 440} \\ \quad\ \ \ \ 336 \\ \quad\ \ \overline{\ (104)} \end{array}$$

What Decimal of a Pound Avoirdupoiſe is 11 Ounces? In one Pound Avoirdupoiſe is 16 Ounces: the Vulgar Fraction, therefore, is $\frac{11}{16}$; then ſay,

As 16 : 11 : : 1000

$$\begin{array}{r} 11 \\ \hline 1000 \\ 1000 \\ \hline \end{array}$$

$$\begin{array}{l} 16\ )\ 11000\ (\ .687 \text{ Anſwer.} \\ \quad\ \ 96 \\ \quad\ \ \overline{140} \\ \quad\ \ 128 \\ \quad\ \ \overline{\ 120} \\ \quad\ \ \ 112 \\ \quad\ \ \overline{\ \ (8)} \end{array}$$

In like Manner the Decimal of $3\frac{1}{4}$ Inches will be found to be .2708 of a Foot. For as 48, the Quarters of Inches in a Foot, is to 13, the Quarters of Inches in $3\frac{1}{4}$ Inches; ſo is 1000 to .2708, the Decimal required. The Vulgar Fraction being $\frac{13}{48}$ of a Foot. An

An Expeditious Method of discovering the Value of a Decimal of a Pound Sterling.

Rule.

Double the Figure which stands next the Point in the Decimals for so many *Shillings*; and if the next Figure be 5, or more, add 1 to the former Shillings; and for what it is under or above 5, reckon so many *Tens*, which added to the Figure in the third Place, will be so many *Farthings*, abating 1 for every 24; which brought into Pence, and added to the Shillings before found, will give the true Value of the Decimal required.

An Example or two will make all plain.

What is the Value of .782 of a Pound Sterling?

Answer 15*s*. $7\frac{3}{4}$*d*.

Here, the *first* Figure 7 being doubled is 14; the *second* being 8, reckon 1 more for 5, which makes 15 Shillings; the remaining 3 above 5 added as so many Tens to the 2 in the third Place is 32; which lessened by 1, because more than 24, makes 31 Farthings, or $7\frac{3}{4}$*d*. this added to the 15 Shillings, gives the Answer as above.

What is the Value of .5789 of a Pound Sterling?

Answer 11*s*. $6\frac{3}{4}$*d*.

Where the Decimal consists of four Figures, the last may be wholly omitted as inconsiderable.

Here

Here it will be useful to remember that

$\frac{1}{4}$. One Quarter of any Thing - - .25
$\frac{1}{3}$. One Third Part is - - - - - .333, &c.
$\frac{1}{2}$. One Half is - - - - - - .5
$\frac{2}{3}$. Two Thirds is - - - - - .666, &c.
$\frac{3}{4}$. Three Quarters is - - - - .75

What relates to the Doctrine of *Repetends* or *circulating Decimals*, with *Contractions* in *Multiplication* and *Division*, as being not absolutely necessary here, will be reserved for a Place in a future Work.

The

The Rule *of* Three *in* Decimals.

THIS Rule is performed the same Way as in whole Numbers, regard being had to cutting off the Decimals right in the Multiplications and Divisions; as in the following Examples.

Example 1.

Suppose $1\frac{1}{4}$ Yard of Cloth cost 17*s.* 6*d.* what will $47\frac{3}{4}$ Yards cost?

Operation.

```
     Yd.          s.           Yds.
As  1.25    :    17.5    ::   47.75
                               17.5
                              ------
                              23875
                             33425
                             4775
                             -------
                  1.25 ) 835.625 ( 668.5 Shillings.
                         750
                         ---
                          856
                          750
                          ----
                          1062
                          1000
                          ----
                            625
                            625
                           -----
                           (...)
```

Answer 668.5 Shillings, which reduced into Pounds by dividing by 20, is 33.425; or 33*l.* 8*s.* 6*d.*

Exam-

Example 2.

If 5 Yards $\frac{1}{2}$ of Cloth $\frac{3}{4}$ wide will make a Suit of Cloaths; how many Yards of Cloth of 1 Yard $\frac{1}{4}$ wide will do?

Operation.

	Yd. Wide.		Yds.		Yd. Wide.
If	.75	:	5.5	::	1.25

```
        5.5
        .75
       ----
        275
       385
      -----
1.25 ) 4.125 ( 3.3 Yards.
       375
       ---
        375
        375
        ---
       (...)
```

Answer 3.3 Yards; or 3 Yards $\frac{1}{4}$ and a little more.

Example 3.

What is the Interest of 325*l.* 15*s.* for a Year, at 4 $\frac{1}{2}$ per Cent. per Annum?

Operation.

	£.		£. Int.		£.
If	100	:	4.5	::	325.75

```
                 325.75
                    4.5
                 ------
                 162875
                130300
               --------     £. Int.
         100 ) 1465.875 ( 14.65875
```

Answer 14.65875, which *Decimal* being reduced by the Rule at Page 16, gives 14*l.* 13*s.* 2*d.*

Exam-

Example 4.

A Merchant bought 436 Yards of Cloth for 8.5*s.* per Yard, and sold it again for 10.75*s.* per Yard; what did he gain by the Sale of it?

Operation.

	s.
Sold at - - - - - -	10.75
Bought at - - - -	8.5
Gain per Yard - - -	2.25

Yard. *s.* Yards.
If 1 gain 2.25, what will 436 gain?

```
          436
         2.25
        -----
         2180
         872
        872
        -----     s.
     1 ) 981.00 ( 981
```

Answer 981*s.* which divided by 20, gives 49*l.* 1*s.*

Example 5.

If 5 Men do a Piece of Work in $4\frac{1}{2}$ Hours, in how many Hours will 12 Men do the same.

```
   Men.  Hours.    Men.
If 5  :  4.5   ::  12
          5
      -------
 12 ) 22.5 ( 1.875 Hours.
      12 00
      -----
      105
       96
      -----
        90
        84
      -----
         60
         60
      -----
        (..)
```

Answer 1 Hour and .875 thousandth Parts of another, which reduced by multiplying it by 60, gives 1 Hour and 52 Minutes and an Half exactly.

Example 6

Two Men, A and B, are Partners: A put in 20*l.* 10*s.* B put in 30*l.* 10*s.* and by trading they gained 8*l.* 10*s.* what is each Man's Share of the Gain?

Operation.

A put in 20.5
B put in 30.5
Stock 51.0 Then, as £. 51. : £. 8.5 : : £. 20.5

```
                    20.5
                     8.5
                    ----
                    1025
                   1640
                   -----
             51 ) 174.25.(3.416 A's Share £.
                  153
                  ---
                   212
                   204
                   ---
                     85
                     51
                     --
                    340
                    306
                    ---
                    (34)

And, as £. 51 : £. 8.5 : : £. 30.5
                            8.5
                           ----
                           1525
                          2440
                          -----
                    51) 259.25(5.083 B's Share
                        255
                        ---
                          425
                          408
                          ---
                           170
                           153
                           ---
                           (17)
```

Proof.

A's Share 3.416 $\frac{34}{51}$.
B's Share 5.083 $\frac{17}{51}$.

8.500 Here the Cyphers make no Addition to the Decimal; consequently the Sum of their Share is exactly 8*l.* 10*s.* as in the Question.

Exam-

Example 7.

Having the Circumference of a Circle or Wheel given; to find its Diameter.

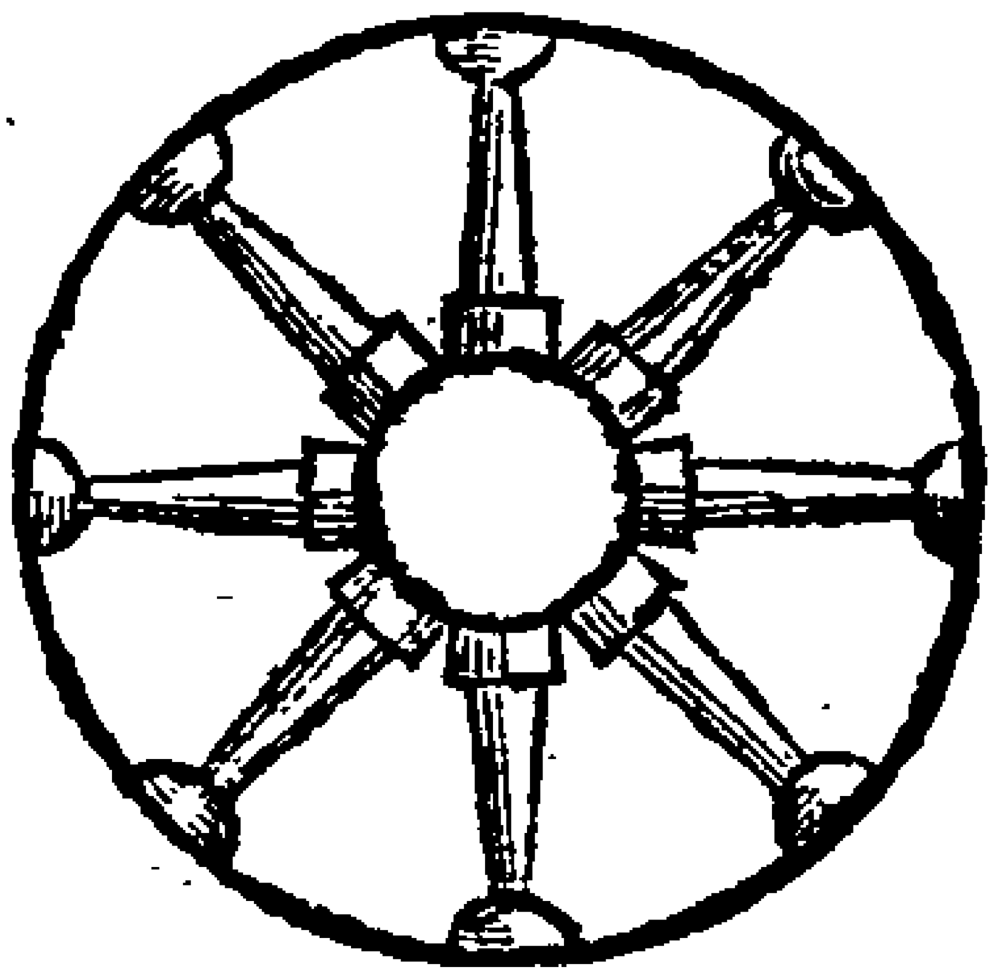
It has been long discovered, that if the Diameter of a Circle be 7, its Circumference will be 22 of the same Measure; and also that the Diameters and Circumferences of all Circles are in a direct Proportion to each other; whence this

Rule.

Dia. Circ.
As 7 is to 22 : so is any other Diameter to the Circum.
Cir. Dia.
And, as 22 is to 7 : : so is any other Circum. to the Diam.

Example.

Suppose a Coach Wheel turns round exactly *twice* in the Length of a Pole or Perch of $16\frac{1}{2}$ Feet; what then is its Diameter?



Operation.

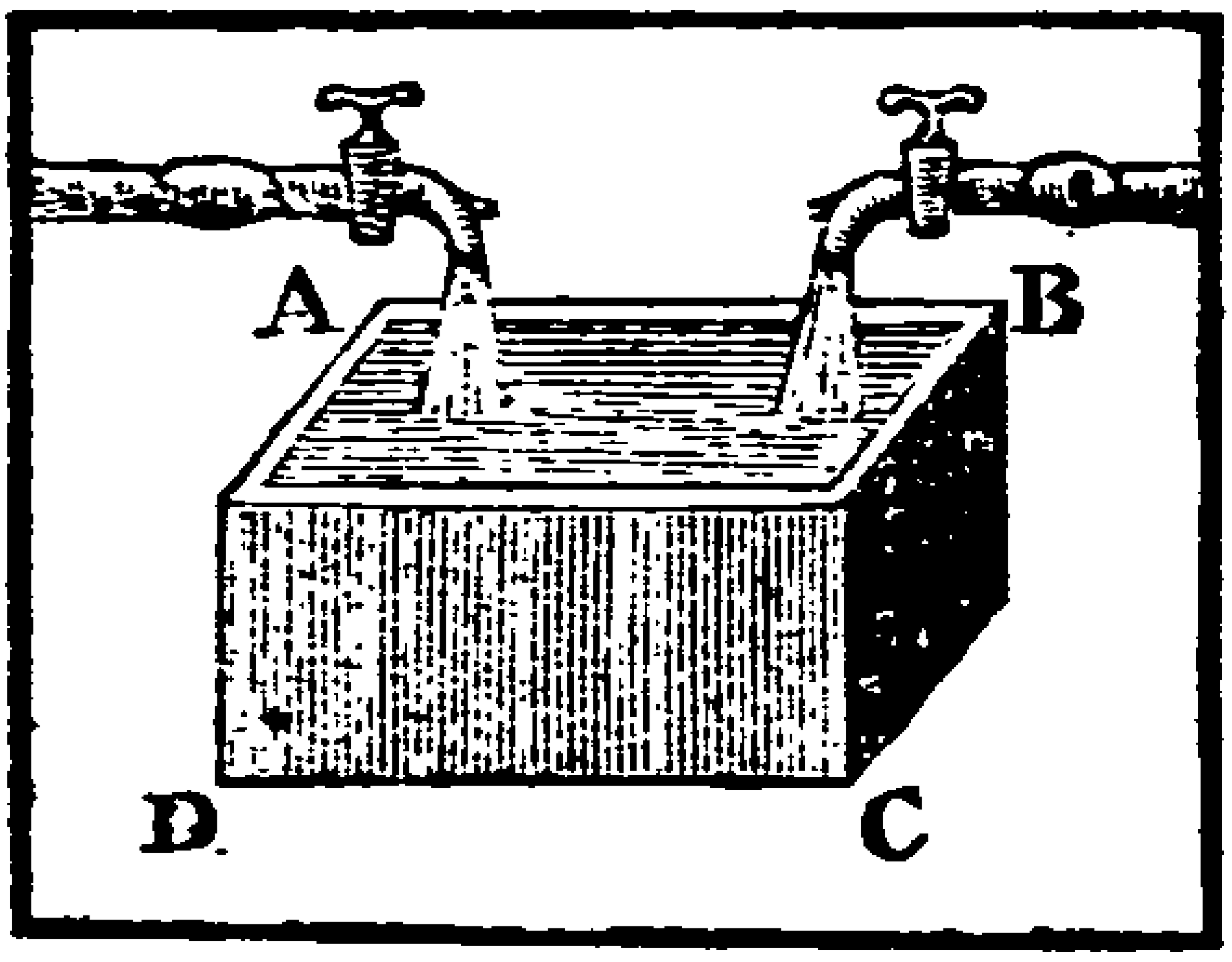
Cir. Dia. Cir.
As 22 : 7 : : 8.25 of the Wheel = Half the Pole.

```
               8.25
                  7
22 ) 57.75 ( 2.625 the Diameter sought.
     44
     ---
     137
     132
     ---
       55
       44
       ---
       110
       110
       ---
       (..)
```

If the Diameter had been given to find the Circumference; we must have made Use of the first Proportion above.

Example 8.

A Ciſtern has two Cocks; by the firſt it is filled in 20 Minutes, and by the ſecond in 30 Minutes; what Time will it take to fill the Ciſtern, when both the Cocks are ſet open at once?



Operation.

If 30 : 20 : : 50 by the Rule of Three Reverſe.

```
       30
50 ) 600 ( 12 Minutes the Time ſought.
      50
     ---
     100
     100
     ---
     (··)
```

Example 9.

If 3 Men and 4 Women (which we will ſuppoſe equal to 3 Men) can do a Piece of Work in 28 Days; how long will 1 Man and 1 Woman be doing the ſame?

Operation.

Here 4 Women being equal but to 3 Men; 1 Woman can be only equal to .75 Parts of a Man; and, as 4 Women are equal to 3 Men in doing the above Work, it may be conſidered as performed by 6 Men; whence this Proportion:

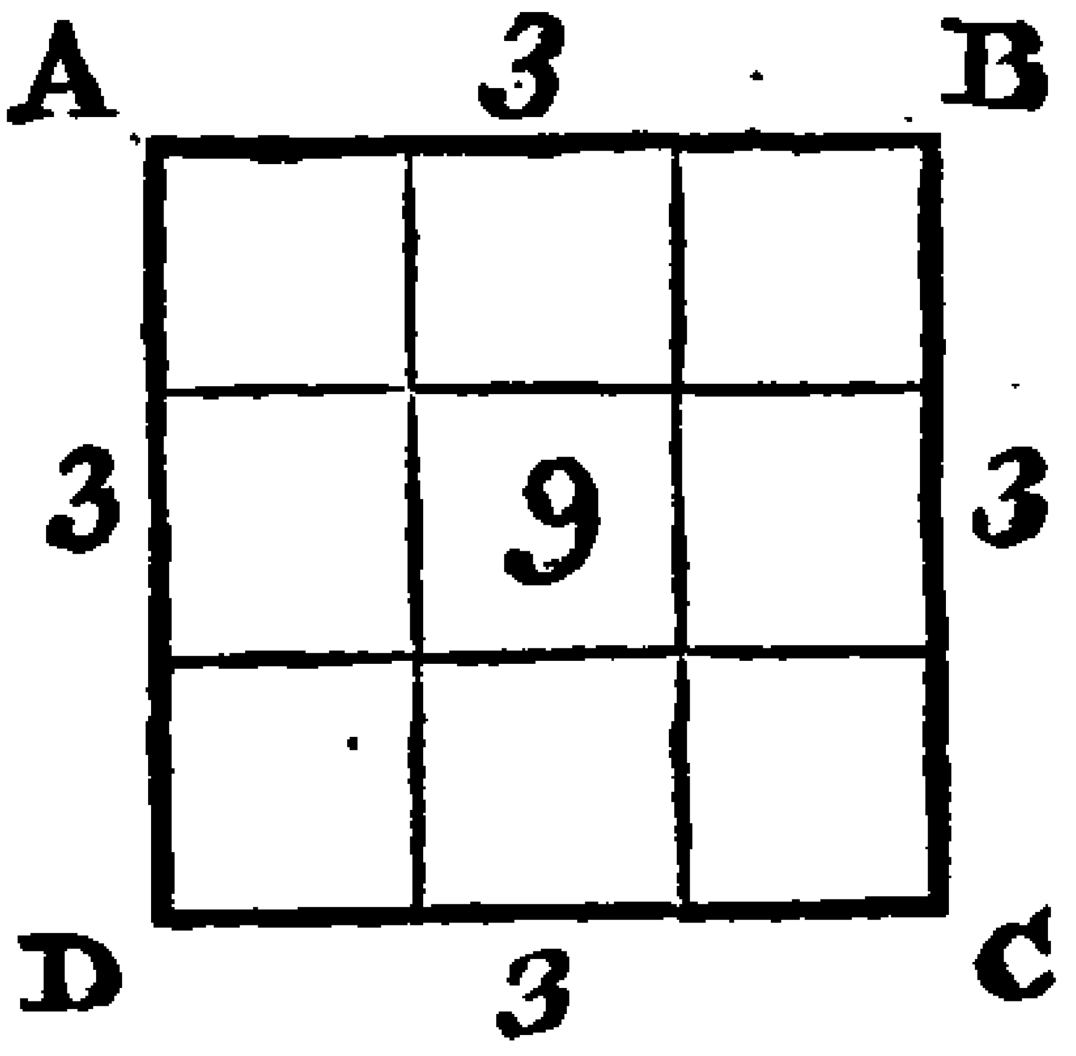
```
=3 M. & 4 W.    Days.      =1 M. & 1 W.
  If 6     :     28     : :    1.75
                  6
              --------
     1.75 ) 168.00 (96
              1575            Anſwer 96 Days.
              ----
              1050
              1050
              ----
              (··)
```

THE

THE EXTRACTION OF THE SQUARE ROOT.

TO extract the Square Root of any Number, is to find out a Number, which being multiplied by itself, the Product shall be equal to the given Number. Thus, suppose 9 the given Number; then will the Root of it be 3; because 3 multiplied by 3 will be equal to 9, the given Number.

And this may be geometrically demonstrated by the following Figure, where each Side contains 3 equal Parts, by which the great Square A B C D is divided into 9 little Squares. The Extraction of the Square Root, therefore, is, by having the Number of little Squares given, that are contained in a greater Square; to find out how many of the less Squares make one Side of the greater.



In order to extract the Square Root of any Number, it will be necessary to have by Heart the following Squares, whose Roots are one Figure.

Root	1	2	3	4	5	6	7	8	9
Square	1	4	9	16	25	36	49	64	81

By this Table you perceive the Square of 1 is 1; the Square of 2 is 4; the Square of is 9; and so of the rest.

This

This being done, we may proceed to extract the Square Root of any given Number by the following

General Rule.

First, set down the given Number; then make a Dot over the Place of Units, and so on, over every second Figure;* (towards the Left Hand in whole Numbers; but towards the Right Hand in Decimals:) and as many Dots as there are in the given Number, (which is usually called the *Resolvend)* so many Figures will there be in the Root.

Next, seek the nearest Square to the first Period or Dot, on the Left Hand; whose Root set in the Quotient, and its Square place under the first Period; then subtract, and to the Remainder bring down the next Period, (as in Division) which will form a Dividend.

Now, double the Root you put in the Quotient for a *Divisor*, and place it on the Left Hand of the Dividend; then seek how oft this Divisor can be had in all the Figures of the Dividend except the last; set it in the Quotient, and also on the Right Hand of the Divisor.

Multiply this increased Divisor, by the Figure last put in the Quotient, set the Product under the Dividend, and subtract it therefrom.

Lastly, to this Remainder bring down another Period for a *new Dividend*; then, double all the Figures in the Quo-

* The Reason for pointing every second Figure is, because the Square of the greatest Number under 10 can consist but of two Places.

tient

tient for a new Divisor; divide with the same Care as before; and so proceed till all the Periods are brought down, and the Operation be finished.

A few Examples will make this plain and easy.

What is the Square Root of 576?

Resolvend.

```
        576 ( 24 true Root.
        4
        ---
Divisor 44) 176
            176
            ---
           (...)
```

☞ To prove if you have performed the Operation right, multiply the Root by itself; to which Product add the Remainder if there be any; then, if this Sum be equal to the *Resolvend*, or Number given, it is right, otherwise not; and the Error must be sought out by performing the Operation over again.

What is the Square Root of 17956?

```
Resolvend .17956 (134 true Root.
           1
           --
     23 ) .79
           69
          ----
    264 ) .1056
           1056
           ----
          (....)
```

If any Thing remains after all the Periods are brought down, its Value may be found to what Exactneſs we pleaſe, by adding two Cyphers at a Time to the Remainder; and for every Pair of Cyphers ſo added, we ſhall have one Decimal Place in the Root; as in this Example.

What's the Square Root of 2268741?

2268741 (1506.23
1
25) 126
125
3006) 18741
18036
30122) 70500
60244
301243) 1025600
903729
(121871)

Note. In this Example, we have added two Pair of Cyphers; therefore we have two Decimal Places in the Root. And thus, by adding Cyphers by Pairs, we may carry on the Work to what Number of Decimals we pleaſe; but ſtill ſomething will remain. All ſuch Numbers are called *Surds*; and their Roots can never be perfectly found.

If the Number to be extracted is either *mixt*, or a Decimal; only make the Number of Decimal Places even, (if they are not so already) by adding a Cypher, (or Cyphers) that the Point may fall on the Units Place of the whole Number.

What is the Square Root of 357.519?

```
   ·  ·  ·  ·
   357.5190 ·( 18.908
   1        00
  ----
28 ) 257
     224
    -----
369 ) 3351
      3321
     ------
37808 ) 309000
        302464
        ------
        (6536)
```

Here we added a Cypher to make the Decimal Places even; and because there was a large Remainder, we annexed two more. Then, as there are three Points over the Decimal, we cut off three Places in the Quotient for Decimal Parts; the other two are Integers or whole Numbers.

To extract the Square Root *of a* Vulgar Fraction.

In the first Place, reduce it to a *Decimal*; and then proceed in all Respects as before.

What is the Square Root of $\frac{37}{340}$ of any Thing?

```
As 340 : 37 : : 10000
             10000
           --------
340 ) 370000 ( .1088 the Decimal Fraction.
      340...
      -----
       3000
       2720
       -----
        2800
        2720
        -----
         (80)
```

```
                        . .
Then extract the Root of .1088.. ( .329 the Square Root
                          9                   required.
                        -----
                     62 ) 188
                          124
                         -----
                    649 ) 6400
                          5841
                          -----
                          (559)
```

In this Extraction, at the second Dividend, we have annexed two Cyphers, for which we obtain one Place more in the Quotient; and, to the last Remainder two Cyphers more may be added, and the Work carried on, if the Root be wanted to greater Precision or Exactness.

The

The Use of the Square Root.

Problem I.

To find a Mean Proportional between two given Numbers.

Rule.

Multiply the two given Numbers together, and extract the Square Root of the Product, which Root will be the Mean Proportional sought.

Example.

What is the Mean Proportional between 10 and 40?

Operation.

```
         40
         10
        ----
         400 ( 20 the Mean required.
         4
        ----
    40) (000)
```

```
Proof.   As 10 : 20 : : 20 : 40 by the Rule of Three
                 20
                ----
         10 ) 400 ( 40
```

In like Manner the Mean Proportional between 16 and 36 is 24. For as, 16 : 24 : : 24 : 40.

Problem 2.

Any Body of Foot Soldiers being given to be formed into a Square Battalion, *i. e.* consisting of the same Number of Men in Rank and File.

Rule.

Only extract the Square Root of the Number, which Root will be the Number to be placed in Rank and File.

Example.

Suppose an Officer has a Body of 1156 Men to form into a Square Battalion; what will be the Number in every Rank and File?

Operation.

```
    1156 ( 34 Number of Men in every Rank and File.
     9
   ----
64 ) 256
     256
    -----
    (...)
```

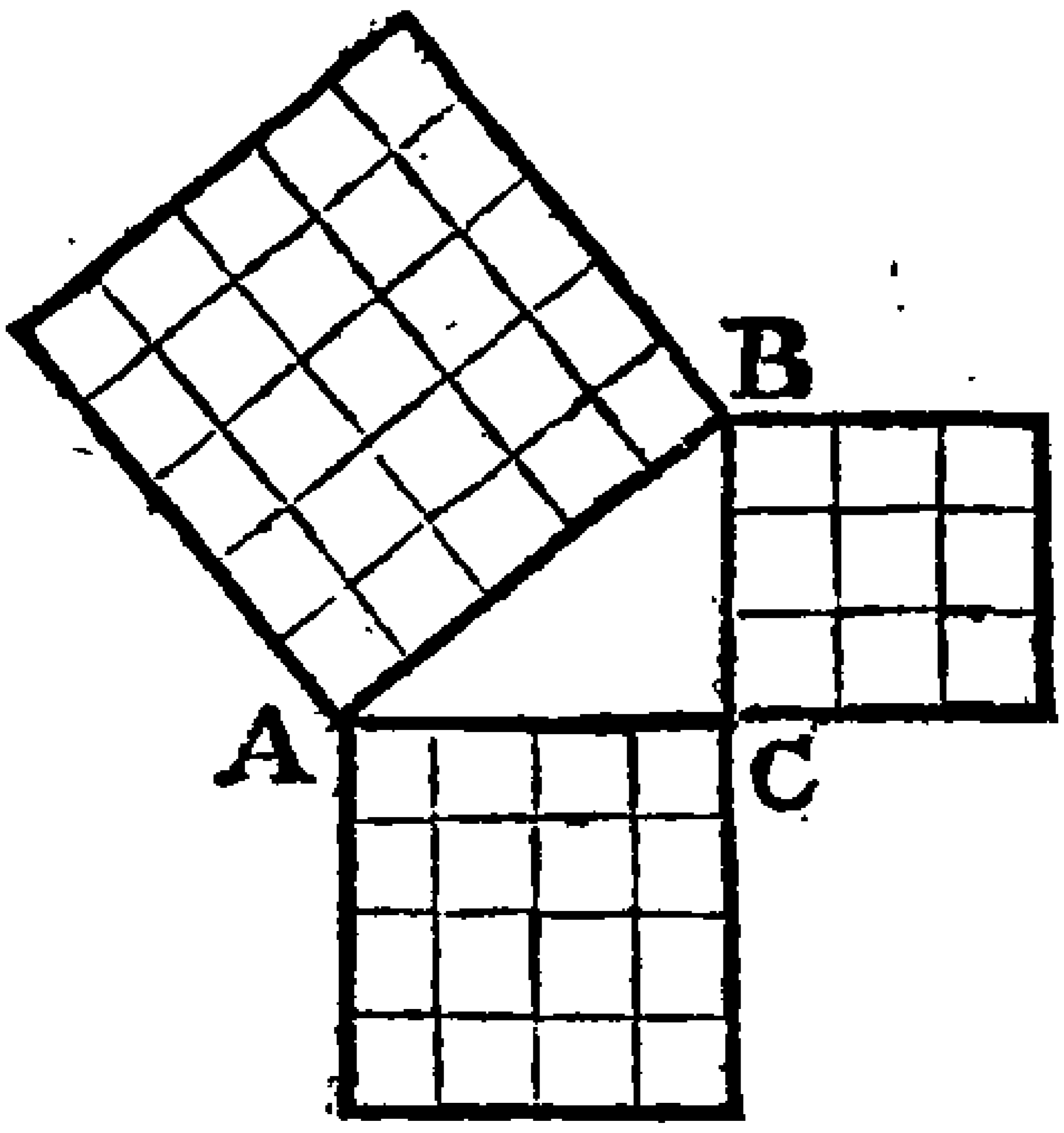
A Battalion thus formed is called a solid Square, to distinguish it from a hollow Square.

Also, if an Army consisting of 23716 Men is to be disposed by the *General* into a Square Battalion; he must place 154 in Rank, and as many in File.

The Property of a Right Angled Triangle.

In every Right Angled Triangle, the Square of the *Hypothenuſe*, or longeſt Side, is equal to the Squares of the *Baſe* and *Perpendicular* added together.

And this may be Geometrically demonſtrated thus.



In the Triangle A B C, we ſay, that the Square made upon the *Hypothenuſe* A B is equal to the Square made upon the Baſe A C; added to the Square made upon the Perpendicular B C.

This Property of a Right Angled Triangle is of the greateſt Uſe in moſt Branches of the *Mathematics*. It is no Wonder then, that *Pythagoras*, a learned Greek Philoſopher, who lived about 500 Years before Chriſt, ſhould offer ſo large a Sacrifice as that of 100 Oxen to the *Muſes*, for inſpiring him with ſuch an uſeful Invention, which he judged beyond the Power of human Ability to diſcover.

Problem 3.

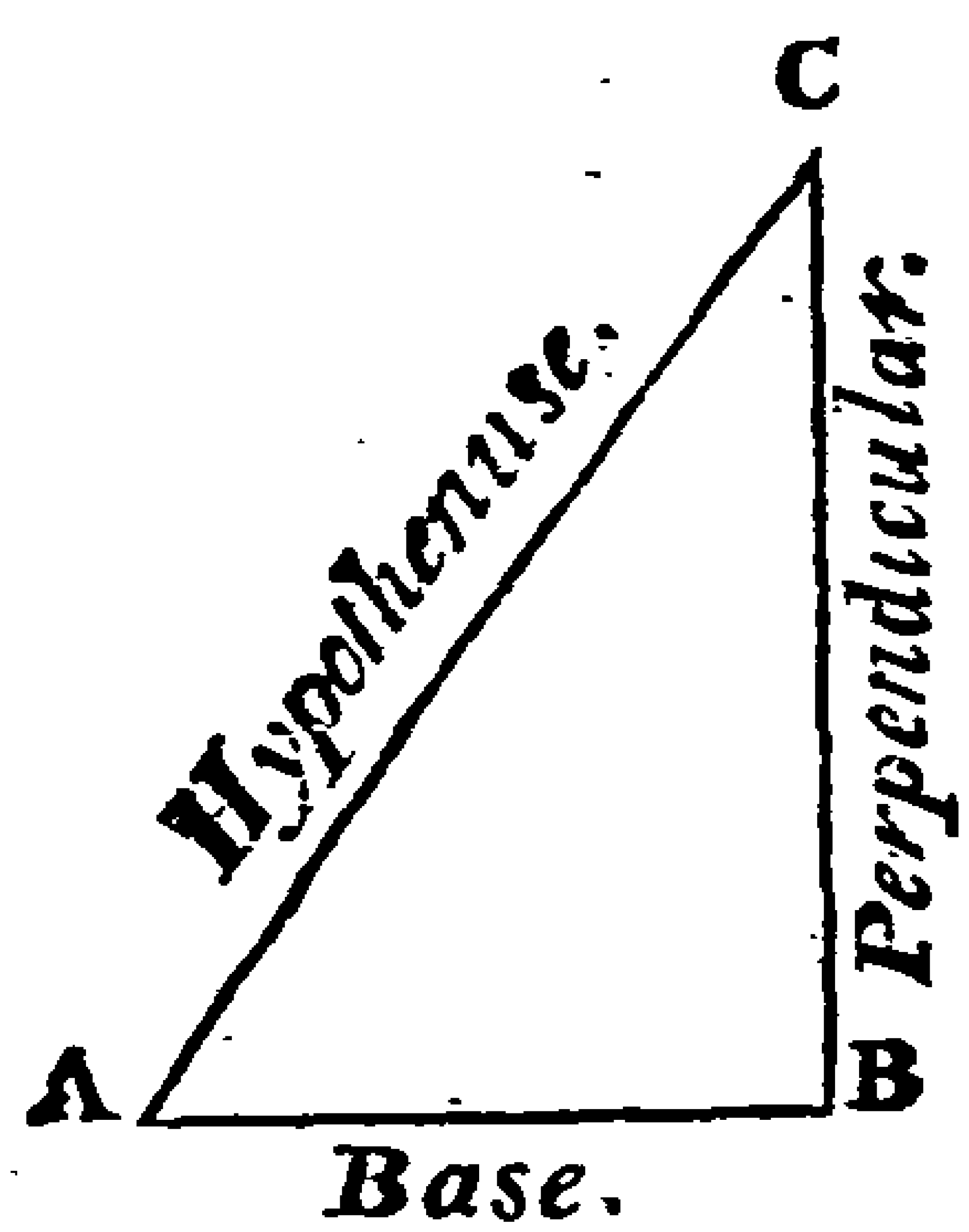
Given the Baſe and Perpendicular of any Right Angled Triangle, to find the Hypothenuſe.

Rule.

Add the Squares of each Side together; extract the Square Root of that Sum; and that Root will be the Hypothenuſe required.

Example.

Suppoſe in the Triangle A B C; the Baſe A B is 30 Yards; and the Perpendicular B C 40 Yards; what is the Length of the Hypothenuſe AC?



Operation.

Baſe ſquar'd.

30
30
——
900.

Perpendicular ſquar'd.

40
40
——
1600

B C □ 1600
A B □ 900
——
Sum 2500 (50 Yards the Length of the Hypothenuſe A C, required.
25
——
100) 0000

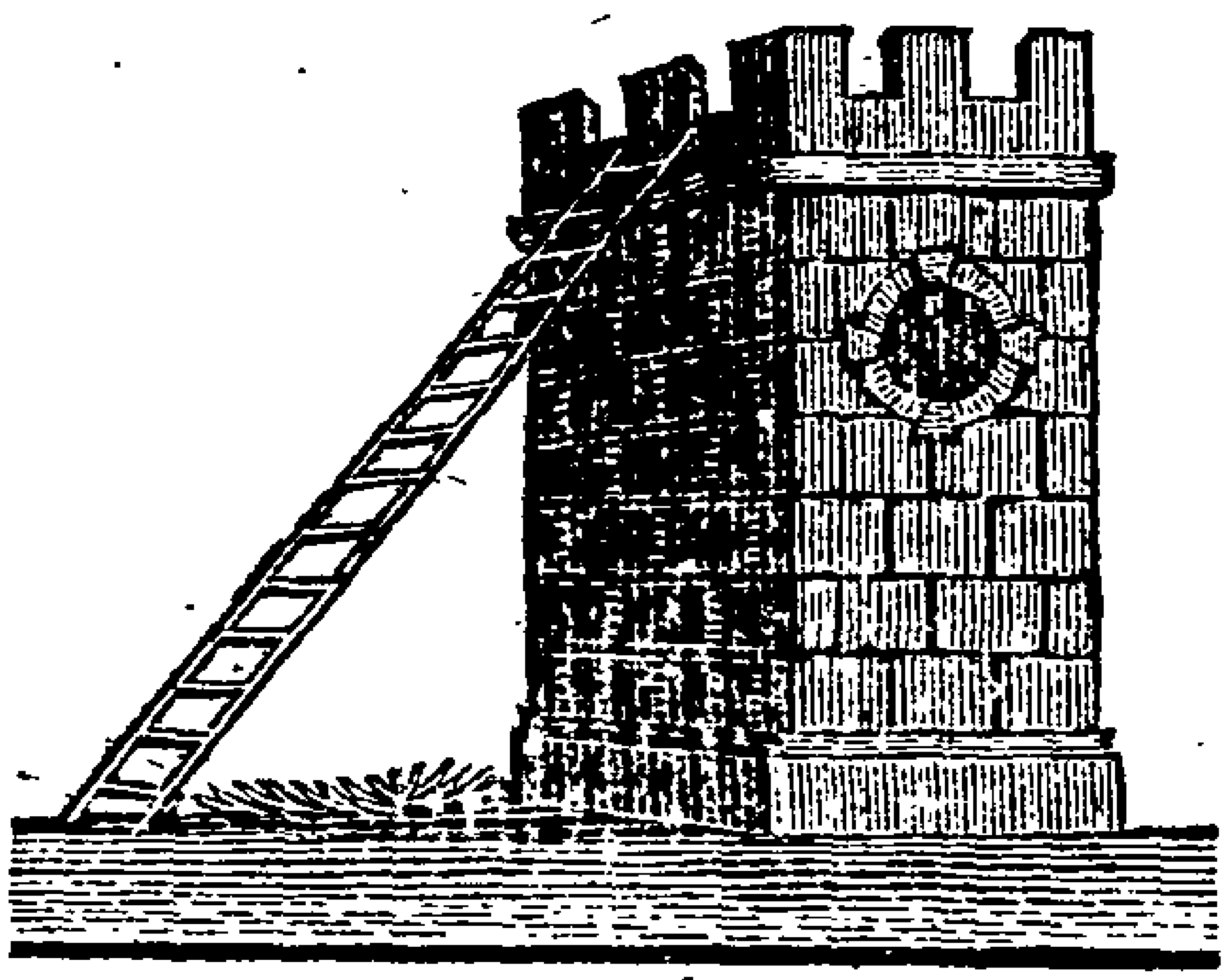
Pro-

Problem 4.

The Hypothenuſe and one Side being given, to find the other Side.

Rule.

From the Square of the Hypothenuſe, ſubtract the Square of the given Side; the Square Root of the Remainder will be the Side required.



Example 1.

Suppoſe the Length of the Ladder in the above Figure be 50 Yards, and it be placed 30 Yards from the Bottom of the Tower, where will the other End touch the Tower, when laid againſt it?

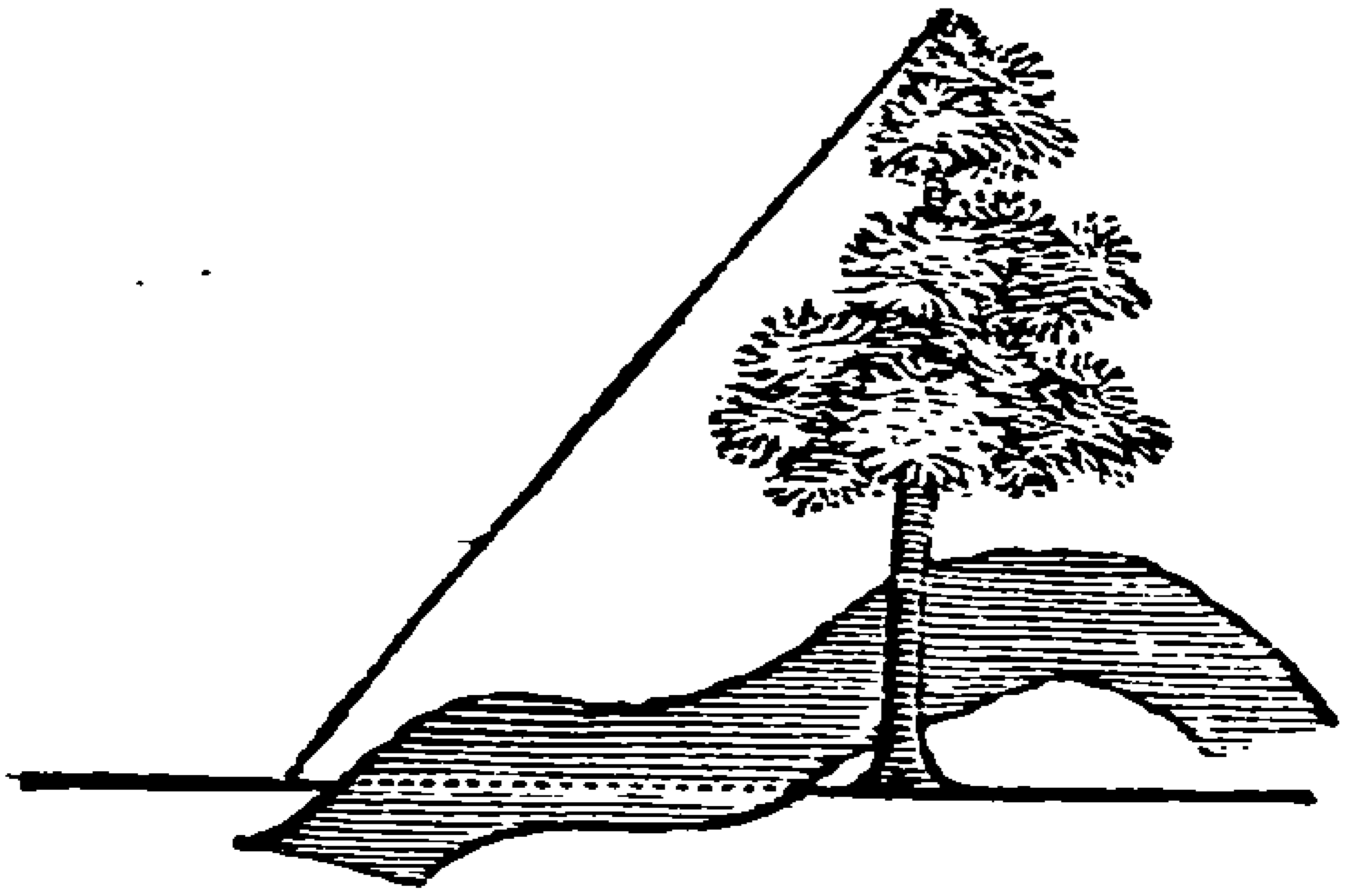
Operation.

From the Square of the Ladder	2500	
Take the Square of the Diſtance	900	
Remains Square of the Height of the Tower - - -	1600 (	40 Yards, the Height from the Baſe.
	16	
	80) 000	

Exam.

Example 2.

A Tree or a Fort ſtanding on the Edge of a *River*, meaſures 40 Feet in Height, and a Line ſtretched out from its Top to the Bank on the other Side the Water is exactly 50 Feet: What is the Breadth of the River, ſuppoſing the Land on both Sides level?



Operation.

From the Square of the Line 2500
Take the Square of the Tree 1600

Remains the Square of the River 900 (30 Root. the Breadth of the River required.
9
60) 000

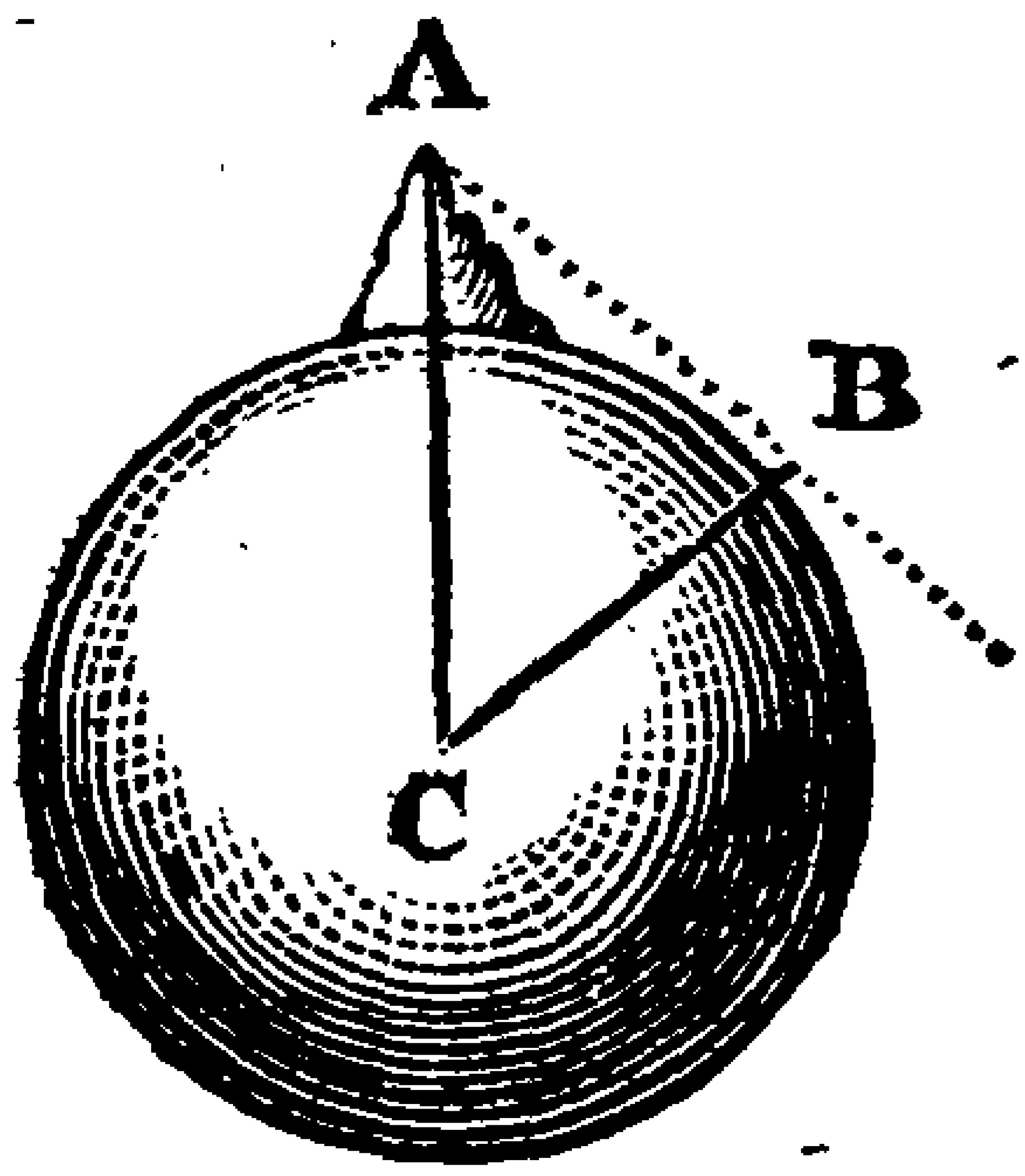
Pro.

Problem 5.

To find how far a *Mountain* of any given Height can be ſeen at Sea or on level Ground.

Example.

How far can the *Pike* of *Teneriff*, (one of the *Canary* Iſles) be ſeen at Sea, whoſe Height is about 4 Miles?



The *Circumference* of the Earth has been found by Admeaſurement to be about 25.020 Miles: Its Diameter therefore muſt be about 7964; and its Semi-diameter 3982 Miles. Then in the Triangle A B C, right angled at B, there is given the Side C B=3982; and alſo the Side A C =3986, to find A B.

Operation.

From the Square of A C 15888196
Subtract the Square of C B 15856324

Remains the Square of A B $\dot{3}18\dot{7}\dot{2}$ (178 Miles; and ſo far can the Mountain be ſeen.

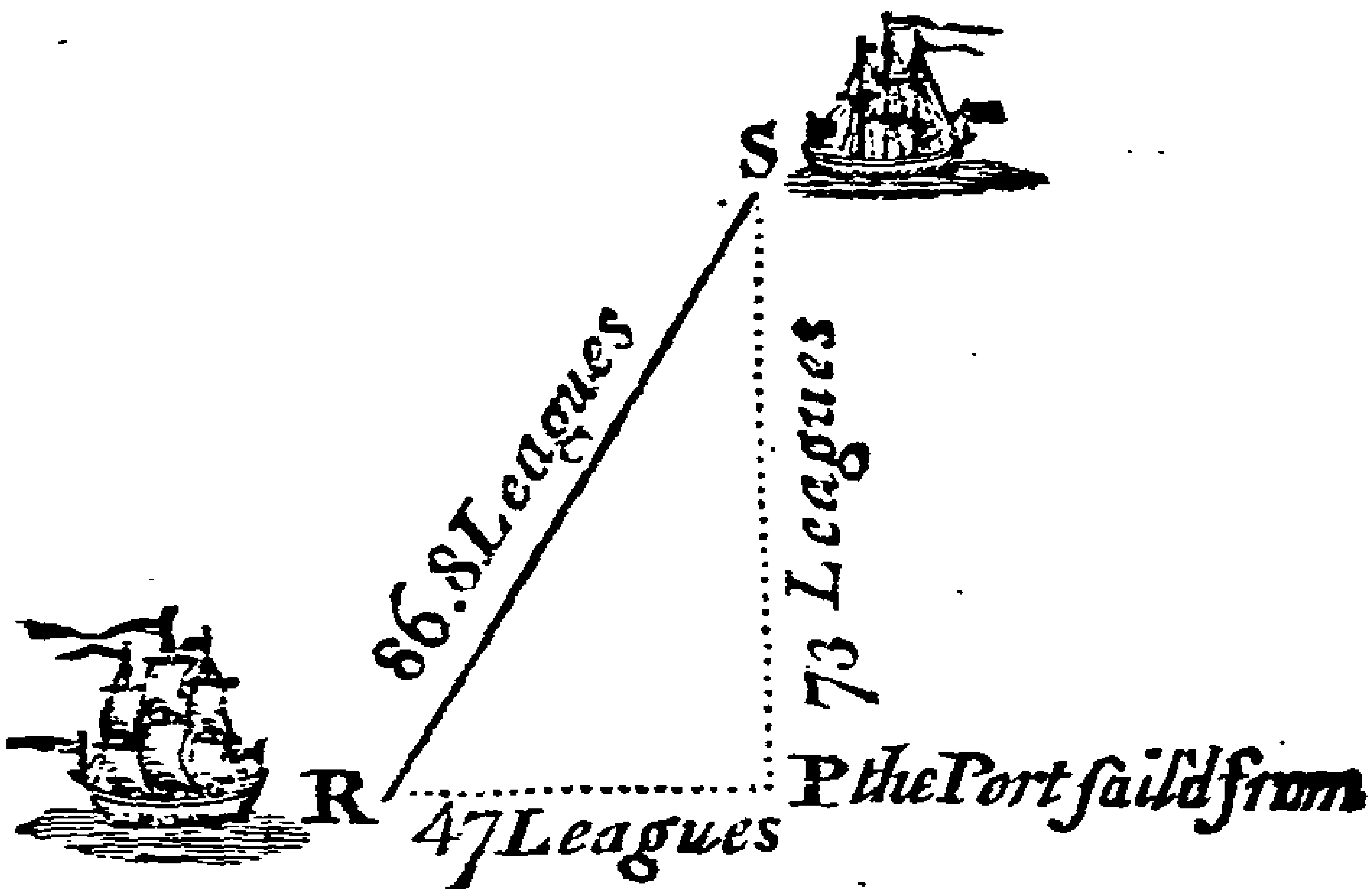
```
       31872 (178
       1
       ----
  27 ) 218
       189
       ----
 348 ) 2972
       2784
       ----
       (188)
```

☞ See an eaſier Method of anſwering this Problem at Page the 28th of my Trigonometry.

Pro-

Problem 6.

Two Ships, A and B, ſet ſail from the ſame Port P; A ſails from P, directly North to S, 73 Leagues; and B ſails directly Weſt from P to R, 47 Leagues; how far are they from each other?



Operation.

To the Square of the Perpendicular P S 5329
Add the Square of the Baſe R P - - 2209

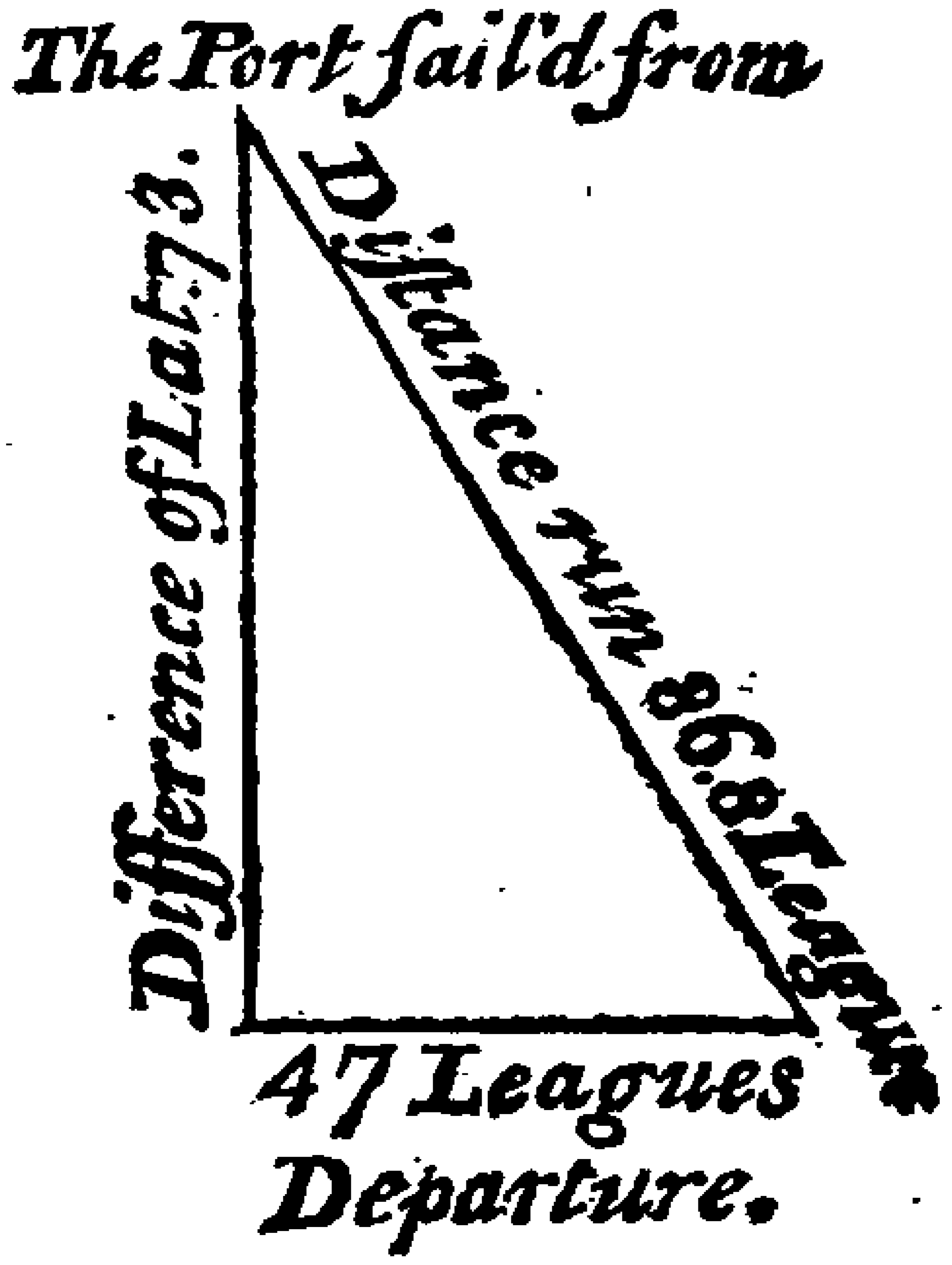
		Root.
The *Sum* is the Square of the Hypothenuſe R S - - -	7538 ·00	(86.8 Leagues, the Diſtance ſought.
	64	
166)	1138	
	996	
1728)	14200	
	13824	
	(376)	

In this Operation two Cyphers are added, conſequently, we have one Decimal Place in the Root. If greater Exactneſs be required, two more Cyphers may be brought down to the Remainder, and the Work be carried on as before.

Prob.

Problem 7.

A Ship failing away in the South-Eaftern Quarter runs 86.8 Leagues, when fhe finds fhe has departed, from the Meridian fhe failed from, 47 Leagues; how much has fhe altered her Latitude?



Operation.

From the Square of the Diftance run 7534.24
Take the Square of the Departure 2209.

Remains the Square of Diff. Latitude 5325.24 (72.9+ or 73 the Diff. of the Lat. (Root.)

```
5325.24 ( 72.9+
49
142 ) 425
      284
1449 ) 141.24
       13041
       (1083)
```

In like Manner, if the Diftance run and Difference of Latitude are given, you may find the Departure from the Meridian.

☞ In all Problems concerning Navigation, we fuppofe the Top of any Book, Paper, or Slate to reprefent the *North*; the Bottom, the *South*; the Right Hand, *Eaft*; and the Left Hand, *Weft*.

Pro-

Problem 8.

Suppose a Ship's *Distance run* in the North-Western Quarter be 110 Leagues; * her *Difference of Latitude*, she then finds to be 88 Leagues, and her *Departure* from her former Meridian 66 Leagues; I demand the Ship's Course, that is, the Angle, or Point of the Compass she sailed upon?

Having shewn how to find the *Distance*, *Difference* of *Latitude*, and *Departure* (from any two of them being given); we will now point out a Method of finding the Course, or Angle the Ship sailed upon, made with the Meridian.

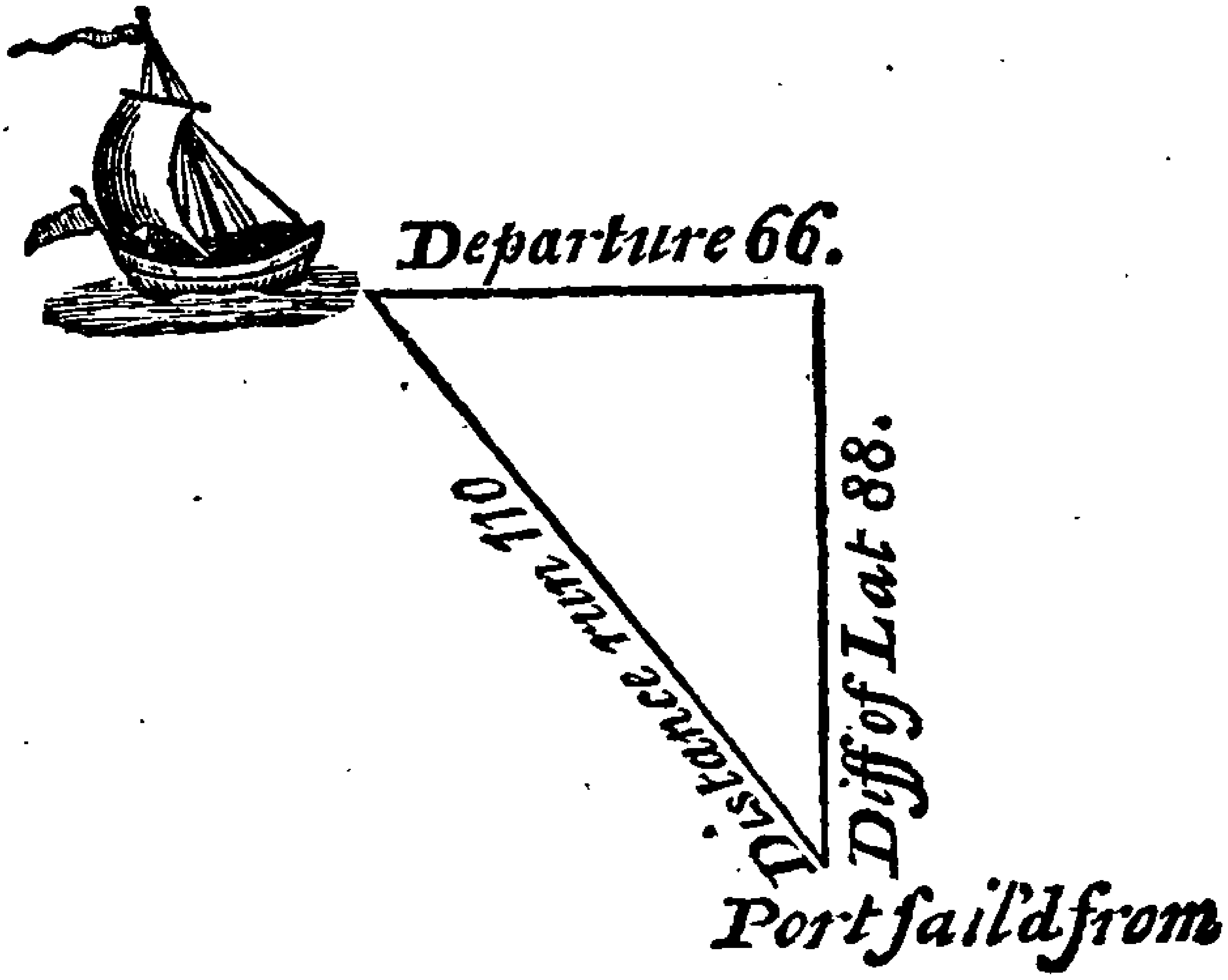
Rule.

As the Sum of the Distance run, and Half the greater of the other two Legs; is to to the less Leg; so is 86, a fix'd Number; to the Angle (in Degrees) opposite to the less Leg; which is the Course, when the Departure is less than the Difference of Latitude; otherwise it is the Complement of the Course; *i. e.* what the Course wants of 90 Degrees. †

* An English League is 3 Miles. A French League is but two Miles.

† For the Reason of this Rule, see my *Trigonometry*.

Opera-



Operation.

The Diſtance run 110 Leagues.
Half the greater Leg 44 Leagues.

Sum 154.

Then, as 154 : 66 : : 86

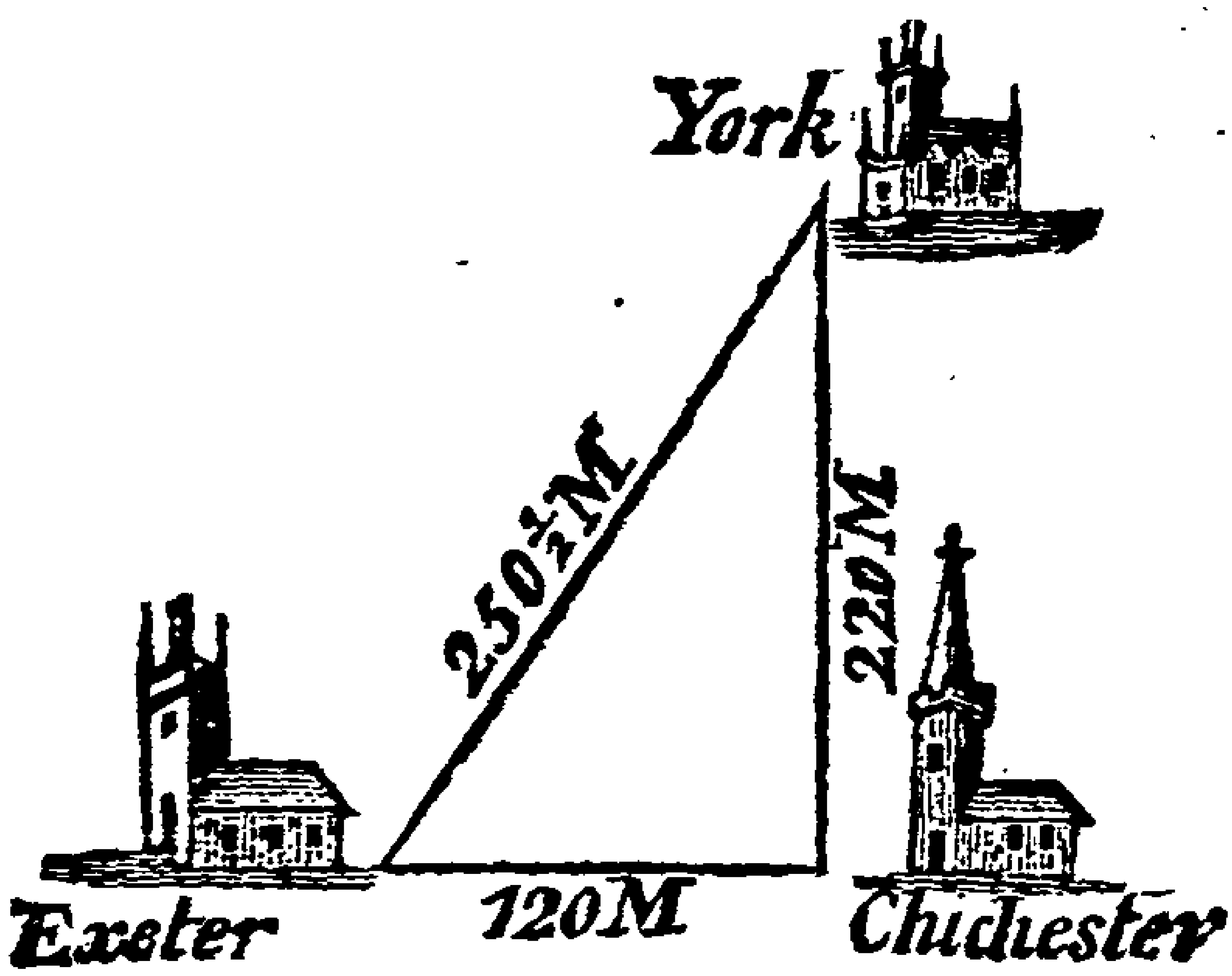
```
                 86
                 66
                ---
                516
               516
               ----
       154 ) 5676 ( 36 Degrees.
             462
             ----
             1056
              924
             ----
            (132)
               60
             ----
       154 ) 7920 ( 51 Minutes.
             770
             ---
              220
              154
             ----
             (66)
```

Anſwer, 36 Deg. 51 Min. the Angle which the Ship's Run made with the Meridian of the Place ſhe failed from.

Pro-

Problem 9.

There are two Cities, as *Chichester* and *York*, which lie North and South from each other, about 220 Miles; and *Exeter*, another City, lies directly West from *Chichester* about 120 Miles; now I desire to know the Distance of *York* from *Exeter*?



Operation.

Square of the Distance from *Chichester* to *York*	48400
Square of the Distance from *Chichester* to *Exeter*	14400
The Sum	62800

Miles.

Then extract the Root of 62800. (250.5

```
            62800. (250.5
            4      00
          -----
    45 )  228
          225
          -----
   505 )  3000
          2525
          -----
          (475)
```

Answer, 250.5 Miles; that is 250 Miles, and Half another, which is the Distance required.

Here are two Cyphers brought down, we have therefore one Decimal Place in the Root.

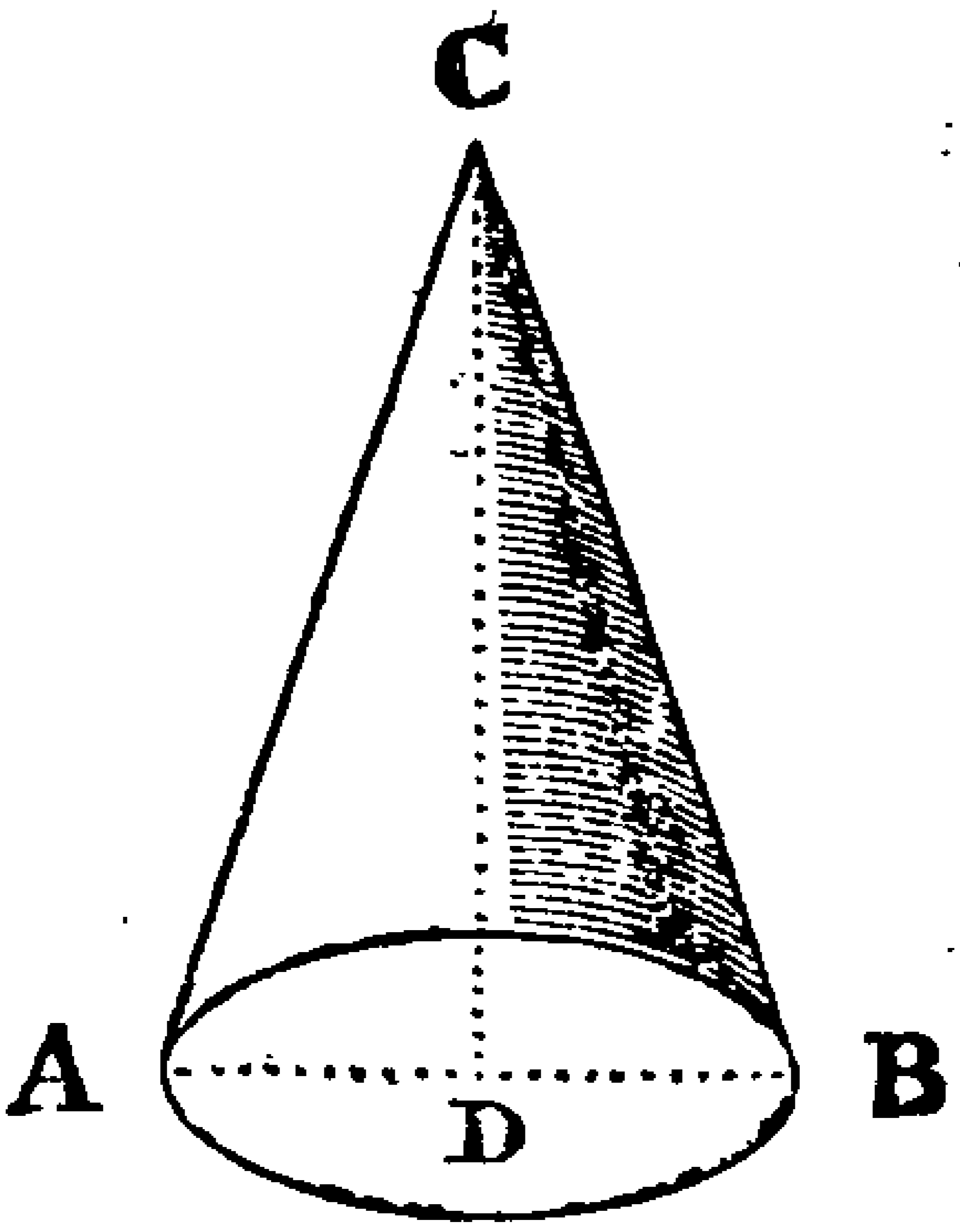
Pro-

Problem 10.

The Slant Side of a Pyramid, or Cone, being given, and likewiſe the (Side or) Diameter of the Baſe; to find the Perpendicular Height?

Example.

Suppoſe the Slant Side A C of the Cone A B C D, be 100 Inches, and the Diameter of the Baſe A B 120 Inches; what is its perpendicular Height?



Operation.

From the Square of the Slant Side A C	-	10000
Subtract the Square of $\frac{1}{2}$ the Baſe A D	-	3600
Remains the Square of the Height C D	-	6400

Then extract the Square Root of $\dot{6}4\dot{0}0$ (80 Root.
64
16.) 000

Anſwer, 80 Inches, the perpendicular Height required.

See more Examples in the *Young Gauger's beſt Inſtructor*.

Pro-

Problem 11.

If a Fathom * of Rope 6 Inches in Circumference weigh 6 Pounds; how much will the fame Quantity of Rope weigh whofe Circumference is 12 Inches?

The Areas and Weight of all Circles are to each other as the Squares of their Diameters or Circumferences, whence this

Operation.

	□ Cir.		lb.		□ Circum.
As	36	:	6	: :	144
					6

36) 864 (24 lb. the Wt. fought
72
———
144
144
———
(. .)

Problem 12.

If a Fathom of Rope weighing 24 lb. be 12 Inches in Circumference; what is the Circumference of a Rope, a Fathom of which weighs 6 lb.

This Problem is but the Reverfe of the former.

lb.		□ Circum.		lb.		□ Circum.
As 24	:	144	: :	6	:	36 whofe □ Root is 6 Inches, the Circumference fought.

* A Fathom is 6 Feet in Length.

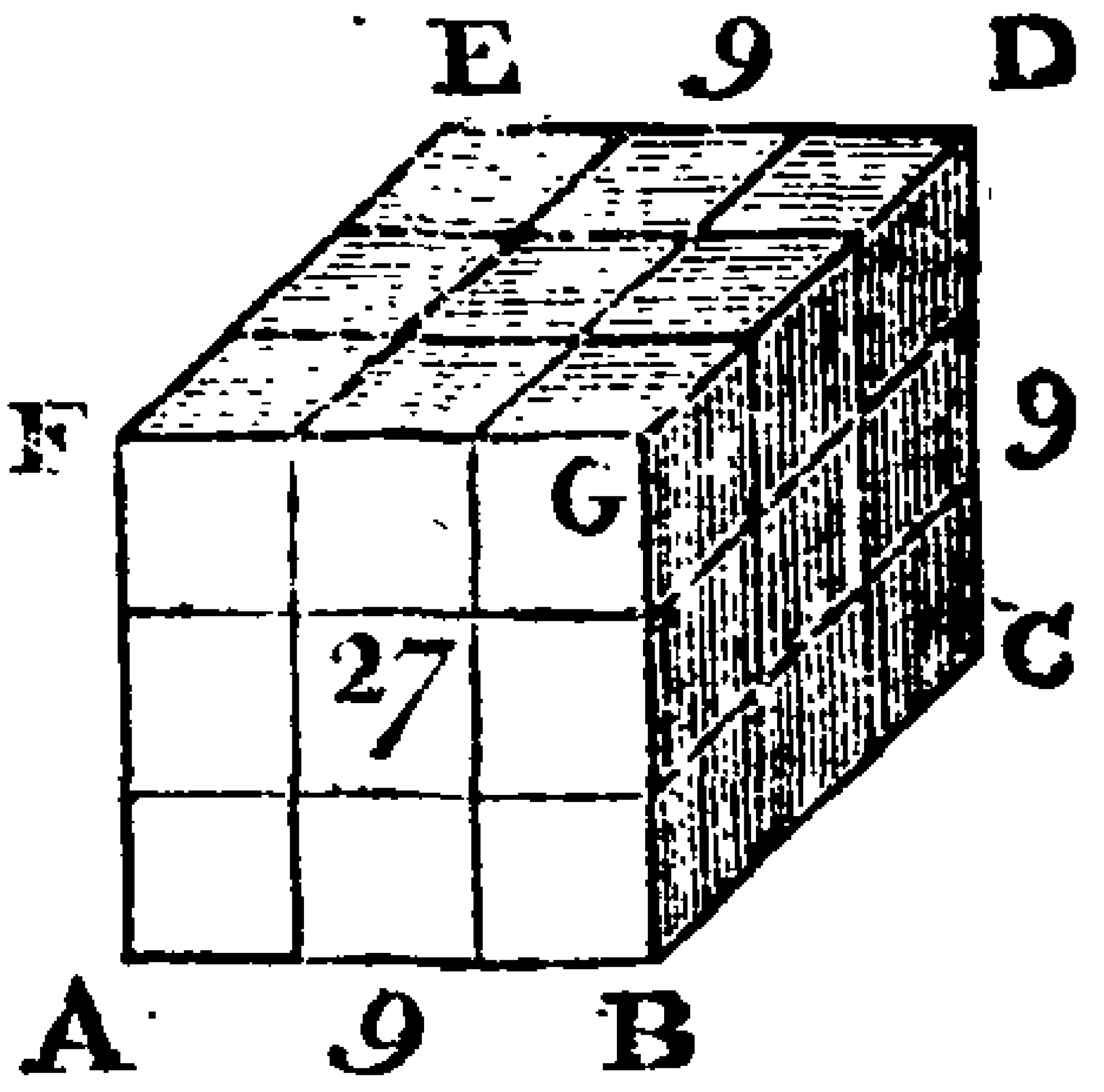
THE

THE EXTRACTION OF THE CUBE ROOT.

BY the Extraction of the Cube Root of any Number, is meant the finding out ſuch a Number, which being multiplied by itſelf, and that Product again multiplied by the *ſame* Number, ſhall produce the given Number. Thus, ſuppoſe 27 were given to have its Cube Root extracted, we ſhall find it to be 3; becauſe 3 múltiplied by 3, is 9; and 9 multiplied by 3, is 27; which is equal to the given Number.

And this may be demonſtrated thus:

Suppoſe we take 9 little Cubes, (or Dies) and lay them down, ſo as to form a Square whoſe Side ſhall be 3; upon theſe let there be laid 9 more Cubes, and upon them let there be laid 9 more; then will there be in all 27 Cubes, which will make one greater Cube, as A B C D E F G, whoſe Length, Breadth, and Depth will be 3 Cubes; and this greater Cube contains juſt 27 leſs Cubes; the Extraction therefore of the Cube Root is by having the Number of littles Cubes given which are contained in a greater Cube; to find how many of the leſs Cubes make one Side of the greater.



To extract the *Cube Root* of any Number, it will be neceſſary to have in Memory theſe Cubes, whoſe Roots are one Figure.

Root	1	2	3	4	5	6	7	8	9
Cube	1	8	27	64	125	216	343	512	729

In this Table you ſee the Cube of 1 is 1; the Cube of 2 is 8; the Cube of 3 is 27; and ſo of the reſt.

Now

Now, the *Cube Root* of any Number, (greater than those expressed in the foregoing Table) may be found out by the following

Rule.

First, having set down the given Number, or Resolvend; make a Dot over the Unit Figure, and so on over every *third* * Figure (towards the Left Hand in whole Numbers; but towards the Right Hand in Decimals); and so many Dots as there are, so many Figures will be in the Root.

Next, seek the nearest Cube to the first Period; place its Root in the Quotient, and its Cube set under the first Period. Subtract it therefrom; and to the Remainder bring down one Figure only of the next Period, which will be a Dividend.

Then, *square* the Figure put in the Quotient, and multiply it by 3, for a Divisor. Seek how oft this Divisor may be had in the Dividend, and set the Figure in the Quotient, which will be the *second Place* in the Root.

Now, cube the Figures in the Root, and subtract it from the two first Periods of the Resolvend; and to the Remainder bring down the first Figure of the next Period, for a new Dividend. Square the Figures in the Quotient, and multiply it by 3, for a new Divisor; then proceed in all Respects as before, till the Whole is finished.

* The Reason for pointing every *third* Figure is, because the Cube of the greatest Number under 10, will consist but of three Places.

The

The following *Examples* will make this difficult Rule plain and easy.

What is the Cube Root of 13824?

	Resolvend.	Quotient.
	13824	(24 true Root.
Divisor.	8	
Square of 2 multiplied by 3 = 12)	58	Dividend.
Subtract Cube of 24 from Resolvend =	13824	
	(0)	Remains nothing.

Having pointed the given Number according to the foregoing Rule, we find there will be two Places in the Root; because there are two Dots in the Resolvend. We then seek the greatest Cube in 13 the first Period, which we find to be 8; whose Root 2, we place in the Quotient for the first Figure of the Root; the 8 we set under the 13, subtract it therefrom, and to the Remainder 5 bring down the first Figure of the next Period for a Dividend. We next divide the Dividend 58 by three Times the Square of 2 (for a Divisor) which makes 12; and the Quotient 4, arising from that Division, is the second Figure in the Root; the Cube of which *whole Root* we subtract from the whole Resolvend, and find the Remainder to be (0) or Nothing. This shows that 24 is the true Root; because its *Cube* is exactly equal to 13824, the given Number.

☞ To prove in all Cases if the Operation be right; multiply the Root by itself, and that Product again by the Root; to which add the Remainder, if any; then, if that Sum be equal to the given Number, the Work is right; otherwise not; and must be performed over again.

Let it be required to extract the Cube Root out of the Number 13312053?

Resolvend.
13312053 (237 true Root.
Cube of = 8

Divisor.
Square of 2 multiplied by 3 = 12) 53 Dividend
Subtract the Cube of 23 = 12167

Divisor.
Sq. of 23 multiplied by 3 = 1587) 11450 new Dividend.
Subtract the Cube of 237 = 13312053

(0)

Here the given Number is a true Cube; for when the Cube of 237 is subtracted from the whole Resolvend, there will remain Nothing.

What is the Cube Root of 48228544?

Resolvend.
48228544 (364 true Root.
Cube of 3 = 27

Divisor.
Square of 3 multiplied by 3 = 27) 212 Dividend.
Subtract the Cube of 36 = 46656

Divisor.
Sq. of 36 multiplied by 3 = 3888) 15725 new Dividend.
Subtract the Cube of 364 = 48228544

(0)

In this Example also, the Number given is a true Cube; for when the Cube of 364 in the Root, is subtracted from the given Resolvend, there will be no Remainder.

If the given Number be not a perfect Cube Number, (that is, hath not a Root expressible exactly by any true Number) we may annex two or three Periods or Cyphers to it; and for every Period so annexed we shall have one Decimal Place in the Root; as in this Example.

What is the Cube Root of 412?

	412. 000.000	(7.44 Root.
Cube of 7 =	343	
Square of 7 multiplied by 3 = 147)	690	
Subtract the Cube of 74 =	405224	
Square of 74 multiplied by 3 = 16428)	67760	
Subtract the Cube of 744 =	411830784	
	(169216)	Remainder.

In this Example, two Periods of Cyphers are annexed to the given Number; we have therefore 2 Decimal Places in the Root.

Note. If the *Cube* of the Root should be greater than the Periods of the Resolvend from which it is to be taken; you must diminish the last Figure put in the Root, till the *Cube* be *equal* to, or *less*, than those Periods of the given Number.

If the Number given be either a *Mixt Number*, or a *Decimal*; make the Number of Decimal Places either *three*, *six*, *nine*, &c. by annexing Cyphers to the Right Hand, that the Point may fall upon the Units Place of the whole Number.

What is the Cube Root of 65.31?

65.310 000 000 (4.027 Root
Cube of 4 = 64

Square of 4 multiplied by 3 = 48) 13
Subtract the Cube of 40 = 64000

Sq. of 40 multiplied by 3 = 4800) 1310
Subtract the Cube of 402 = 64964808

Sq. of 402 multiplied by 3 = 161604) 3451920
Subtract Cube of 4027 = 65304767683

(5232317)

There were, in this Example, but two Decimal Places; we therefore annexed 7 Cyphers to the given Number, by which Means a Point not only falls on the Unit's Place in the Whole Number, but we gain 3 Decimal Places in the Root.

To extract the Cube Root *of a* Vulgar Fraction.

The beſt Way will be to reduce it to a Decimal Fraction equal to it; and then proceed in all Reſpects to find its Root as before.

Let $\frac{4}{9}$ be a Vulgar Fraction given, whoſe Cube Root is required.

First, reduce it to a Decimal thus.

As 9 : 4 :: 100000000
                                    4

9) 400000000 (4 over.
   .444444444 the Decimal Fraction.

Then extract the Cube Root of .444444444 (.763 Root.
Cube of 7 = 343

Square of 7 multiplied by 3=147) 1014
Subtract the Cube of 76 = 438976

Square of 76 multiplied by 3=17328) 54684
Subtract the Cube of 763 = 444194947

(249497)

In like Manner the Cube Root of $\frac{184}{240}$ will be found to be .912, with a Remainder of 8116138, which the Learner may try at his Leiſure.

The Use of the Cube Root.

Problem I.

To find two Mean Proportionals between any two Numbers given.

Rule.

Divide the greater Extreme by the leſs; and the Cube Root of that Quotient multiplied by the leſs Extreme, will give the leſſer Mean; then multiply the ſaid Cube Root by the leſſer Mean, and the Product will be the greater Mean Proportional required.

Example.

What are the two Mean Proportionals between 8 and 216?

Operation.

Firſt, 216 divided by 8 = 27; whoſe Cube Root is = 3; which multiplied by 8 is = 24, the *leſſer Mean.*

And, the Cube Root 3 multiplied by 24 = 72, the *greater Mean.*

For, As 8 : 24 :: 72 : 216

In like Manner, the two Mean Proportionals between 9 and 243 are found to be 27 and 81.

For, As 9 : 27 :: 81 : 243

Pro-

Problem 2.

The Solid Content of any Veſſel, or other Solid Body being given, to find the Side of a *Cube*, which ſhall be equal in Solidity thereto.

Rule.

Extract the Cube Root of the Solid Content of the given Body, and that Root will be the Side of the Cube required.

Example 1.

Suppoſe the Solid Content of a *Globe*, or *Cylinder*, *Pyramid*, or *Cone*, &c. be 157464 cubic Inches, what is the Side of a *Cube* of equal Solidity?

Operation.

157464 (54 Root; the Side of the Cube required.

Cube of 5 = 125

$5 \times 5 \times 3 = 75$) 324

Subtract $54 \times 54 \times 54 = 157464$

(0)

Example 2.

In digging a Cellar of a Cubic Form, about 3375 Solid Feet of Earth were carried out; how many Feet was it every Way, *viz.* in Length, Breadth, and Depth.

Anſwer 15 Feet, as the Learner may try at his Leiſure.

Problem 3.

The Dimensions, Capacity, or Weight of a Solid being given; to find the Dimensions, Weight, &c. of a like Solid of a different Capacity and Dimension.

Rule.

Like Solids are in a (Triplicate or) *Cubic* Proportion to their like Sides; it will therefore always be; as the Cube of the Dimension given, is to its given Weight; so is the Cube of any like Dimension, to the Weight required; *and the contrary.*

Example 1.

Suppose a Ball or Bomb of 4 Inches Diameter weighs 9 Pounds; what is the *Diameter* of another of the same Shape and Metal that weighs 84 Pounds?

Operation.

	lbs.		Cube of 4.		lbs.
As	9	:	64	::	84
					64
					336
					504
					9)5376.000

Then extract the Cube Root 597.333(8.4 Root in Inchs.
Cube of 8 = 512

$8 \times 8 \times 3 = 192.$) 853
Subtract $84 \times 84 \times 84 =$ 592704

4529
&c.

Answer, 8 Inches, 4 Tenths, and something over.

Exam-

Example 2.

Suppose a Ship of 300 Tons Burthen be 75 Feet long in the Keel; what is the Burthen of another Ship, whose Keel is 100 Feet long?

Operation.

	Cube of 75.		Tons.		Cube of 100.
As	421875	:	300	: :	1000000
					300

```
421875) 300000000 (711.1 Tons, the Bur-
        2953125    .then sought.
        -------
         468750
         421875
         ------
          468750
          421875
          ------
           468750
           421875
           ------
          (46875)
```

Example 3.

If a Globe of solid Silver 1 Inch in Diameter be worth 14*s.* 4*d.* what is the Diameter of another Globe of the same Metal, whose Value is 4*l.* 17*s.* 4*d.* ?

Operation.

	Shill.		Cube of 1.		Shillings.
As	14.333	:	1	:	917.333
					1

```
14.333) 917.333 ( 64 the Cube Root of
        85998      which is 4, the Diame-
        -----      ter required.
         57353
         57332
         -----
          (21)
```

Problem 4.

If a Cannon whoſe Diameter at the Bore is 3.75 Inches, require 4 Pounds of Powder to charge it; what is the Diameter of the Bore of another Cannon that takes 20.8 Pounds to charge it?

Operation.

	lb.		Cube of the Bore.		lb.
As	4	:	52.734375	:	20.8.
			20.8		
			421875000		
			10546875co		
			4) 1096.8750000 (		

Extract its Root 274.2187500 (6.49 Inches, the Diameter required.

216

$6\times6\times3=108)582$

Cube of 64 262144

$64\times64\times3=4096)120747$

Cube of 649 = 273359449

(859301)

The Reaſon of the whole Operation of the Extraction of Roots, cannot be perfectly conceived, till the Learner has made ſome Progreſs in his *Algebra.*

A General *Theorem*, or Rule for extracting the *Roots* of *all Powers*, how high ſoever.

Make a Dot over the Units Place, and ſo over every *ſecond*, *third*, *fourth*, *fifth*, &c. Figure, according as it is a *Square*, *Cube*, *Biquadrate*, *Surſolid*, &c. to be extracted.

Seek the greateſt *Square*, *Cube*, *Biquadrate*, *Surſolid*, &c. to the firſt Period; ſet its Root in the Quotient; and ſubtract its *Square*, *Cube*, *Biquadrate*, &c. from the firſt Period; and to the Remainder bring down the *firſt* Figure of the next Period for a *Dividend*.

Raiſe the Root to a Power leſs by Unity or 1; then the given Power whoſe Root is required; whether it be *Square*, *Cube*, *Biquadrate*, &c. and multiply it by the Index of that Power; as by 2 for a Square; 3 for a Cube; 4 for a Biquadrate, &c. for a Diviſor. Divide the Dividend by this Diviſor, and ſet the Figure in the Quotient.

Then raiſe the Root to the given Power, whether it be *Square*, *Cube*, *Biquadrate*, &c. ſubtract it from the two firſt Periods; * and to the Remainder bring down one Figure of the ſucceeding Period, for another Dividend; proceed then to form the Diviſor, and carry on the Operation in all Reſpects as before.

* If this Number be greater than the Periods from which it is to be ſubtracted; a leſs Figure muſt be (taken in the former Diviſion, than that laſt) put in the Quotient or Root; and the Work muſt be done over again.

For the more easy Procedure in this business, we have annexed a Table of the Powers of the several Roots to the sixth Power, which the Learner by a continued Multiplication may carry on through the nine Digits himself.

Root, or first Power	1	2	3	4	5	6	7	8	9
Square, or second Power	1	4	9	16	25	36	49	64	81
Cube, or third Power	1	8	27	64	125	216	343	512	729
Biquadrate, or fourth Power	1	16	81	256	625	1296	2401	4096	6561
Surfolid, or fifth Power	1	32	243	1024	3125	7776	16807	32768	59049
Squared Cube, or sixth Power	1	64	729	4096	15625	46656	117649	262144	531441*

A few Examples will make all plain.

Example 1.

What is the Square Root of 204304?

Operation.

204304 (452 the Square Root required.
Square of 4 = 16

4 × 2 = Divisor 8) 44 Dividend.

45 × 45 = 2025 Subtrahend.

45 × 2 = 90) 180 Dividend.
452 × 452 = 204304 Subtrahend.

(0)

* Note, the seventh Power is called the *second Surfolid*; the eighth Power is the *Biquadrate squared*; and the ninth Power is the *Cube cubed*.

Exam-

Example 2.

What is the Cube Root of 92345408?

Operation.

92345408 (452. Cube Root required.

Cube of 4 = 64

4 × 4 × 3 = Divisor 48) 283 Dividend.

45 × 45 × 45 = 91125 Subtrahend.

45 × 45 × 3 = Divisor 6075) 12204 Dividend.

452 × 452 × 452 = 92345408 Subtrahend.

(0)

Example 3.

What is the Biquadrate Root of 41740124416?

Operation.

41740124416 (452.

4 × 4 × 4 × 4 = 256

4×4×4×4 = Divisor 256) 1614 Dividend.

45 × 45 × 45 × 45 = 4100625 Subtrahend.

45 × 45 × 45 × 4 = 364500) 733874 Dividend.

452 × 452 × 452 × 452 = 41740124416 Subtrahend.

(0)

☞ The *Biquadrate Root* of any Number may be found by extracting the Square Root of the given Number first, and then extracting the Square Root of that Root.

But

But as the Extraction of Roots higher than the *Biquadrate* are difficult in common Numbers, on Account of their high Involutions (or Multiplications); we will now point out an easier as well as shorter Method of extracting the Roots of all Powers and Numbers, how high and large so ever, by the *Logarithms*.

Rule.

Seek the *Logarithm* of seven, or (if possible) eight of the leading Figures of the Number given, and prefix to that *Logarithm* an Index answerable to the Number of Places in the whole Number, (*i. e.* one less than the Number of Places) that *Logarithm* being divided by the Index of the Power, whose Root is sought, as by 2 for the *Square*; 3 for the *Cube*; 4 for the *Biquadrate*; 5 for the *Sursolid*, &c. the Quotient will be the *Logarithm* of the Root required.

Example.

What is the *Cubed Cube Root* (or ninth Power) of 4722366482869645213696?

Operation.

We find the *Logarithm* of the seven leading Figures 4722366 to be 67415963, and consequently the Index to be prefixed is 21; because the given Number consists of 22 Places. This intire Number (*Logarithm* and *Index*) 21.67415963 being divided by 9, the Index of the *Cubed Cube* or ninth Power, the Quotient will be 2.40823995, the *Logarithm* of 256, the Root required.

A

A COMPENDIOUS COURSE

OF

PRACTICAL GEOMETRY.

GEOMETRY is that Part of Mathematical Learning, which teaches how to meaſure the Earth, and determine the Magnitude and Diſtance of all Bodies contained therein. It is of the utmoſt Uſe in Life; for here nothing can deceive us, by appearing *bigger* or *leſs*,—*higher* or *lower*,—*nearer* or *further off*, than it really is.

This Science is of very remote Antiquity, and ſuppoſed to take its firſt Riſe in *Egypt*.* The Inhabitants of that Country were in a Manner compelled to invent it, to Remedy the Confuſion, which generally happened in their Lands, from the Overflowings of the River *Nile*, which carried away all their Boundaries, and totally effaced the Limits of their Poſſeſſions. Thus this Invention, which at firſt conſiſted only in laying out and meaſuring the Lands, that every Perſon might have what juſtly belonged to him, was called Geometry, and it is probable, that the Draughts and *Schemes* which they were annually compelled to make,

* It is generally allowed that the *Chaldeans* were firſt poſſeſſed of the Mathematical Sciences, which muſt imply a Knowledge of *Geometry*. Whether *Abraham* (as ſome learned Men think) taught theſe Sciences firſt to the *Ægyptians*, when he went from *Ur* of the *Chaldees*, is not clear; but on this we may depend, that the *Ægyptians* were the firſt People that cultivated *Geometry*, and applied it, as the Word expreſſes, to *Land Meaſure*: they being compelled thereto by a Kind of Neceſſity, in order to aſcertain every Man his legal Property and Eſtate, in a Country where Boundaries and Land Marks were ſwept away and confounded by yearly Inundations.

helped

helped them to discover many excellent Properties of these Figures, which Speculations continued to be gradually improved, and are still improving to this Day.

From Egypt *Geometry* passed over into *Greece*, where it continued to receive new Improvements in the Hands of the Learned; * and at Length it grew into such Value and Estimation, that *Plato*, who flourished about 300 Years before Christ, would admit none to his Lectures who had not made some Advances in it, thinking them not capable or fit Hearers. Whence that famous Inscription (said to be written) over his School Doors, †—*Let none ignorant of Geometry enter here.*

For the Pursuit of *Geometrical* Speculations will not only inure us to attend closely to any Subject; to seek and gain clear Ideas; to distinguish Truth from Falshood; to judge justly and argue truly; but do by their own Nature more directly furnish us with all the various Rules of those useful Arts and Sciences of Life, viz. *Mensuration*, *Architecture*, *Fortification*, *Navigation*, *Dialling*, *Perspective*, *Optics*, *Mechanics*, *Astronomy*; and, in short, with a perfect Knowledge of (all Things measurable in) the *Heavens* and the *Earth*.

* *Thales*, *Pythagoras*, *Archimedes*, *Euclid*, &c.

† οὐδεὶς ἀγεωμέτρητος εἰσίτω.

Geo-

Geometrical Definitions.

DEFINITIONS are ſhort Deſcriptions of Things expreſſive of their ſeveral Properties.

A *Point*, which is the very Beginning of Magnitude, is ſuppoſed to be ſo ſmall as to have no Parts; neither Length, nor Breadth, nor Thickneſs, as the Point A.*

(A)

.

Point

A *Line* is ſuppoſed to be made by the Motion of a Point, and hath Length, without Breadth or Thickneſs.

If a Line be quite ſtrait, it is called a *Right Line*, as A B.

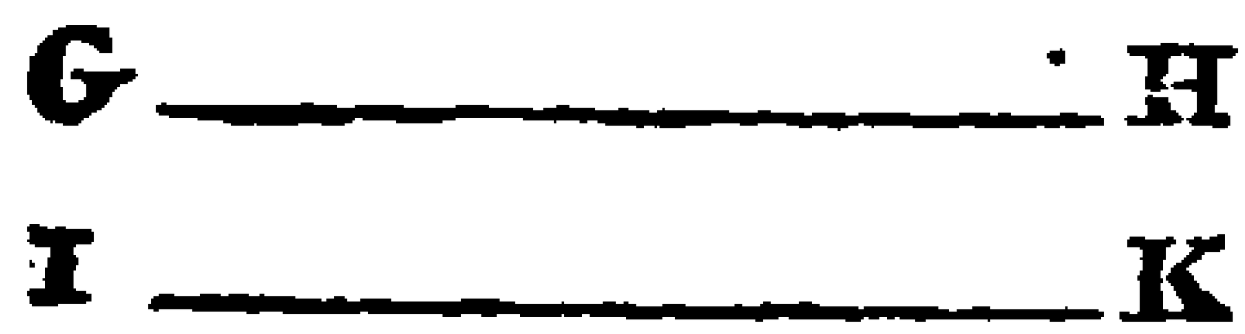
If the Line be bent or crooked, it is called a *Curve Line*, as C D.

* A *Point* is the ſmalleſt Object viſible to the Eye; it is ſuppoſed to be ſo ſmall as to have no Geometrical Magnitude, and is made by the Point of a Pin, Pen, or Pencil, as the Point A above.

If

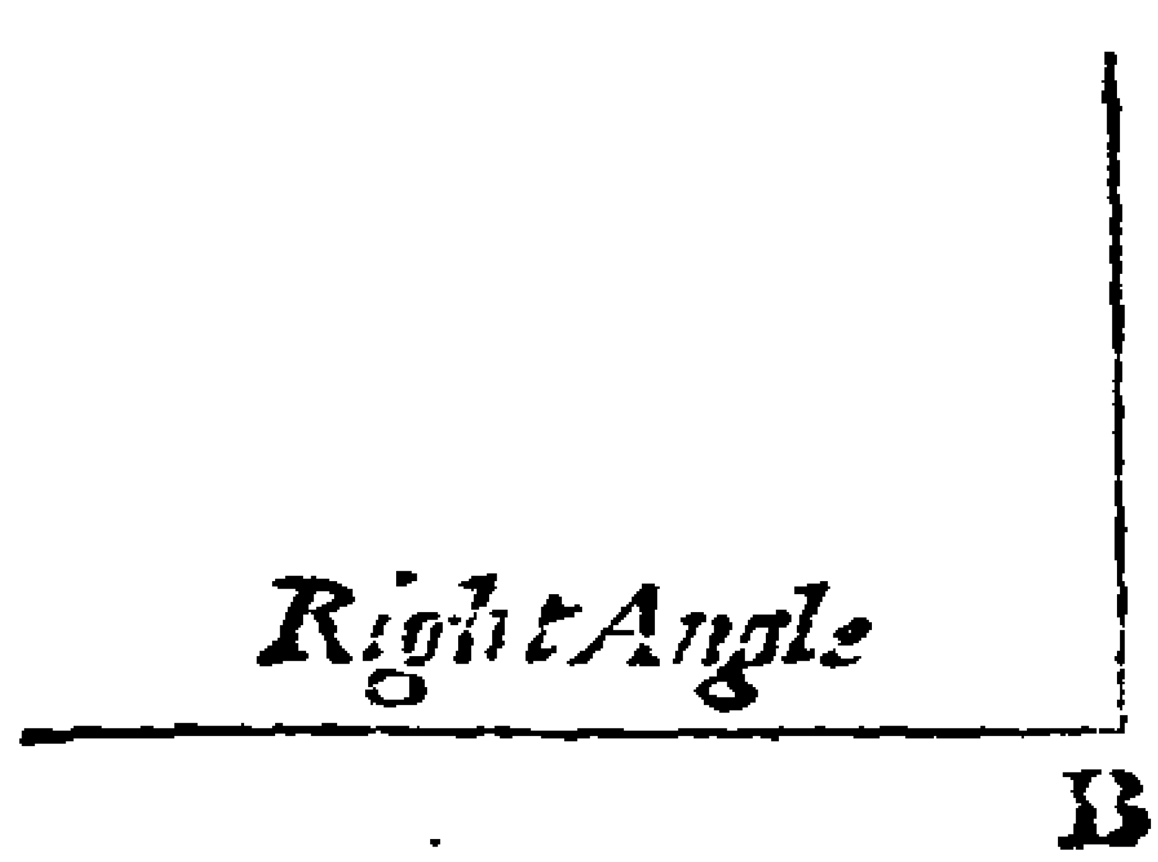
If a Line turns backward and forward, it is called a *Serpentine Line*, as E F.

If two Lines run equally diſtant from each other, they are called *Parallel Lines*, as G H and I K.



If two Lines lean or incline towards each other, they will at laſt meet, which Place of Meeting is called an *Angle*, as A.

If one Line falls perpendicuarly on another, the Angle made by their Meeting is called a *Right Angle*, as at B.

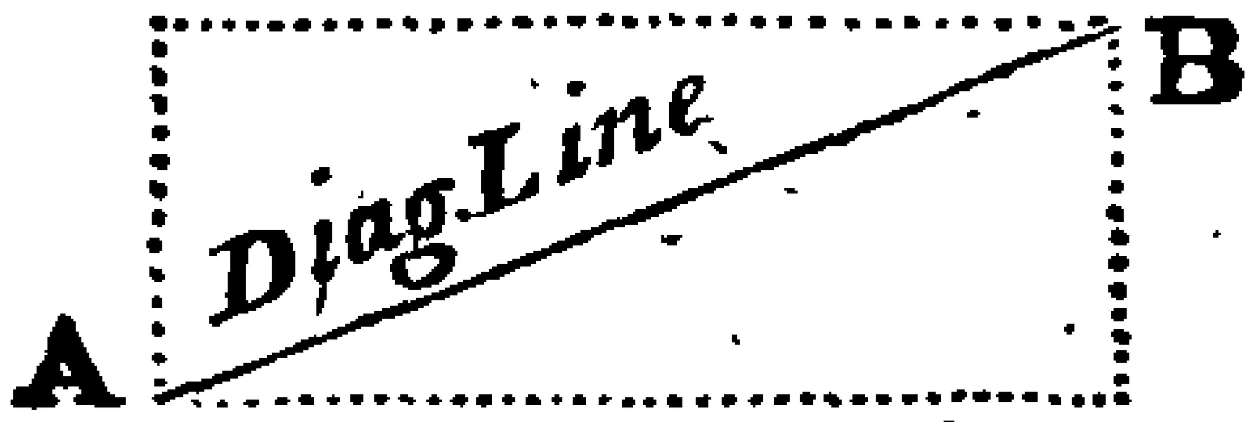


If

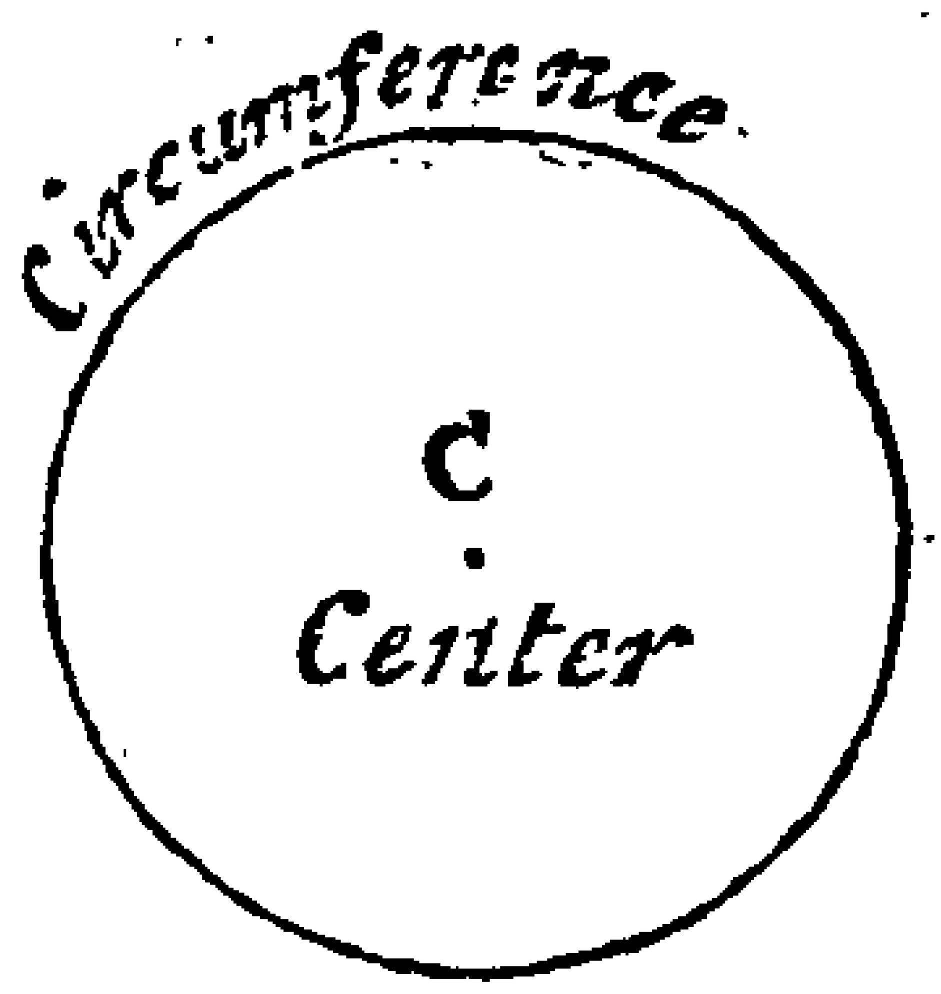
If the Lines incline to each other, the Angle made is leſs than a Right Angle, and is ſaid to be *Acute*, as the Angle C.

If one of the Lines fall backward, the Angle is greater than a Right Angle, and ſaid to be *Obtuſe*, as the Angle D.

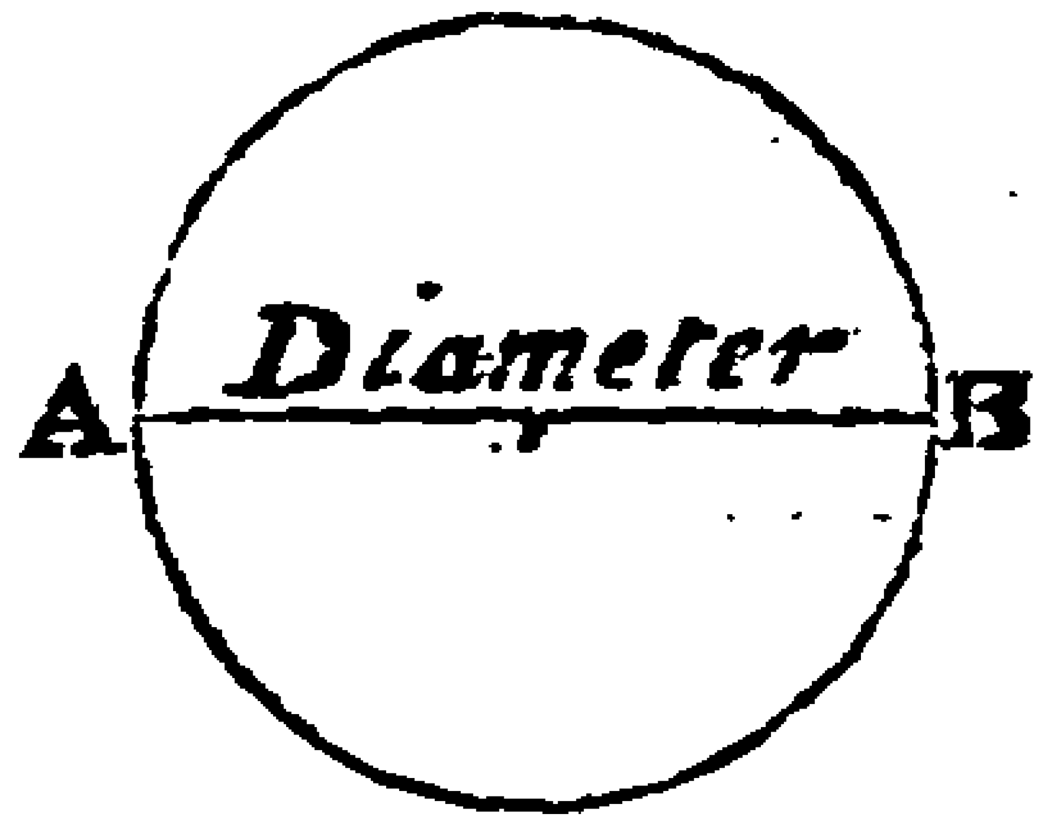
A Line going from one Corner of a Figure to the other is called a *Diagonal*, as A B.



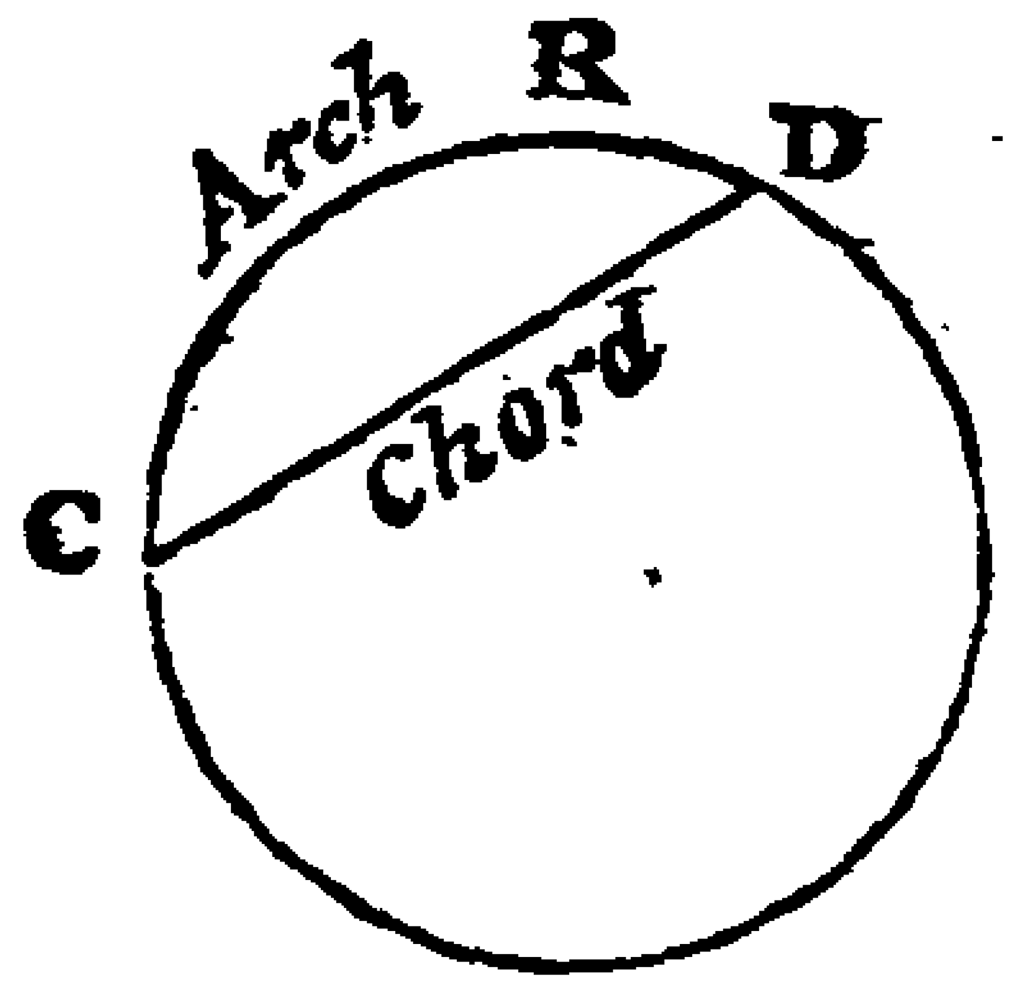
If a Line bends round regularly returning into itſelf, it is called a *Periphery*, or *Circumference*; and all the Space within is called the *Circle*. The Point in the Middle is called the Center, as at C.



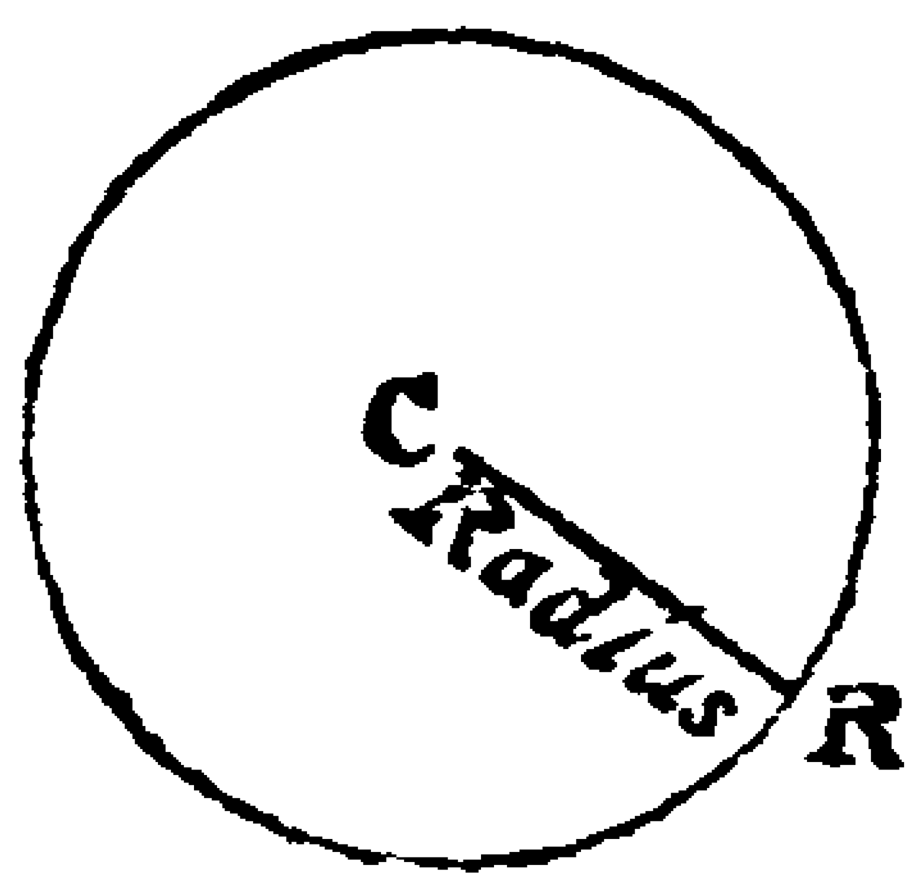
If a Line paſſes exactly acroſs the Middle of a Circle, from one Part of the Circumference to the Part oppoſite, dividing it into two equal Parts, it is called a *Diameter*, as A B.



A Right Line paſſing acroſs a Circle in any Part, except it be exactly in the Middle, divides it into two unequal Parts, which Parts are called *Segments* of a Circle. Such Line is called a *Chord*, as C D, and the Part of the Circumference lying between is called an *Arch*, as C R D.



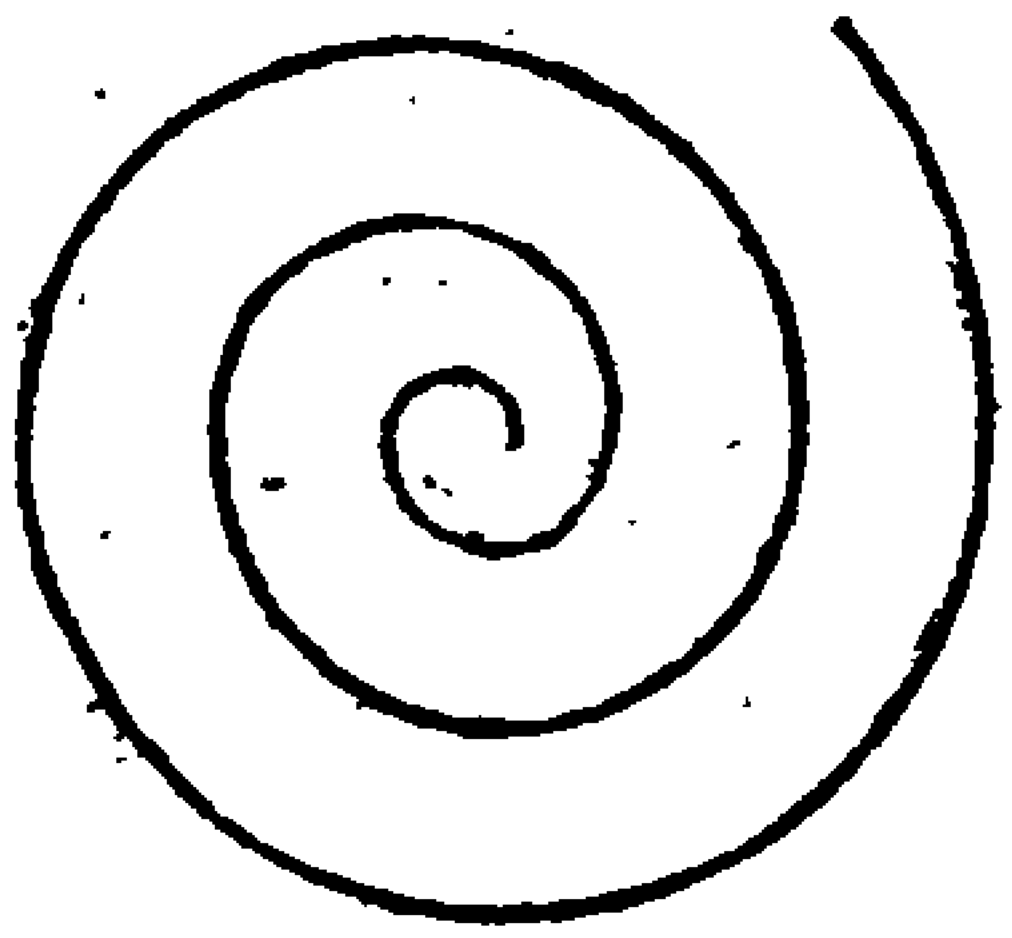
A Right Line being drawn from the Middle or Center of a Circle to any Part of the Circumference, is called a *Radius* or *Semi-diameter*, as C R.



If

If a Line bends or coils round like the Spring of a Watch, it is called an *Helix* or *Spiral* Line, as B.

B

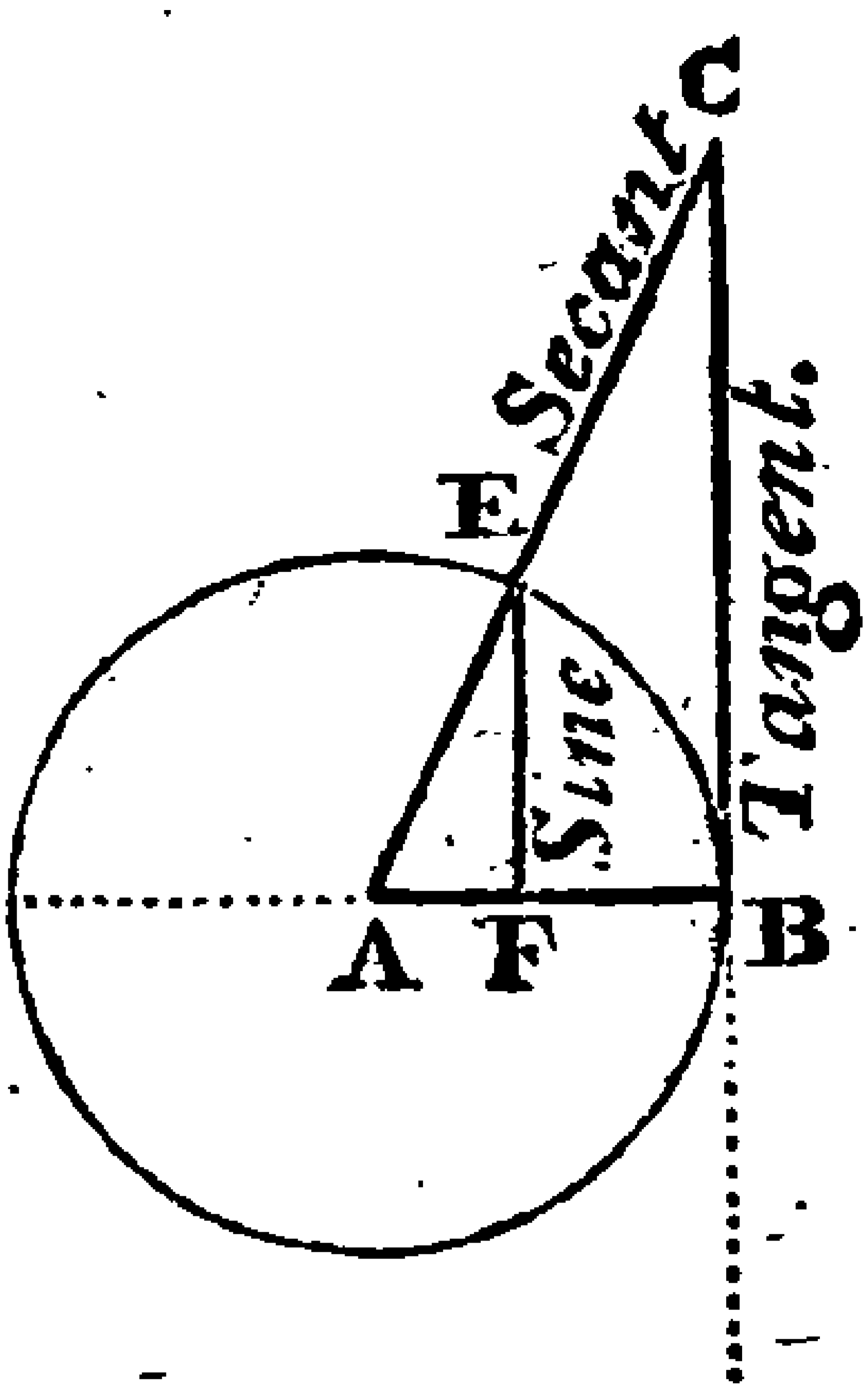


A Right Line falling perpendicularly upon the End of the Diameter, so as just to touch the Arch of a Circle, is called a *Tangent*, as B C in the Figure below.

A Right Line drawn from the Center of the Circle through the Arch till it meet the End of the Tangent, is called the *Secant* of that Arch, as A C.

A Right Line falling perpendicularly from any Part of the Arch upon the Diameter within the Circle, is called a *Sine*, as E F.

That Part of the Diameter intercepted between the *Sine* and Tangent, is called the *versed Sine*, as F B.



Geo-

Geometrical Axioms.

Axioms are Propositions which contain self-evident Truths: The Principal are these that follow:

Axiom 1.

The Whole is greater than any of its Parts.

Axiom 2.

All the Parts taken together are equal to the Whole.

Axiom 3.

Two Things that are equal to a third, are equal between themselves.

Axiom 4.

If two equal Things are *equally* increased, or diminished, they still continue to be equal.

Axiom 5.

If two equal Things are increased or diminished *unequally*, they will become unequal.

Axiom 6.

From Nothing, Nothing can arise; nor hath it any Properties or Dimensions of *Length*, *Breadth*, or *Thickness*.

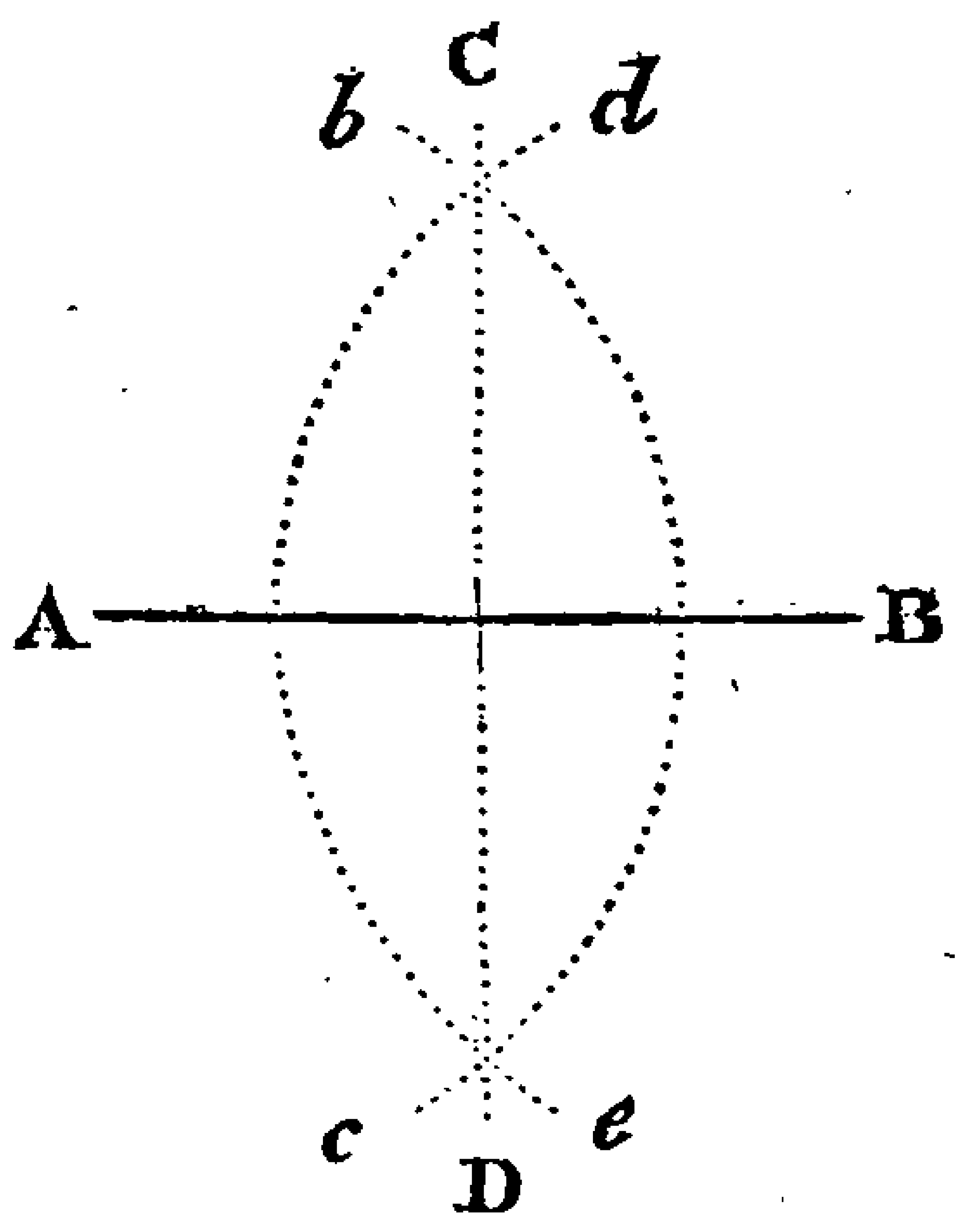
All these Truths hold good, not only in Numbers, but also in *Lines*, *Superficies*, and *Solids*.

Geometrical Problems.

Def. *Problems* in Practical Geometry are Propositions wherein some Operation or Construction is required or proposed to be done; as to divide Lines, Angles; erect or fall Perpendiculars, &c.

Problem 1.

To divide the Right Line A B into two equal Parts.



Construction. First open the Compasses to more than Half the Length of the given Line; with that Wideness, setting one Foot in A, describe the Arch *b c*; then set one Foot in B, and describe the Arch *d e*, intersecting the former in the Points C and D. Lastly, through the Points C and D draw the Right Line C D, and it will divide the given Line A B into two equal Parts, which was required.

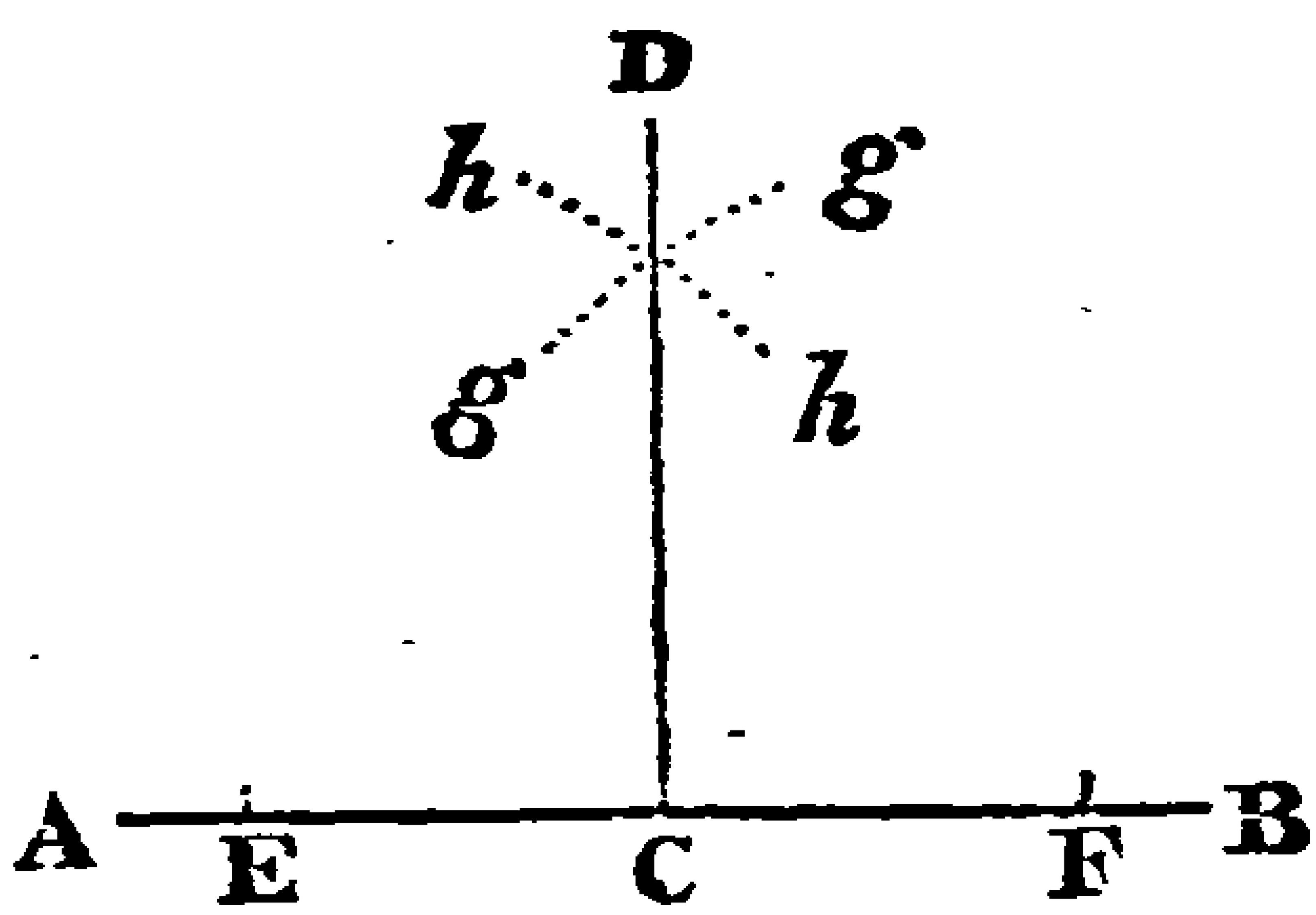
☞ This Problem is useful in dividing Measures into small equal Parts.

Pro-

Problem 2.

To *erect* a *Perpendicular* on any Point in a Right Line given.

Suppose upon the Point C on the Right Line A B.



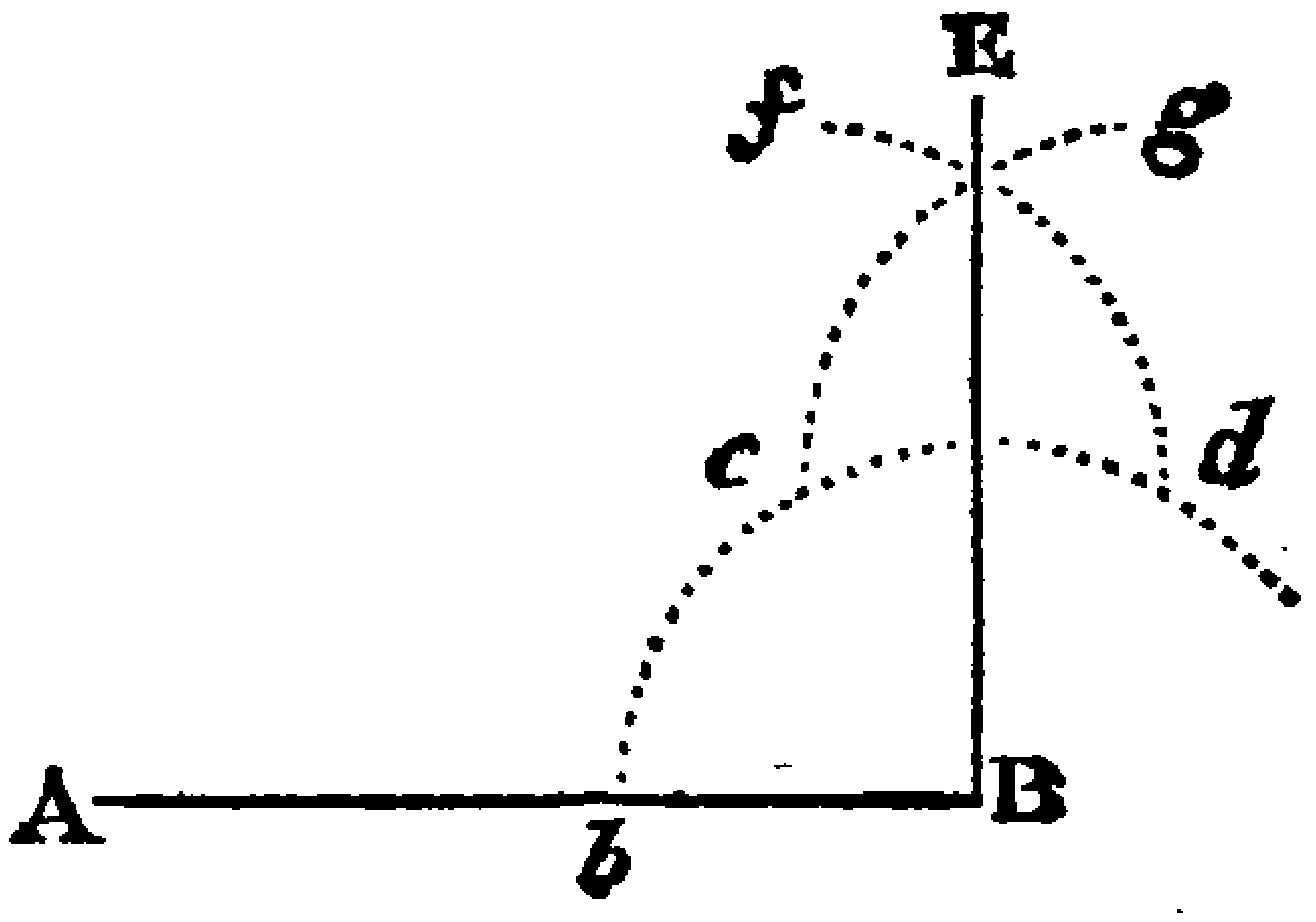
Construction. First open your Compasses any Wideness, and setting one Foot in the Point C, with the other make two Dashes at E and F; then opening your Compasses any Distance greater than the former, set one Foot in E, and with the other describe the Arch *h h*; with the same Extent, setting one Foot in F, with the other describe the Arch *g g* intersecting the former at D. Lastly, through the Points D and C draw the Line D C, and it will be the Perpendicular required.

☞ This Problem is not only useful to Mathematicians, but also almost to all Artificers, especially those who are obliged to make use of *true Squares*.

Problem 3.

To erect a Perpendicular on the End of a Right Line.

Let A B be the Line given; and let B be the Point on which the Perpendicular is to be erected.



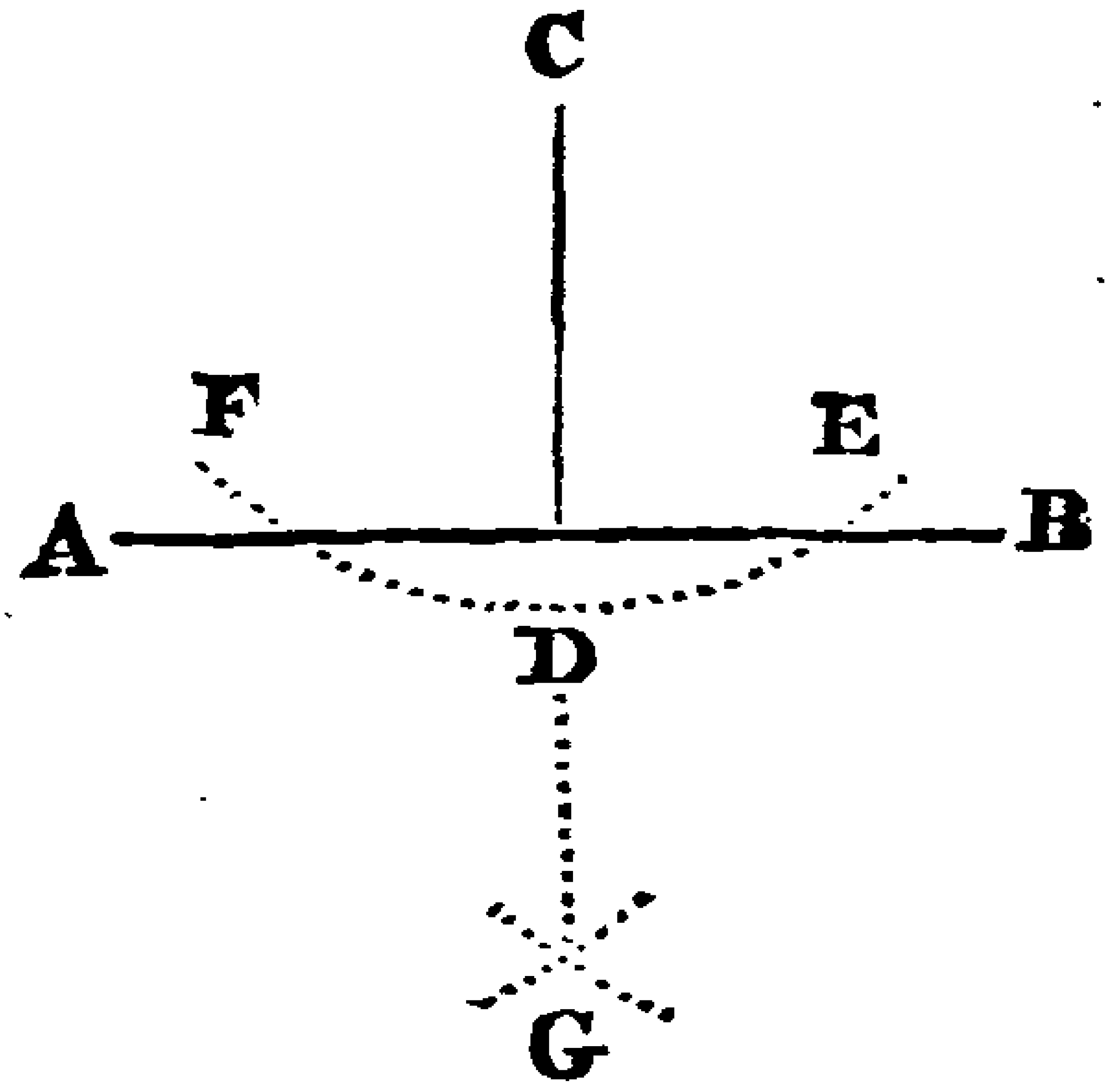
Construction. First open your Compasses to any small Distance, and setting one Foot in the Point B, describe the Arch *b c d*. The Compasses remaining at the same Wideness, set one Foot in *c*, and describe the Arch *f d*; with the other in *d*, describe the Arch *c g*, intersecting the former in E. Lastly, from E draw the Line E B, which will be the Perpendicular required.

☞ *Surveying*, *Dialling*, &c. cannot be carried on without the continual Use of this Problem.

Problem 4.

To let fall a Perpendicular upon a Right Line given from a Point at any Diſtance above it.

Let A Bbe the given Line; and let C be a Point above it, from whence the Perpendicular is to fall.



Conſtruction. Firſt, with the Compaſſes opened ſomething wider than the Diſtance which the Perpendicular is to fall, ſet one Foot in the Point C, and with the other deſcribe the Arch E D F, interſecting the given Line at E and F. Then, from the Points E and F ſtrike the croſs Daſhes at G. Laſtly, draw the Line C D, and it is the Perpendicular required.

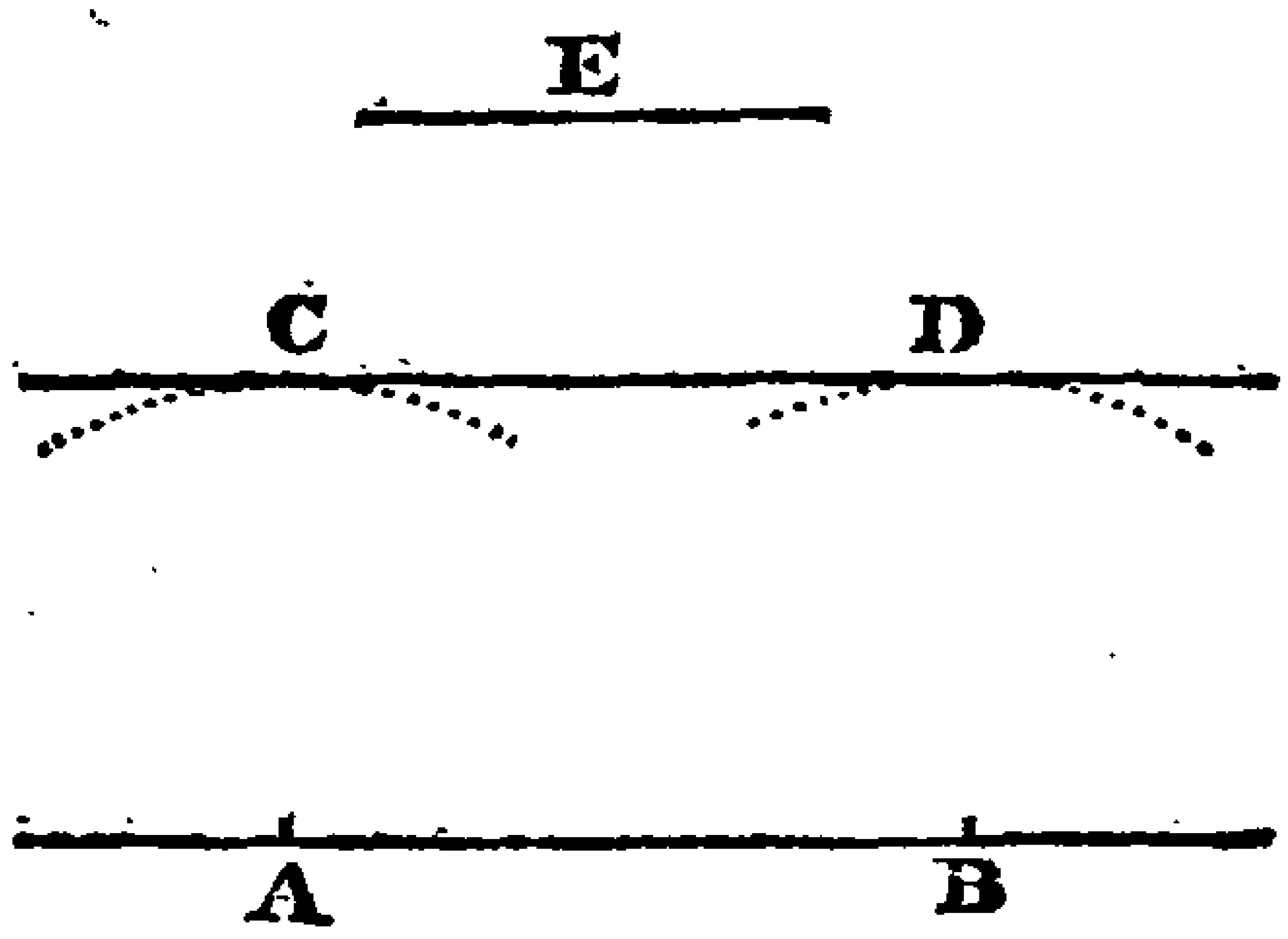
☞ The greateſt Part of the Practice of *Mechanics* conſiſts in the Knowledge of drawing (both upward and downward) *Perpendicular Lines*.

Pro-

Problem 5.

To draw a Line parallel to another Line given, at any Diſtance propoſed.

Let the Line given be A B, and the Diſtance of the Parallels equal to the Line E.



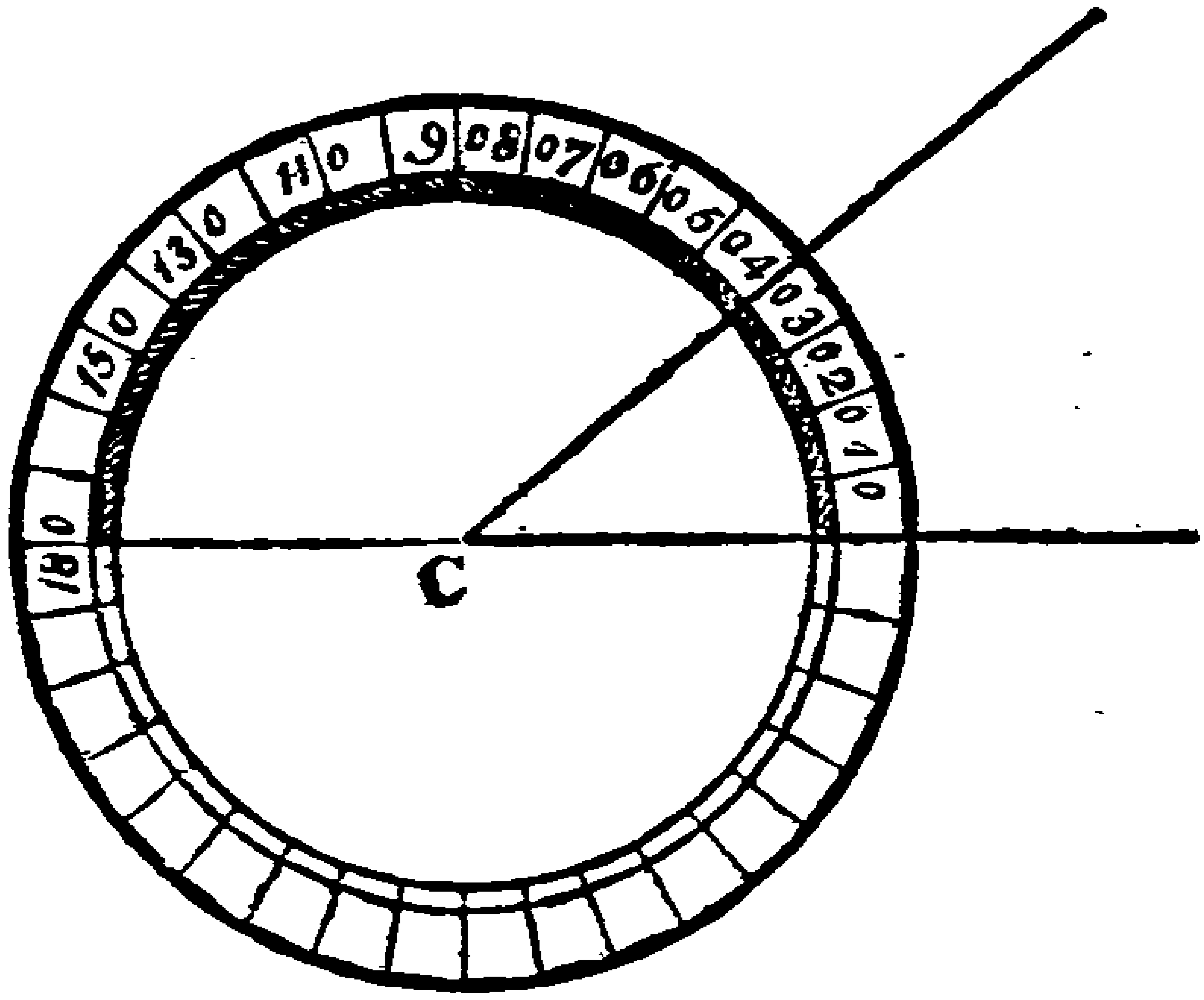
Conſtruction. Firſt, open your Compaſſes to the Length of the Line E, the Diſtance required. Set one Foot in the Point A, and deſcribe an Arch on the Side you are to draw the Parallel on, as at C. Do the like from B to D. Laſtly, by the Convexity of theſe two Arches draw the Line C D, which will be the Parallel required.

☞ The Uſe of *Parallel Lines* is very great in *Navigation*, and the Conſtruction of ſome Kinds of *Dials*.

Problem 6.

To protract, or lay down an Angle of any Number of Degrees.

Let the Angle to be protracted or delineated consist of 40 Degrees.



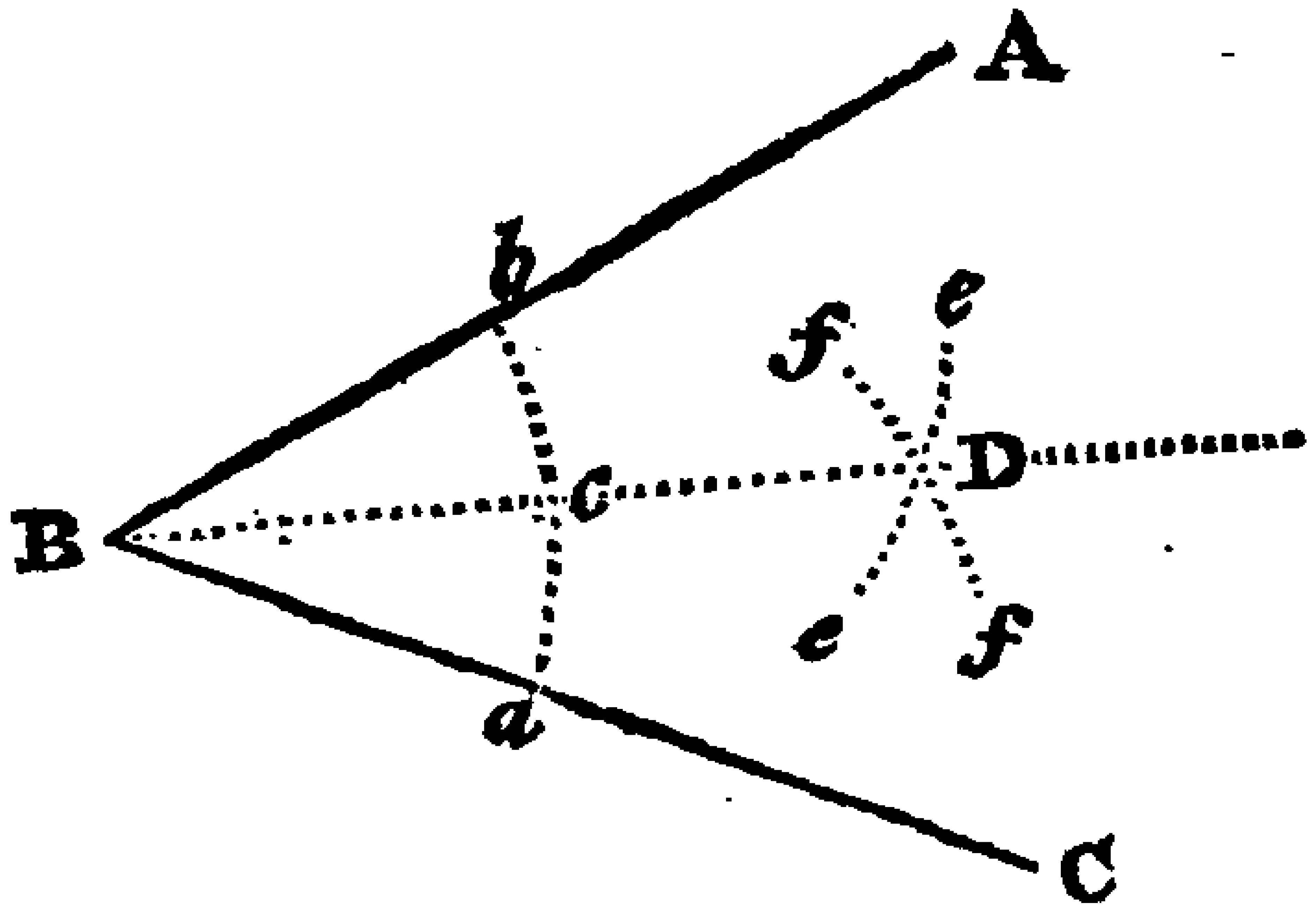
Construction. First, with any Radius, or Opening of the Compasses upon the Point C, describe a Circle, which divide into 360 equal Parts, called Degrees; then from the Center C draw two Lines, one through the Beginning of the Degrees, and the other through 40, and it is done: Because all Angles are estimated or measured by the Number of Degrees contained in the Arch of the Circle intercepted between the Legs that form that Angle.

Note. If the Angle contains less than 90 Degrees, it is said to be *Acute*. If exactly 90 Degrees, it is a *Right Angle*. If more than 90, it is *Obtuse*; and so continues to 180, at which Place the Angle vanishes; the Legs becoming a *Right Line*.

Problem 7.

To divide an Angle given into two equal Parts.

Let A B C be the given Angle, and let it be required to (biſect or) divide it into two equal Parts.



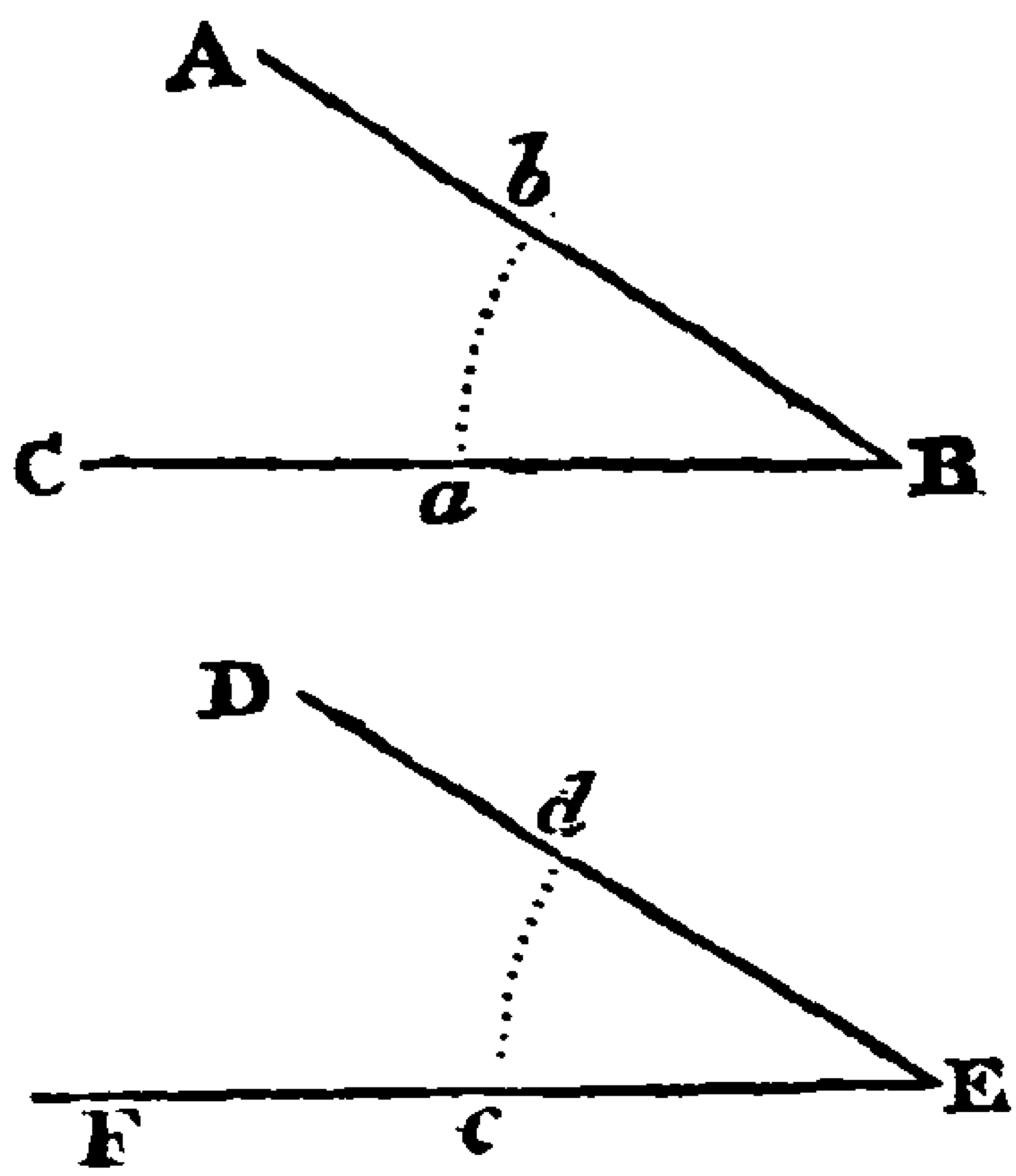
Conſtruction. Firſt, having opened the Compaſſes to any convenient Wideneſs, ſet one Foot in the Point B, and deſcribe the Arch *a c b*, cutting the Sides in *a* and *b*; then with the ſame or any Extent at Pleaſure, ſetting one Foot in *a*, deſcribe the Arch *f f*; with the ſame Extent ſet one Foot in *b*, and deſcribe the Arch *e e*, cutting the former at D. Laſtly, draw the Line B D, and it will biſect, or divide the Angle into two equal Parts, as required.

☞ By this Problem, a *Quadrant* may be divided into certain Degrees; and the *Seaman's Compaſs* expeditiouſly divided into 32 Points.

Problem 8.

To make an Angle equal to an Angle given.

Let A B C be the given Angle; and let it be required to make another Angle equal to it.



Conſtruction. Firſt, upon the Angular Point B, with any Opening of the Compaſſes, deſcribe the Arch *a b*. Then, upon the Point E, (having drawn the Line F E) with the ſame Extent of the Compaſſes deſcribe the Arch *c d*; next, take the Arch *a b* in the Compaſſes, and ſet it from *c* to *d*. Laſtly, draw the Line E *d* D, and it will make the Angle F E D, equal to the Angle A B C, as was required.

⁂ When an Angle is expreſſed by three Letters, the *middle Letter* expreſſes the *Angular Point*, and the firſt and laſt the End of the Legs.

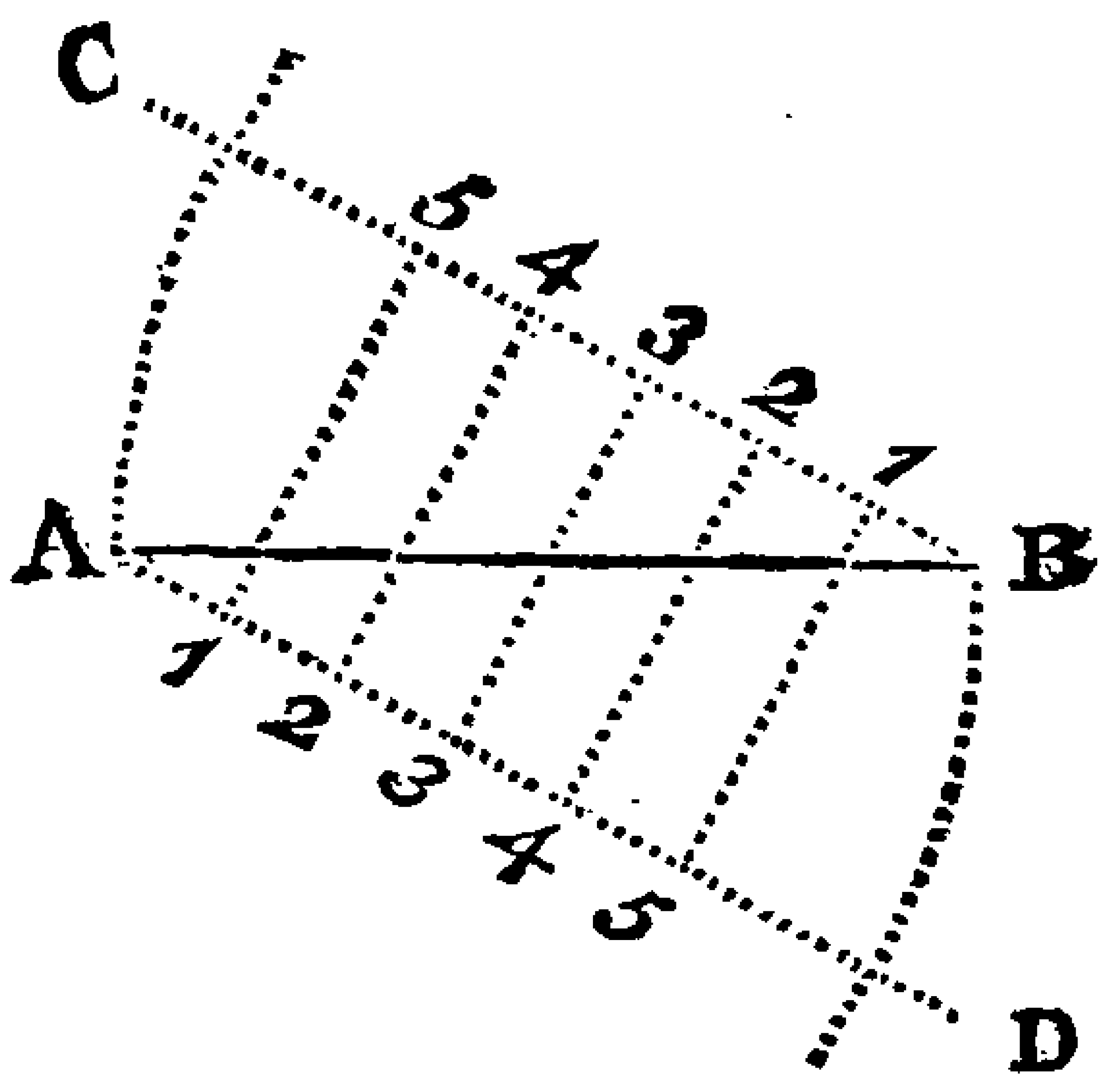
☞ This Problem is ſo neceſſary, that *Surveying*, *Fortification*, *Perſpective*, &c. cannot be performed without it.

Pro-

Problem 9.

To divide a given Right Line into any Number of equal Parts.

Let A B be the Line given, and let it be propoſed to be divided into *ſix* equal Parts.



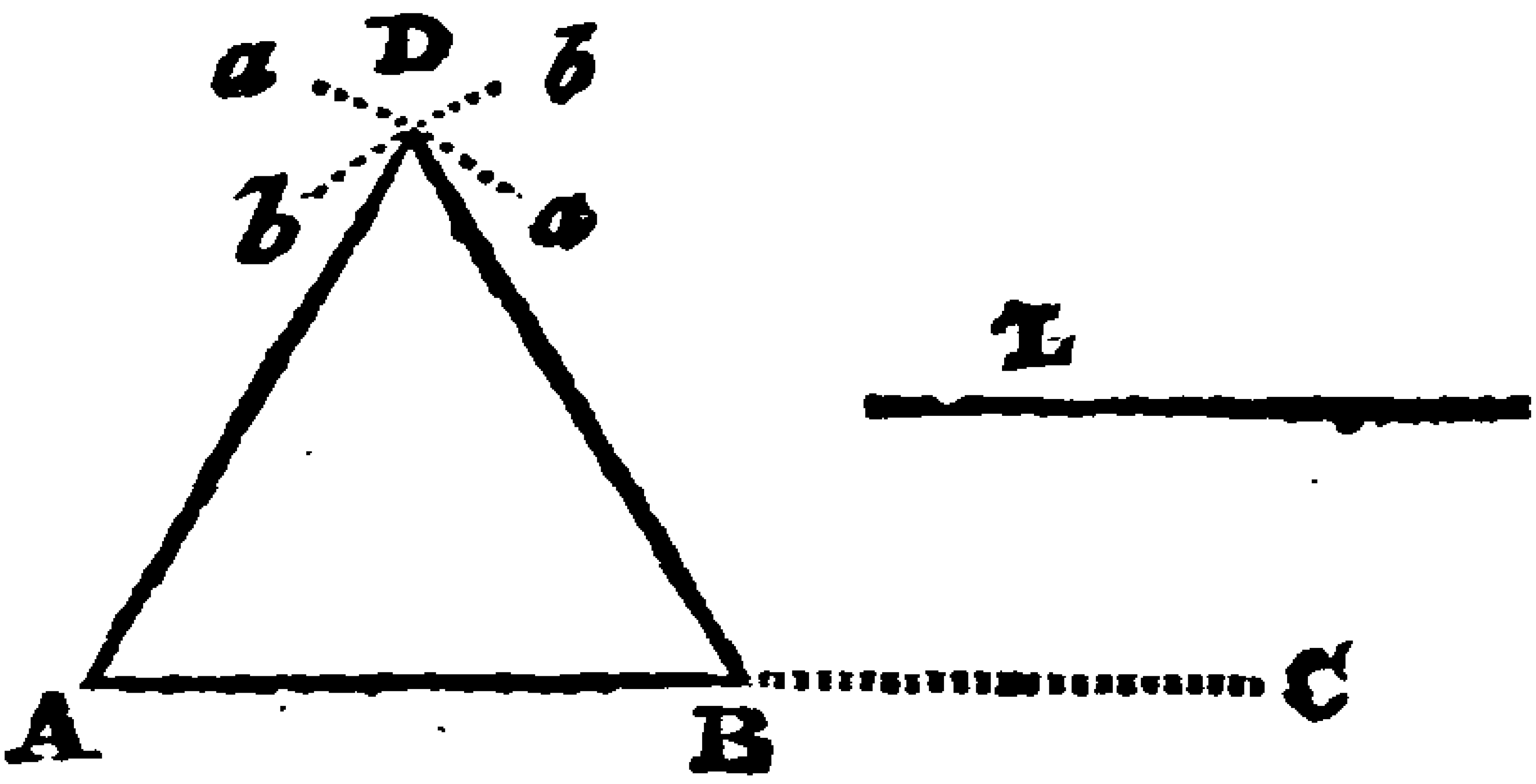
Conſtruction. Firſt, from the End B of the given Line draw the Line B C, making an Angle of any Quantity with the Line A B. Then from the other End A draw the Line D A (parallel to B C by Problem 5, or) making the Angle D A B equal to the Angle C B A by the laſt Problem. Next, on the Line B C, with any ſmall Opening of the Compaſſes, beginning at B, make the five equal Diſtances at 1, 2, 3, 4, 5. Alſo ſet off the ſame Diſtances on the Line A D, beginning at the Point A. Laſtly, draw Lines from 5 to 1; from 4 to 2; from 3 to 3, &c. as in the Figure, and they will divide the given Line A B into 6 equal Parts, as required.

☞ By this Problem, a Line may be divided after the ſame Manner as another Line is divided, with great Exactneſs and Preciſion.

Problem 10.

To make a *Triangle*, each Side of which ſhall be equal to a given Right Line.

Let L be the Line given, and let it be required to make a Triangle, having each Side equal to the ſaid Line.



Conſtruction. Firſt, draw the Line A C ſomething longer than the given Line L. Take the Line L in your Compaſſes, and ſet it from A to B. With the ſame Extent of the Compaſſes, place one Foot in A, and deſcribe the Arch *a a*. Then place one Foot in B, and ſtrike the Arch *b b*, interſecting the former in the Point D. Laſtly, from D, draw Lines to A and to B, ſo will the Triangle be formed, having each Side equal to the Line L, as was required.

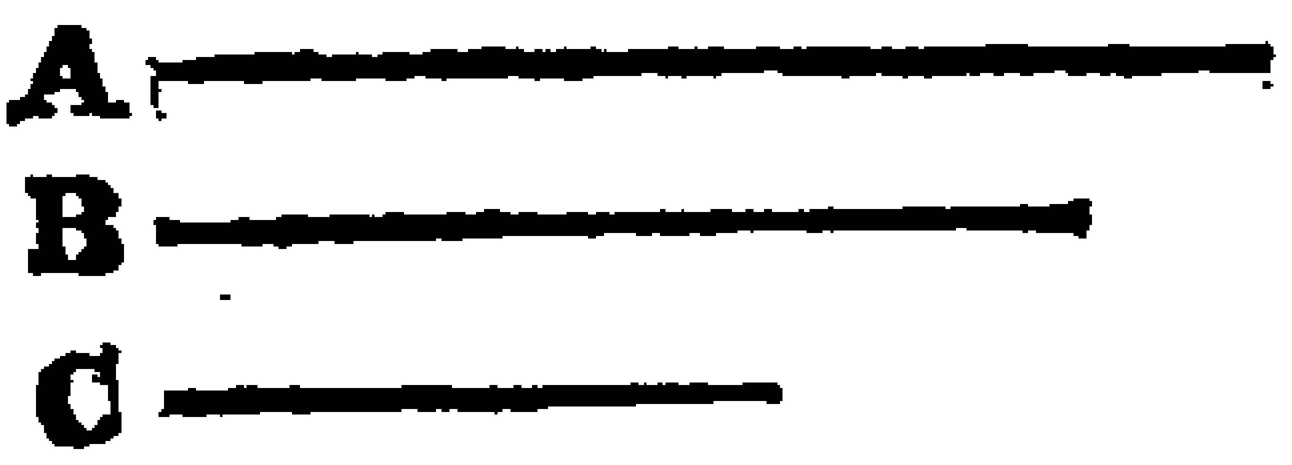
☞ This Figure is called an *Equilateral Triangle*, becauſe all its Sides are equal, and may be uſefully applied (when deſcribed on a Board) in taking *inacceſſible Diſtances*.

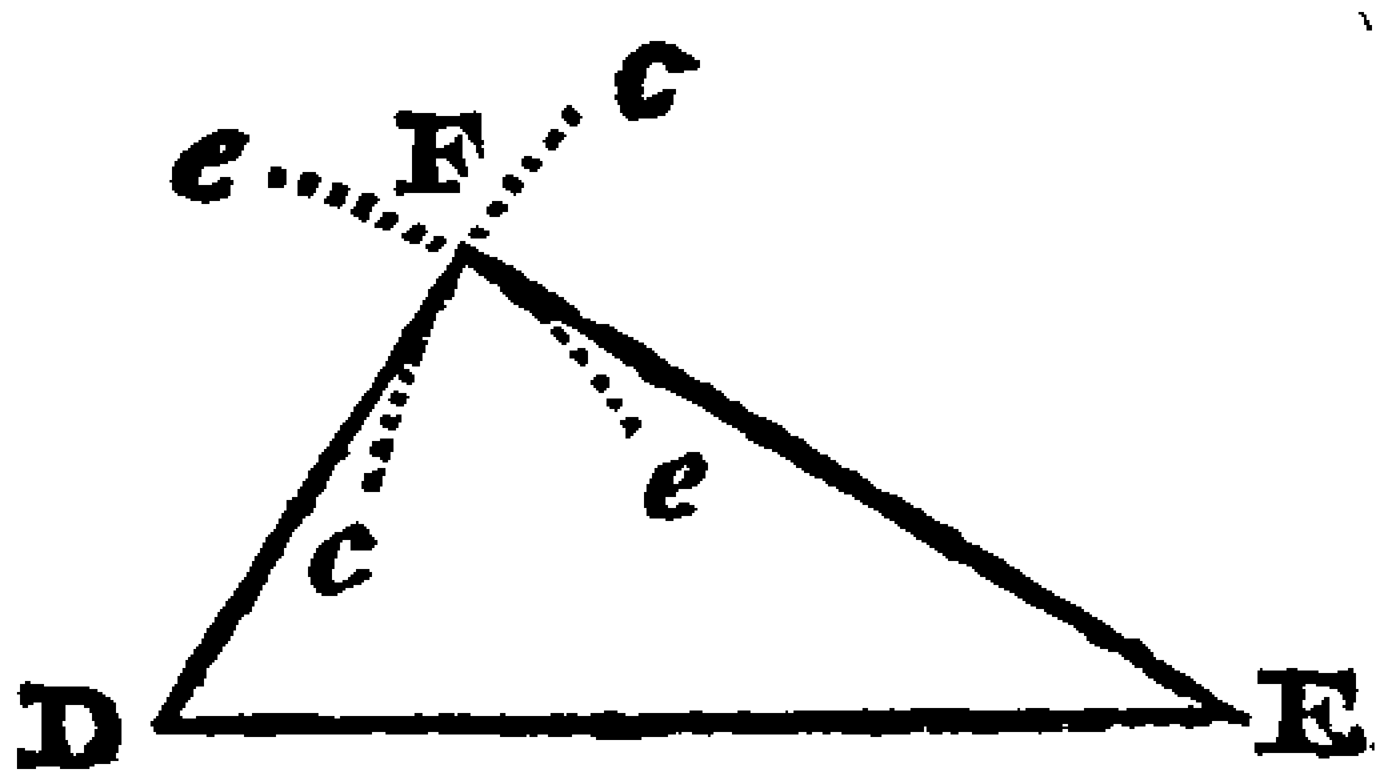
Pro-

Problem 11.

To make a Triangle whoſe three Sides ſhall be equal to three given Right Lines, provided that any two of them be greater than the third.

Let A, B, and C, be the three Lines given, with which it is required to make a Triangle.





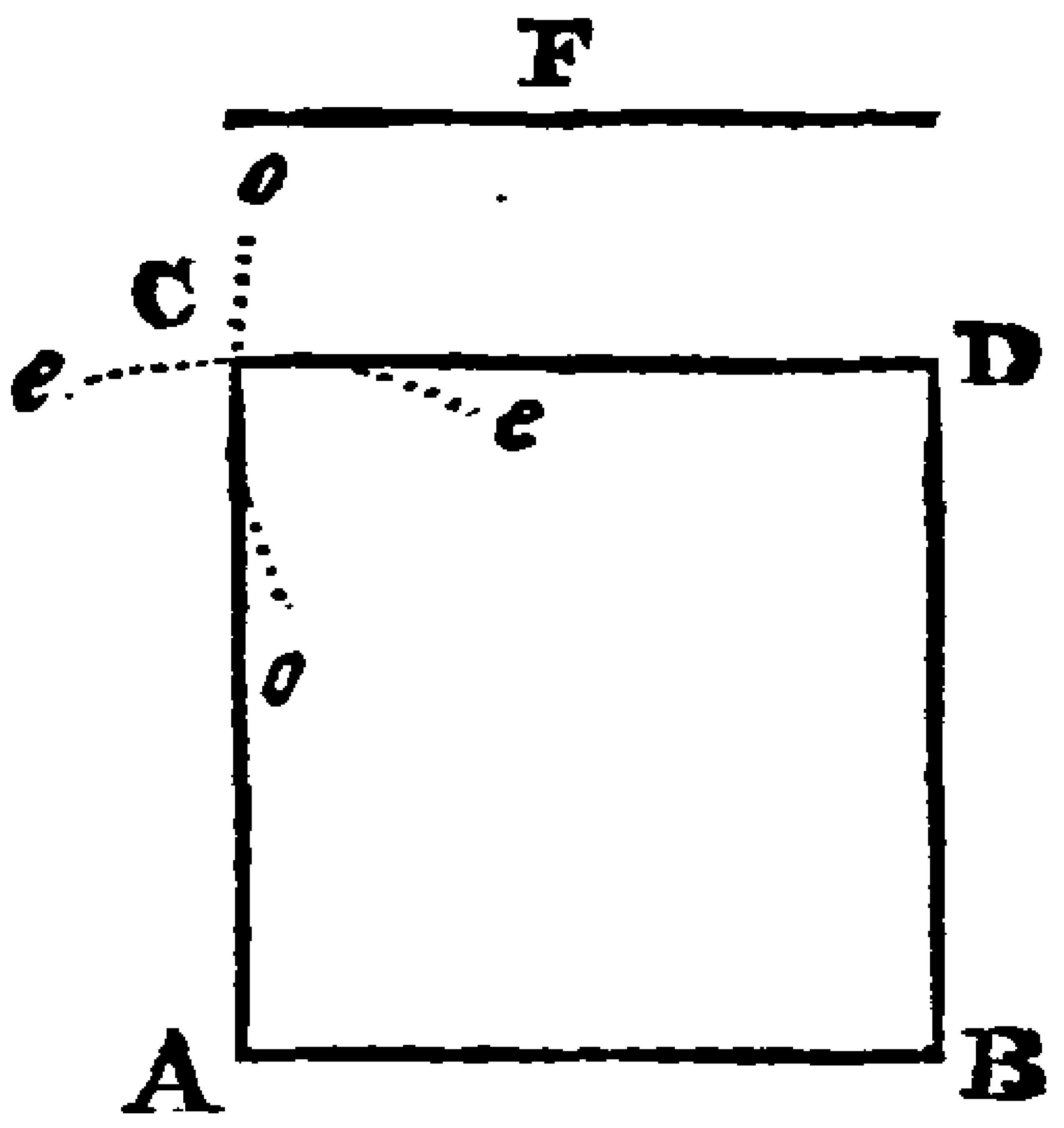
Conſtruction. Firſt, take the Line A in the Compaſſes, and ſet it from D to E; then take the Line B in the Compaſſes, and ſetting one Foot in E, deſcribe the Arch *c c*. That done, take the Line C in the Compaſſes, and ſetting one Foot in D, deſcribe the Arch *e e*, interſecting the other in F. Laſtly, from F draw the Lines F D and F E, ſo ſhall the Triangle be formed, whoſe three Sides are equal to three Lines, A, B, and C, reſpectively.

☞ We make Uſe of this Problem in *Surveying*, &c. to make a Figure equal to another given, by dividing it into *Triangles*, and taking off the Sides as above.

Problem 12.

To make a Square whose Sides shall be equal to a given Right Line.

Let F be the Right Line, with which it is required to make the Square A B C D.



Construction. First, take the Line F in the Compasses, and set it from A to B. Next, on the Point B erect the Perpendicular B D, by Problem 3, equal to the Line F. With the same Wideness of the Compasses, setting one Foot in D, describe the Arch *o o*; and with the same Extent of the Compasses, setting one Foot in A, describe the Arch *e e*, cutting the former in C. Lastly, from C draw Lines to A and D, which will complete the Square required.

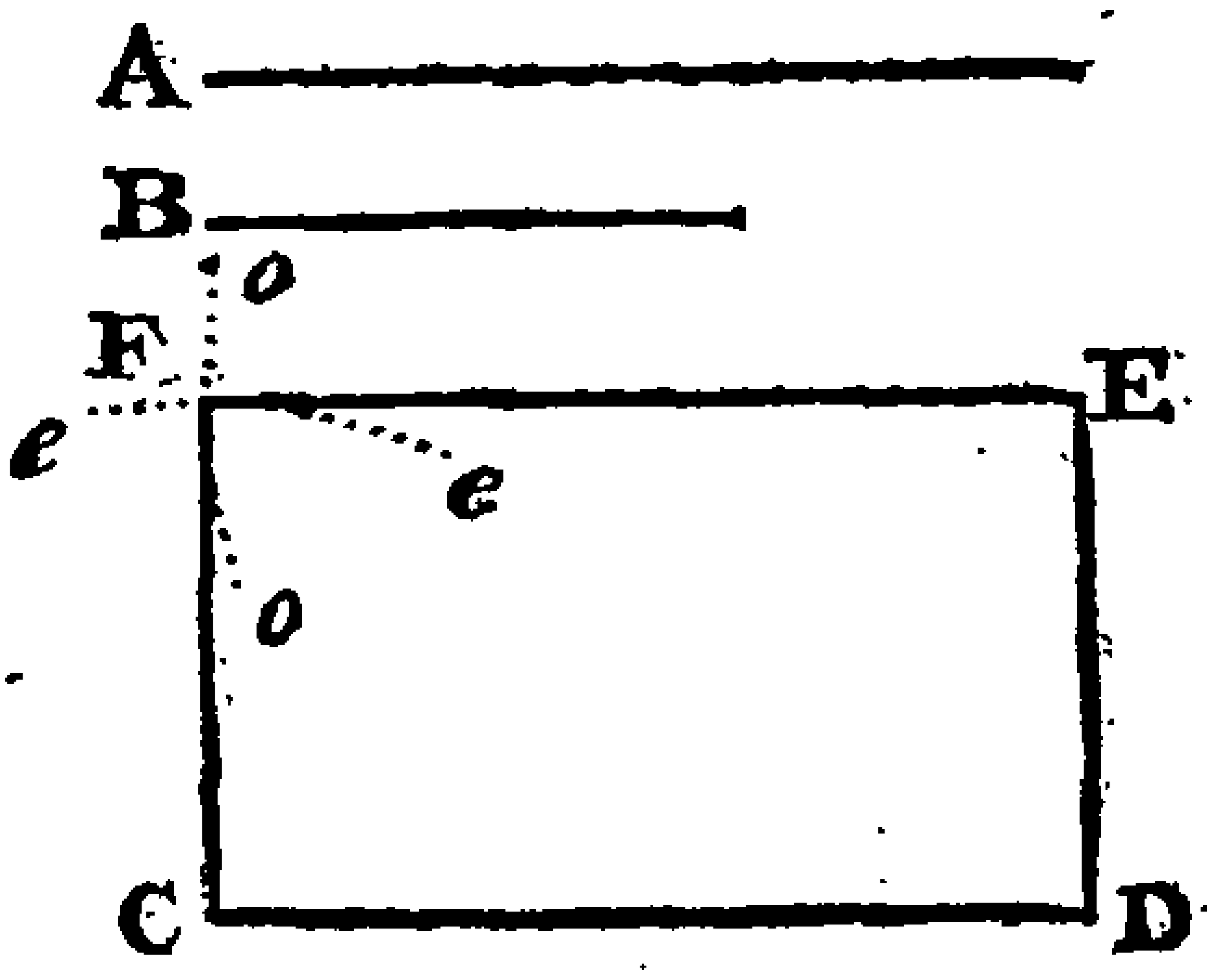
☞ *Architects*, *Surveyors*, &c. make Use of this Problem in laying out Ground for *Buildings*, or for *Pleasure*.

Pro-

Problem 13.

To make a Parallelogram, or long Square, whoſe Length and Breadth ſhall be equal to two Right Lines given.

Let the two Lines given be A and B; the former the Length, and the latter the Breadth of the Parallelogram required to be made.



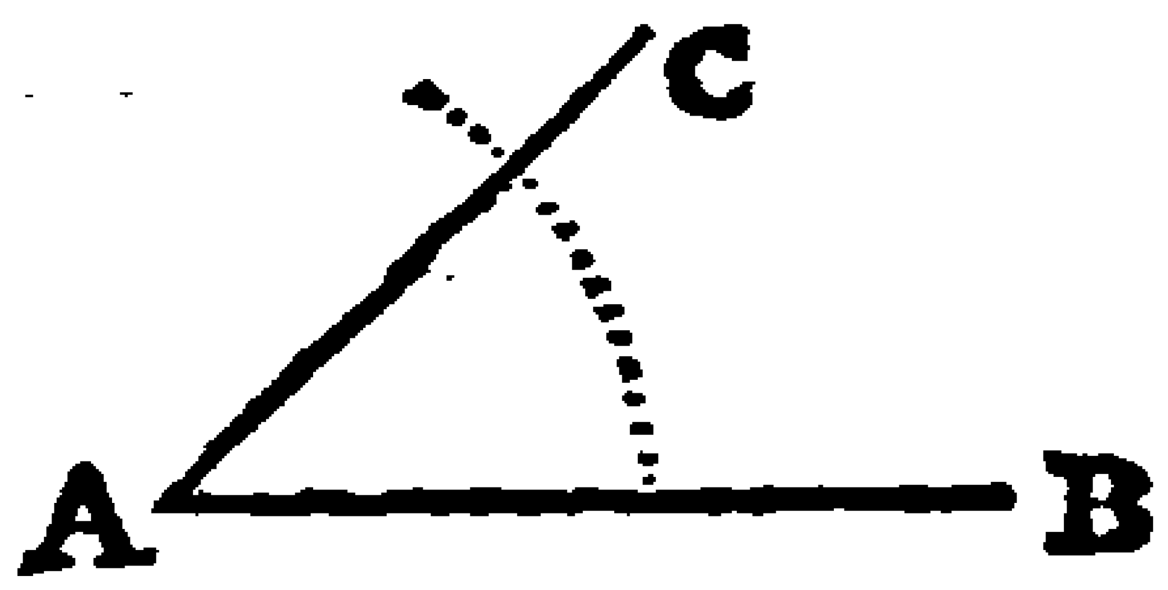
Conſtruction. Firſt, take with the Compaſſes the Length of the Line A, and ſet it from C to D; upon the Point D erect a Perpendicular D E, equal to the Line B. Then take the Line A again in the Compaſſes, and ſetting one Foot in E, deſcribe the Arch *o o*; next, take the Line B in the Compaſſes, ſet one Foot in C, and deſcribe the Arch *e e*, interſecting the other in F. Laſtly, draw the Lines F C and F E; and C D E F will be the Parallelogram required.

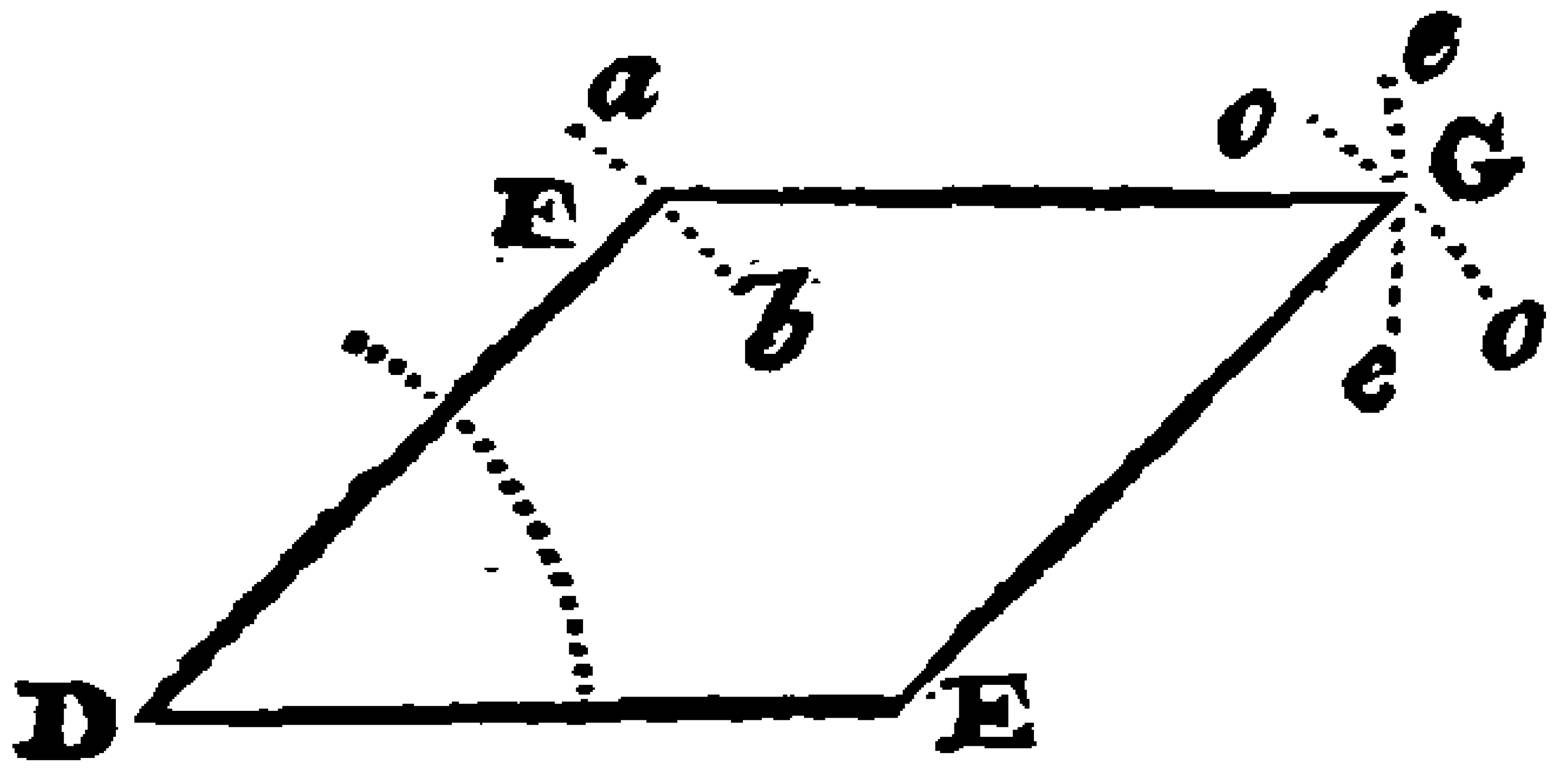
☞ This Problem, like the laſt, is equally uſeful to *Surveyors*, *Architects*, and *Mechanics*.

Problem 14.

To make a *Rhombus*, each of whose Sides shall be equal to a given Right Line; and whose acute Angle shall also be equal to an Angle given.

Let the Line given be A B, and the Acute Angle C A B, to delineate a Rhombus, whose Sides and Acute Angles shall be equal thereto.





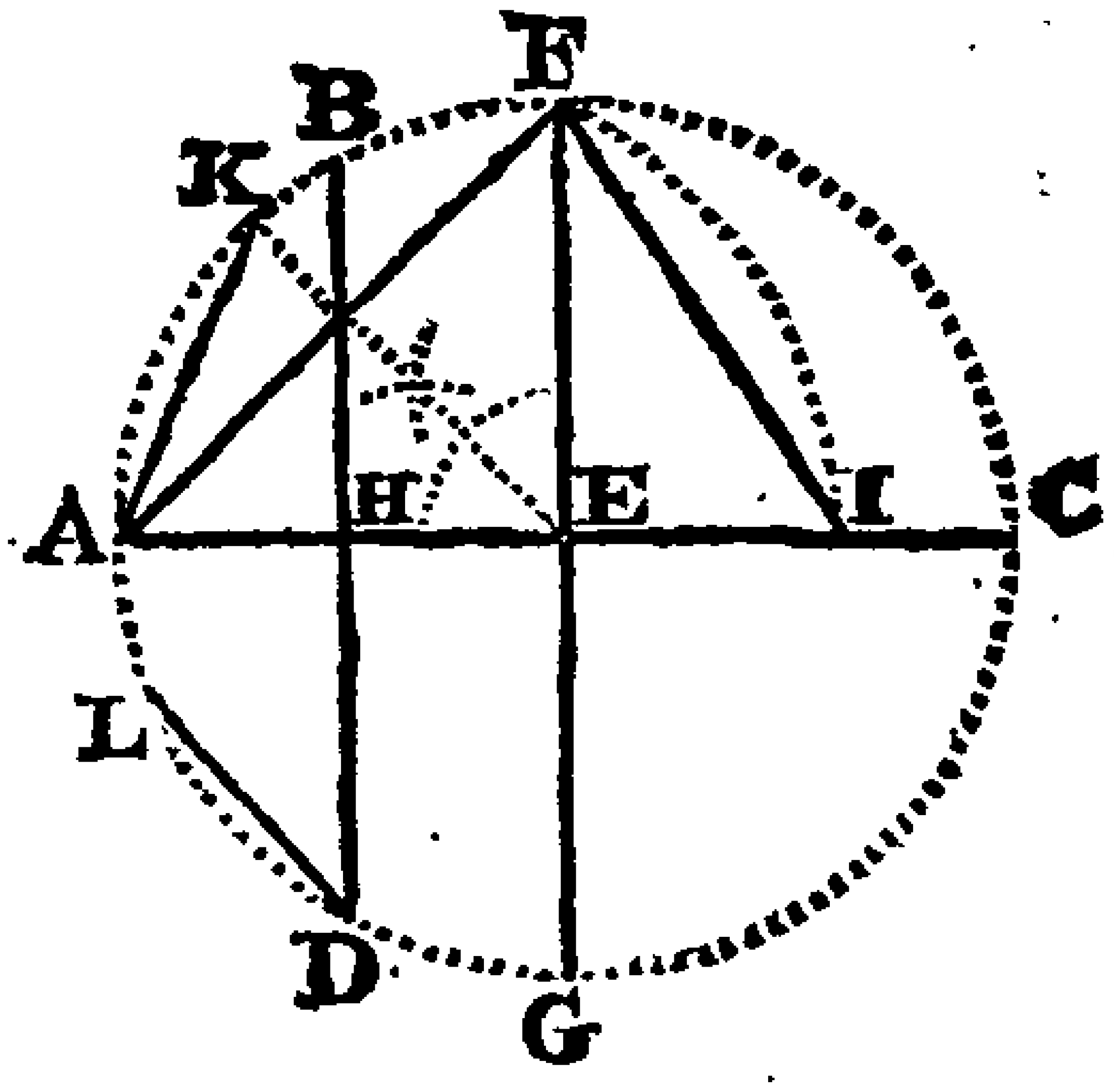
Construction. First, make the Angle at D equal to the Angle A, by Problem 8. Then take the Line A B in the Compasses, and setting one Foot in D, describe the Arch *a b*, cutting the Side D F in F. Next, on the Points F and E, with the same Extent A B, describe the two Arches *o o*, and *e e*, intersecting each other at G. Lastly, draw the Lines F G and E G, so will the Rhombus D E G F be formed, corresponding with the Line and Angle given.

Note. In like Manner any *Rhomboides*, whose Length, Breadth, and Acute Angle are given, may be easily constructed.

Problem 15.

To divide the *Circumference* of a Circle into any Number of Parts not exceeding *ten*.

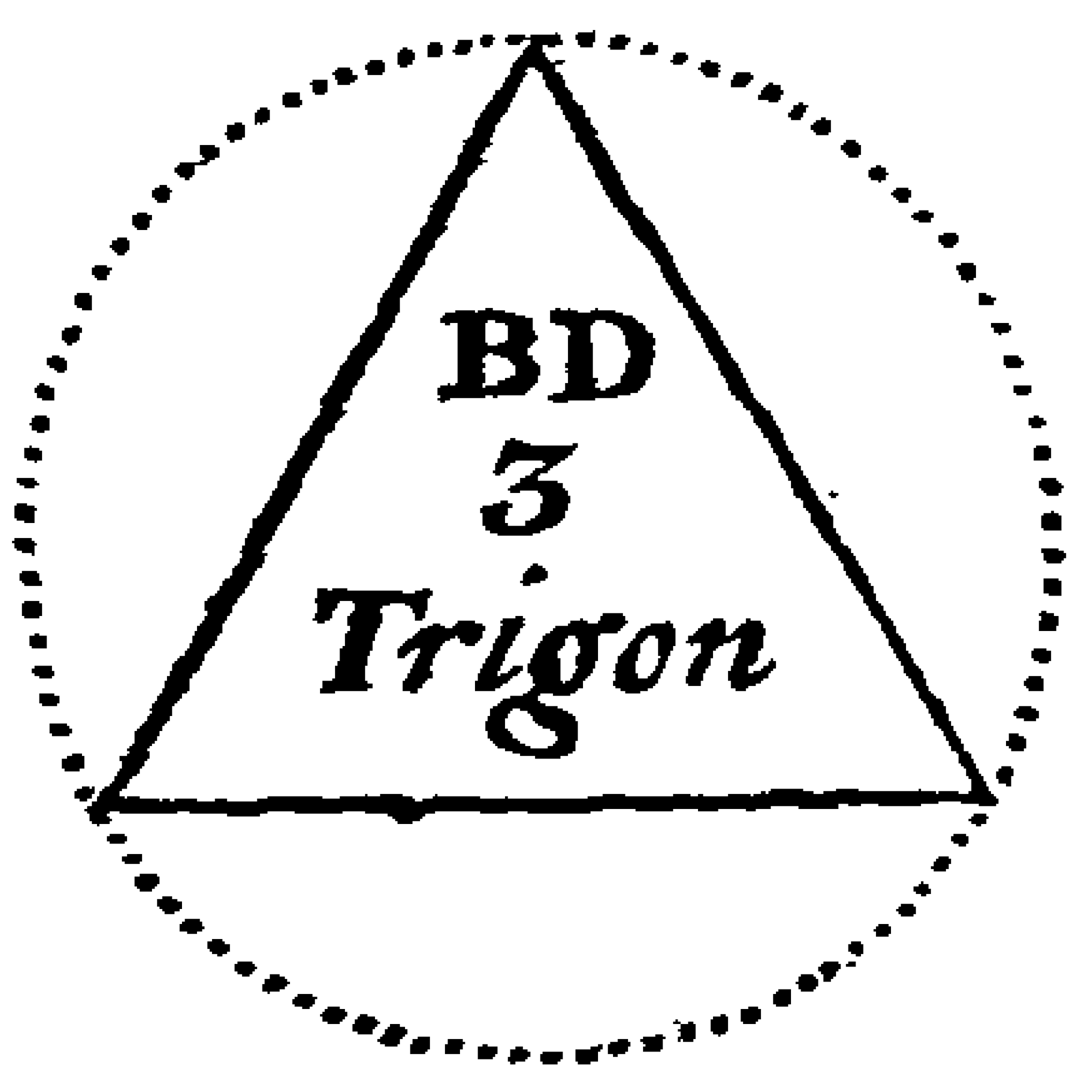
Let A F C G be a Circle given to be ſo divided.

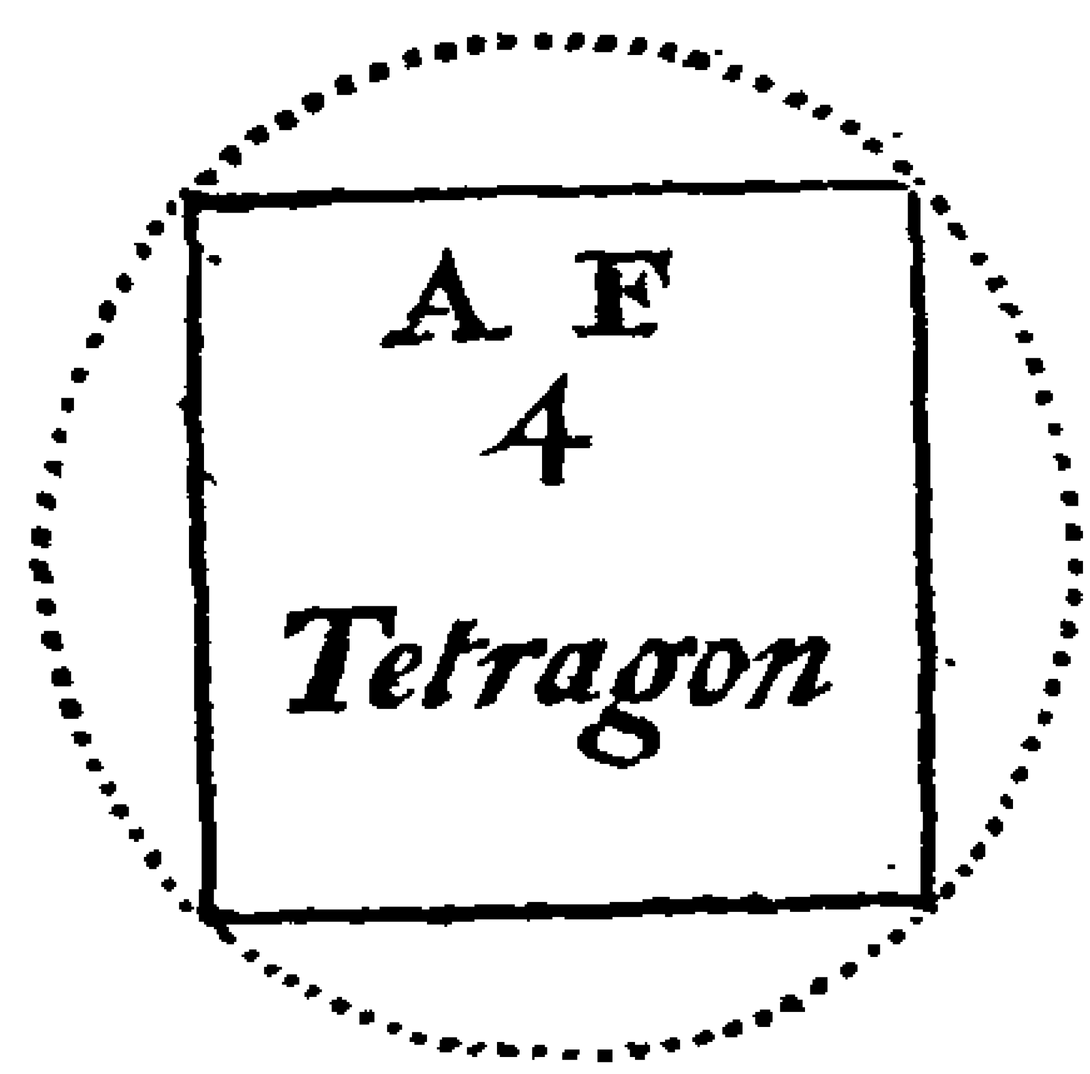


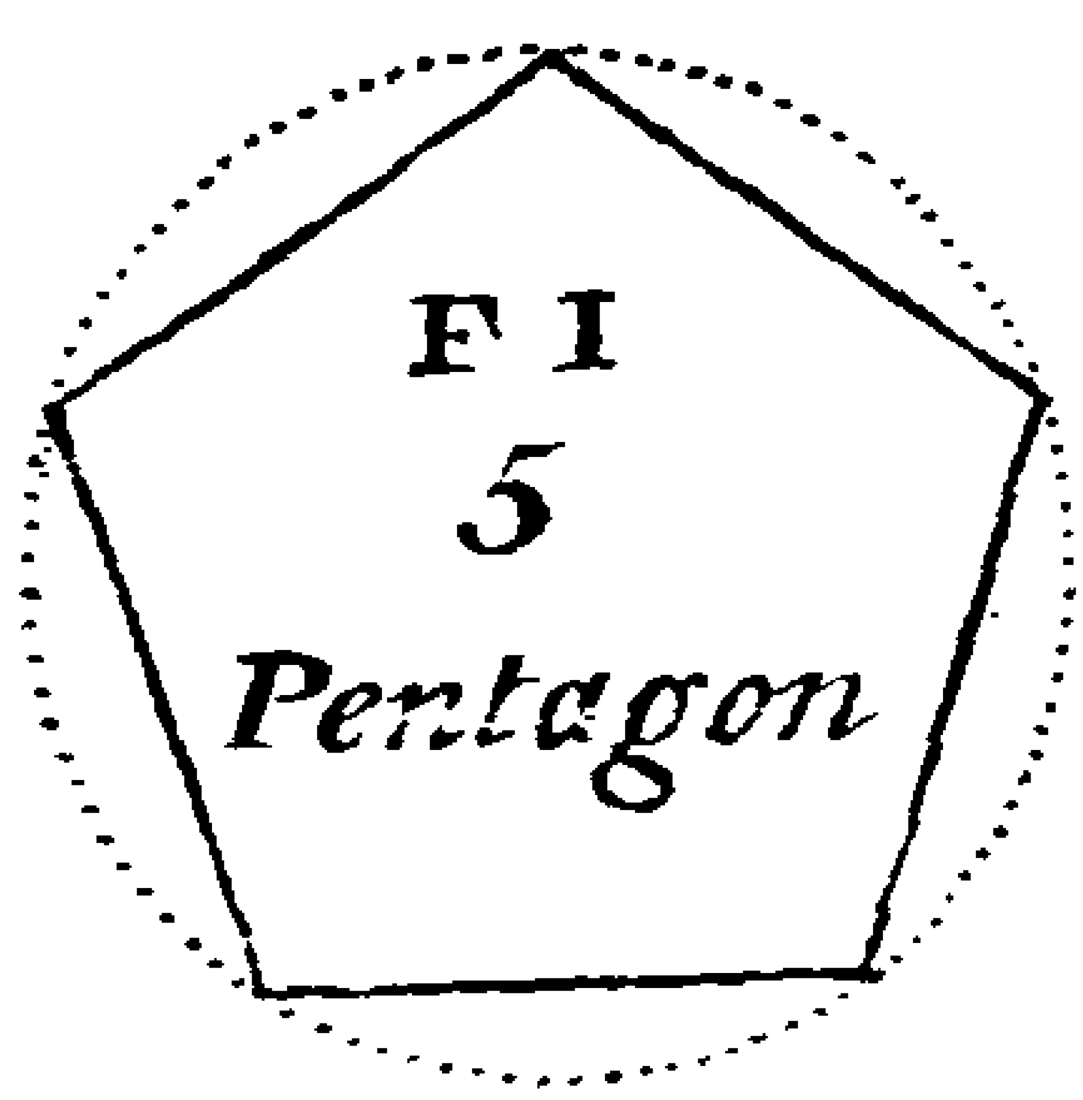
Conſtruction. Firſt, draw the Diameter A E C, which will divide the Circumference into two equal Parts. Take the Semi-diameter A E or C E in the Compaſſes, and ſetting one Foot in A, with the other make in the Circumference the Points B D, which Line B D will divide the Circle into 3 equal Parts. Divide the Diameter A C into two equal Parts by Problem 1. at right Angles with F G, and draw A F, which will divide the Circle into 4 equal Parts. Next, ſet one Foot of the Compaſſes in H, where the third Part B D cuts the Diameter A C, and extend the other to F, and deſcribe the Arch F I; then draw the Line F I, which will divide the Circle into 5 equal Parts. The Semi-diameter A E or C E will always divide the Circle into 6 equal Parts. Half the third Part, *i. e.* B H or H D, will divide the Circle into 7 equal Parts. Divide the Angle A E F, by Problem 7, into two equal Parts with the Line E K, cutting the Limb in K; then draw A K, and that Line will divide the Circle into 8 equal Parts. Take one

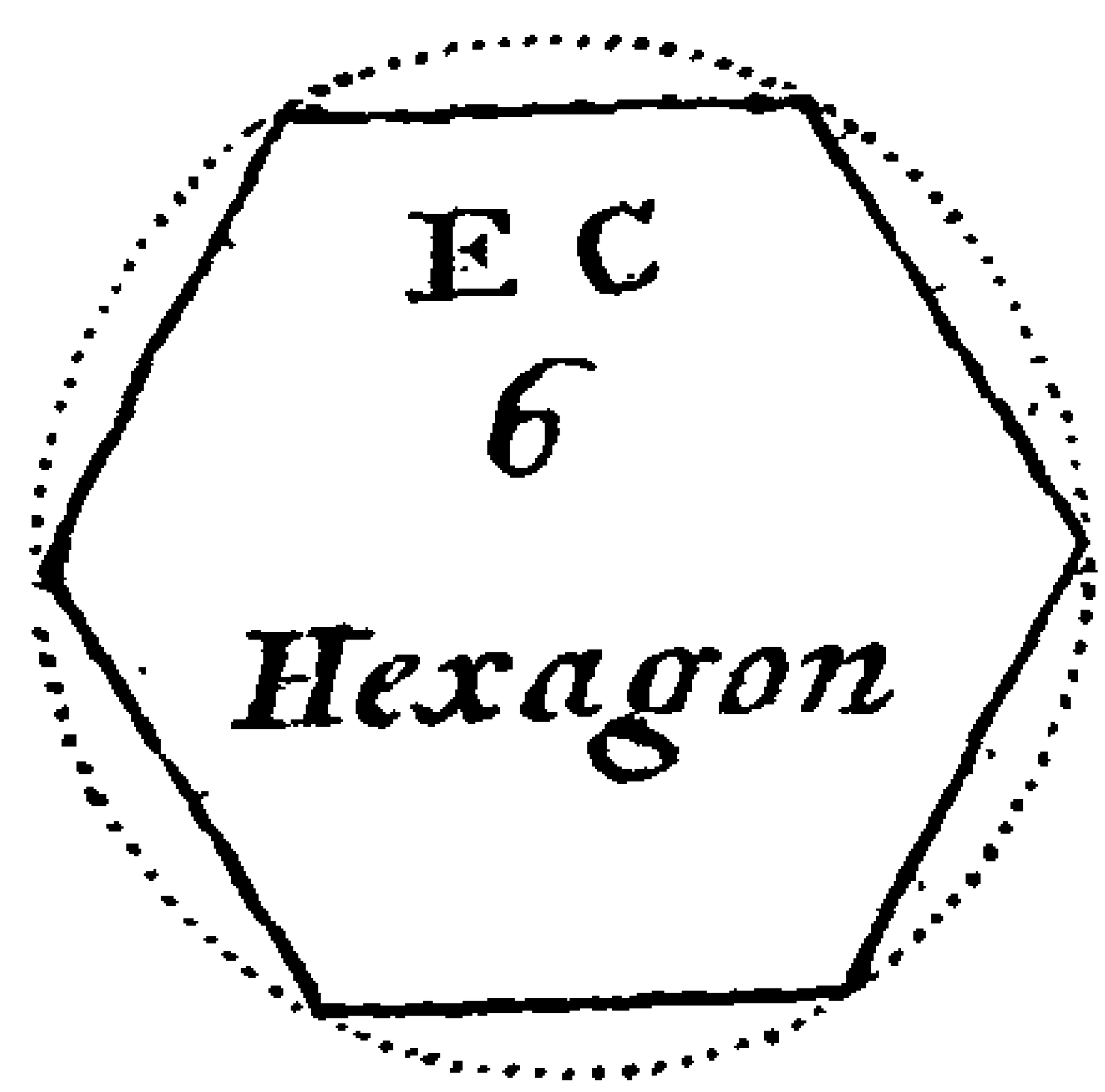
one third of the Arch B A D, as D L, and that Space will divide the Circle into 9 equal Parts. Laſtly, take the Line E I in the Compaſſes, and that Diſtance will divide he Circle into 10 equal Parts.

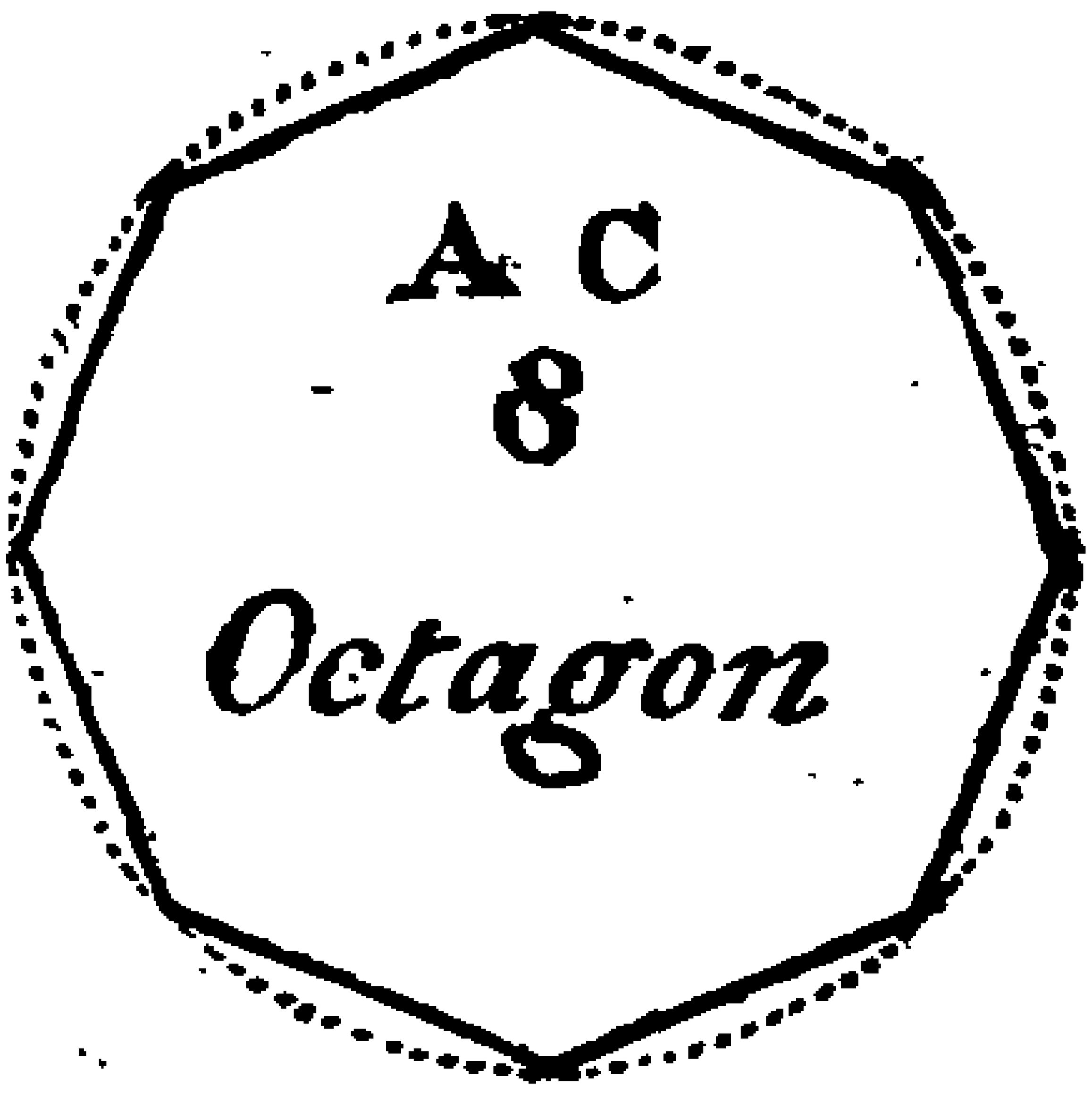
Now, if ſeveral Circles be deſcribed with the ſame Radius or Semi-diameter as the foregoing, and the Lines be taken off with the Compaſſes and applied to the Circumferences of thoſe Circles, they will mark out exactly the Diviſions intended, as is very evident from the following Figures.

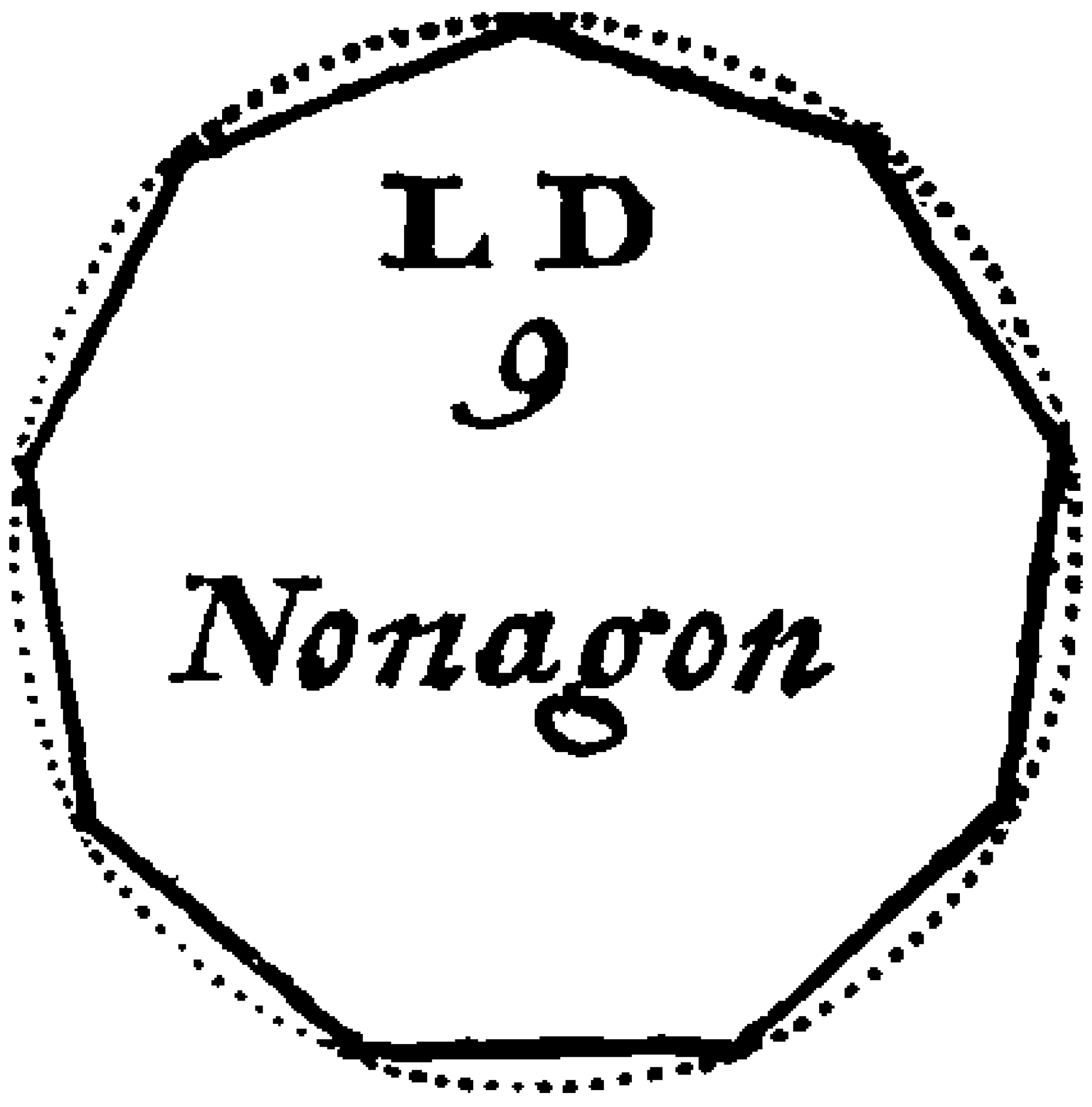












Note. The inſcribed Figures are called *Regular Polygons*, and take their Names from the Number of Sides and Angles that bound them.

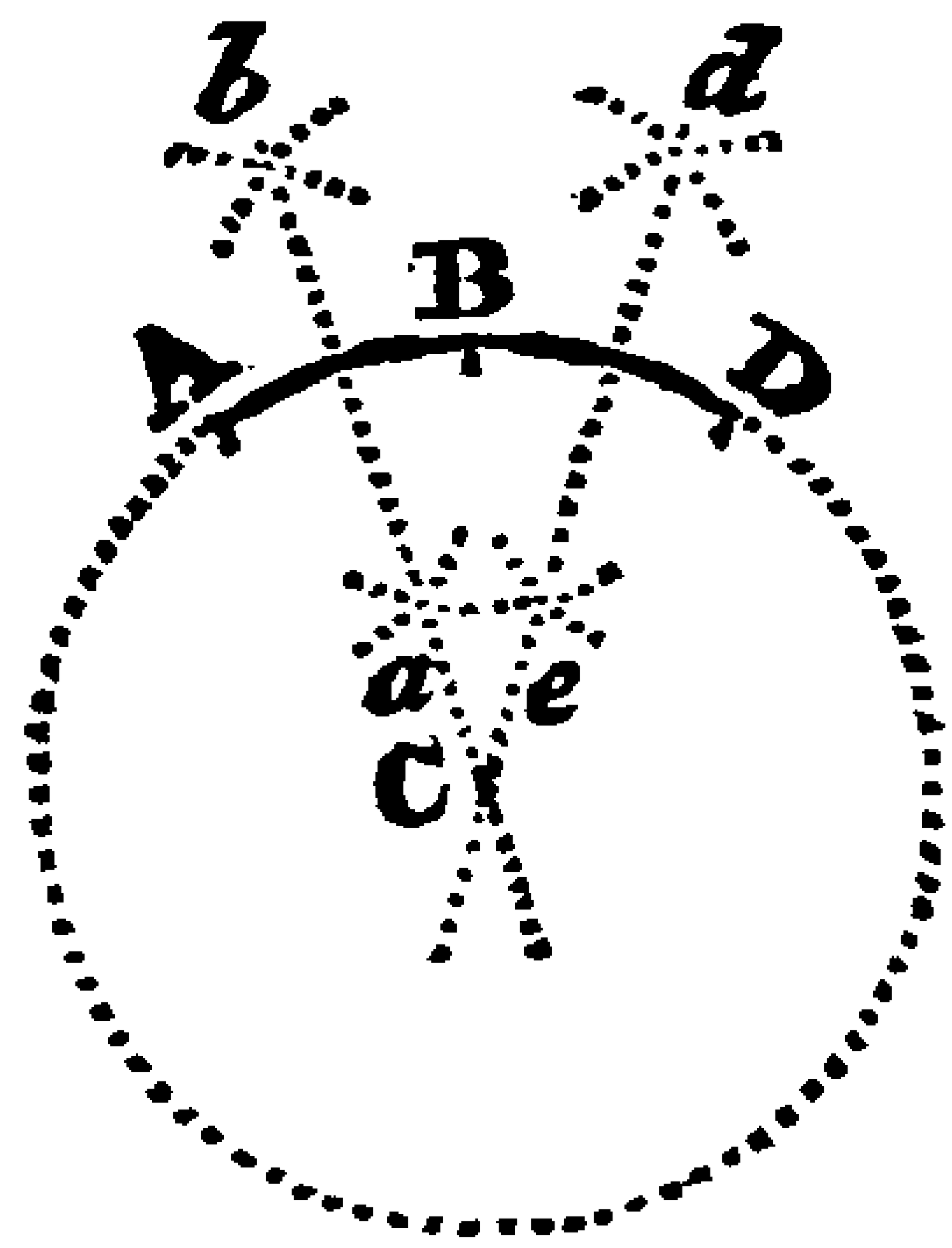
☞ Hence we ſee, that *Polygons* deſcribed in Circles degenerate at laſt into *Circles*. This Problem is very uſeful in tracing Ground for *Forts* and *Citadels*; and may be of Help in *ſquaring the Circle*.

Pro-

Problem 16.

The Arch of a Circle being given, to find the Center; or, three Points being given not ſituated in a Right Line, to find the Center of a Circle, which ſhall paſs through thoſe three Points.

Let the Arch given be A B D, and it be required to find its Center.



Conſtruction. Firſt, make three Points in the Arch at any Diſtance from each other, as at A, B, D. Next, open the Compaſſes from one Point to the other, as from A to B; and with that Diſtance, ſetting one Foot in A, deſcribe an Arch above and below, at *b* and *a*; then, ſetting the other Foot in B, ſtrike Arches acroſs the other, and draw the Line *b a*. Next, ſet the Compaſſes in B and C, and ſtrike Arches interſecting each other at *d* and *e*. Laſtly, lay a Ruler from *d* to *e*, and draw the Line *d e* continued; and where it interſects the other Line *b a* continued, that Point of Interſection will be the Center of the Circle required.

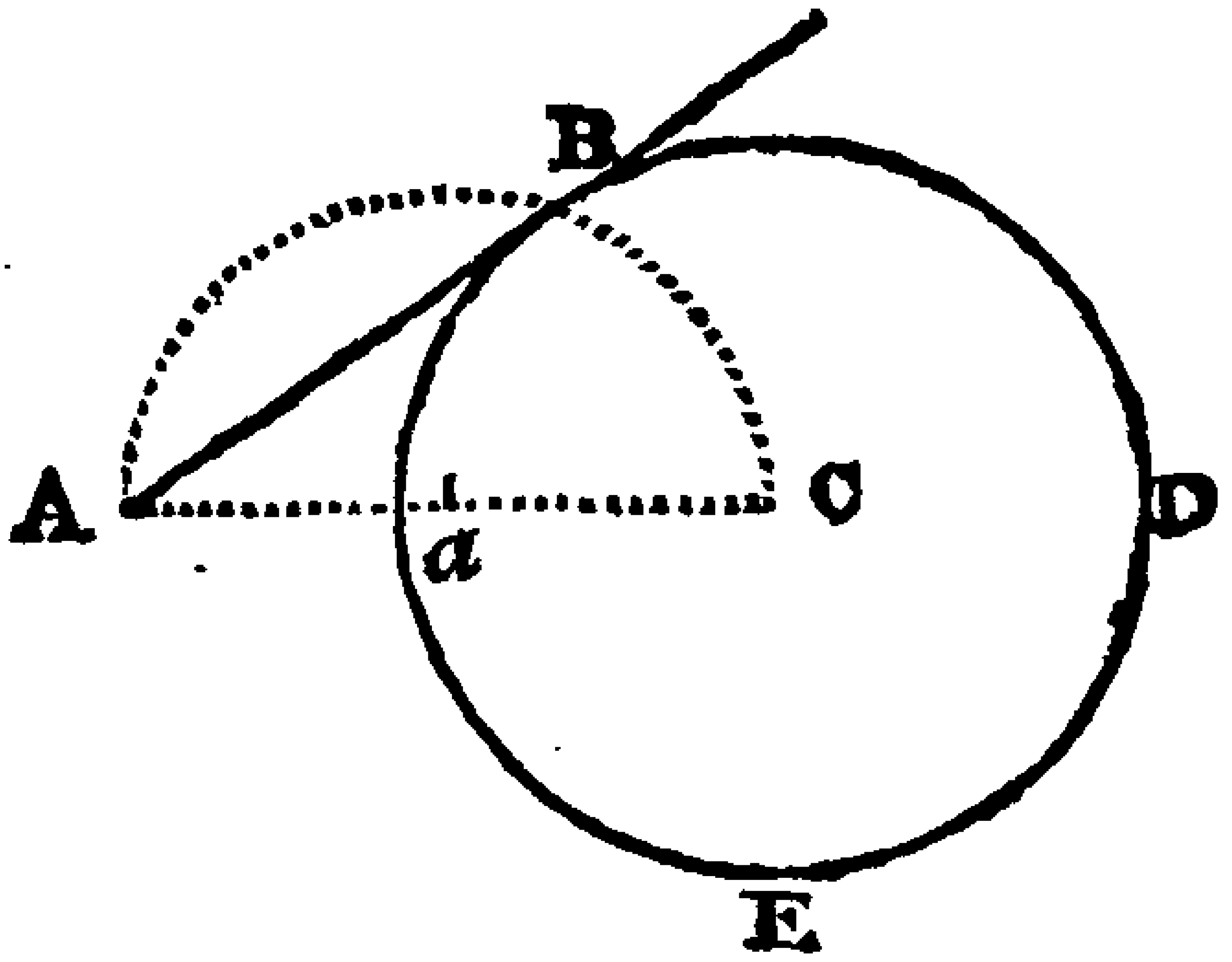
☞ This Problem is very uſeful in deſcribing *Curve Lines* in a *Quadrant*; and completing *Circles*, of which we have but a Part given.

Pro-

Problem 17.

To draw a Tangent to a given Circle, that ſhall paſs through a given Point.

Let B D E be the given Circle, C its Center, and A the Point from whence the Tangent is to be drawn.



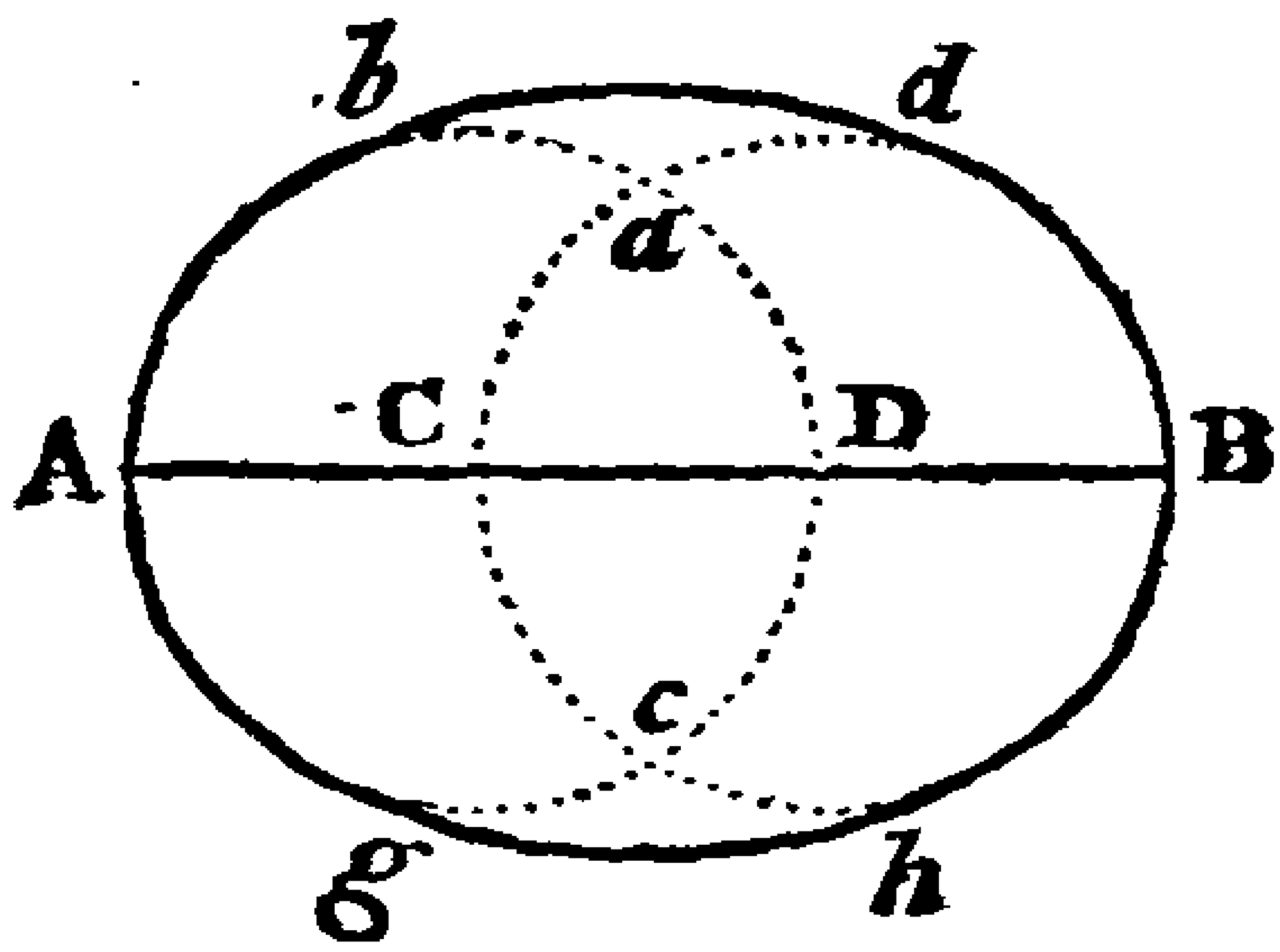
Conſtruction. Firſt, from the Center of the Circle C, draw the Line C A, and divide it into two equal Parts in the Point *a*. Then, upon the Point *a*, with the Diſtance A *a* or *a c*, taken in the Compaſſes, deſcribe the Semicircle A B C, cutting the given Circle in B. Laſtly, through the Points A and B, draw the Line A B, and it will be the Tangent required.

☞ The Uſe of a *Tangent Line* is very great in all Parts of the *Mathematics*; and ſo great in *Trigonometry*, that the whole Branch is founded upon it.

Problem 18.

To describe a Geometrical Oval; the Length only being given.

Let A B be the Length of the Oval required to be made.



Construction. First, draw the Line A B equal in Length to that of the Oval. Divide it into 3 equal Parts, as A C, C D, and D B. Then, opening the Compasses to the Distance of one of those Parts, describe upon the Points C and D two Circles, intersecting each other in the Points *a* and *c*. Next, taking the whole Diameter of one of the Circles, as A D or C B, set one Foot in *c*, and describe the Arch *b d*. Lastly, with the same Wideness, set one Foot in *a*, and describe the Arch *g h*, so will the Figure A *b d* B *h g* be the Oval required.

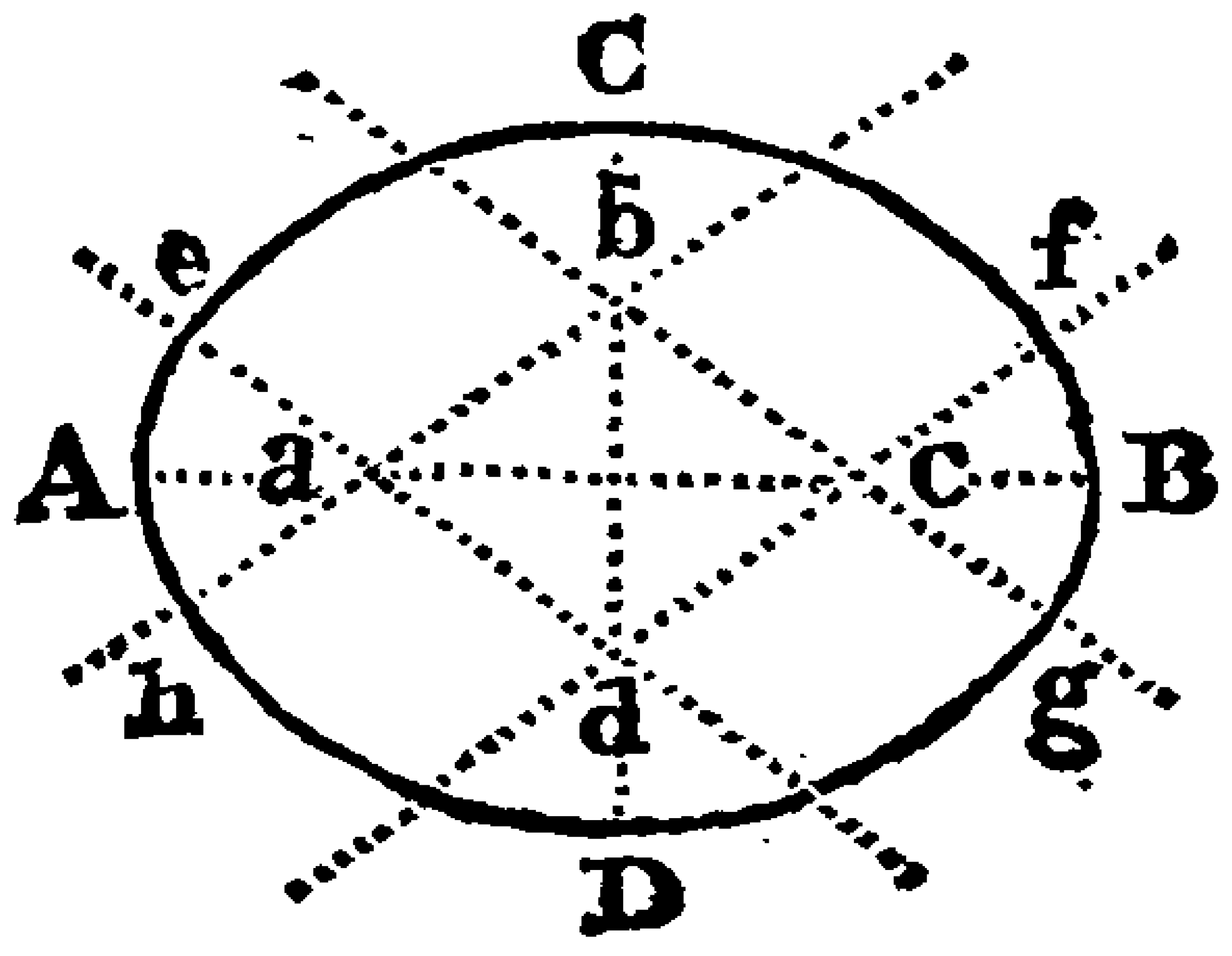
☞ This Method (as well as the following) of describing *Ovals*, is very necessary for *Joiners*, *Cabinet Makers*, and other *Mechanics*.

Pro-

Problem 19.

To describe a Geometrical Oval by another Method.

Let A B be the Length, and C D the Breadth of the required Oval.



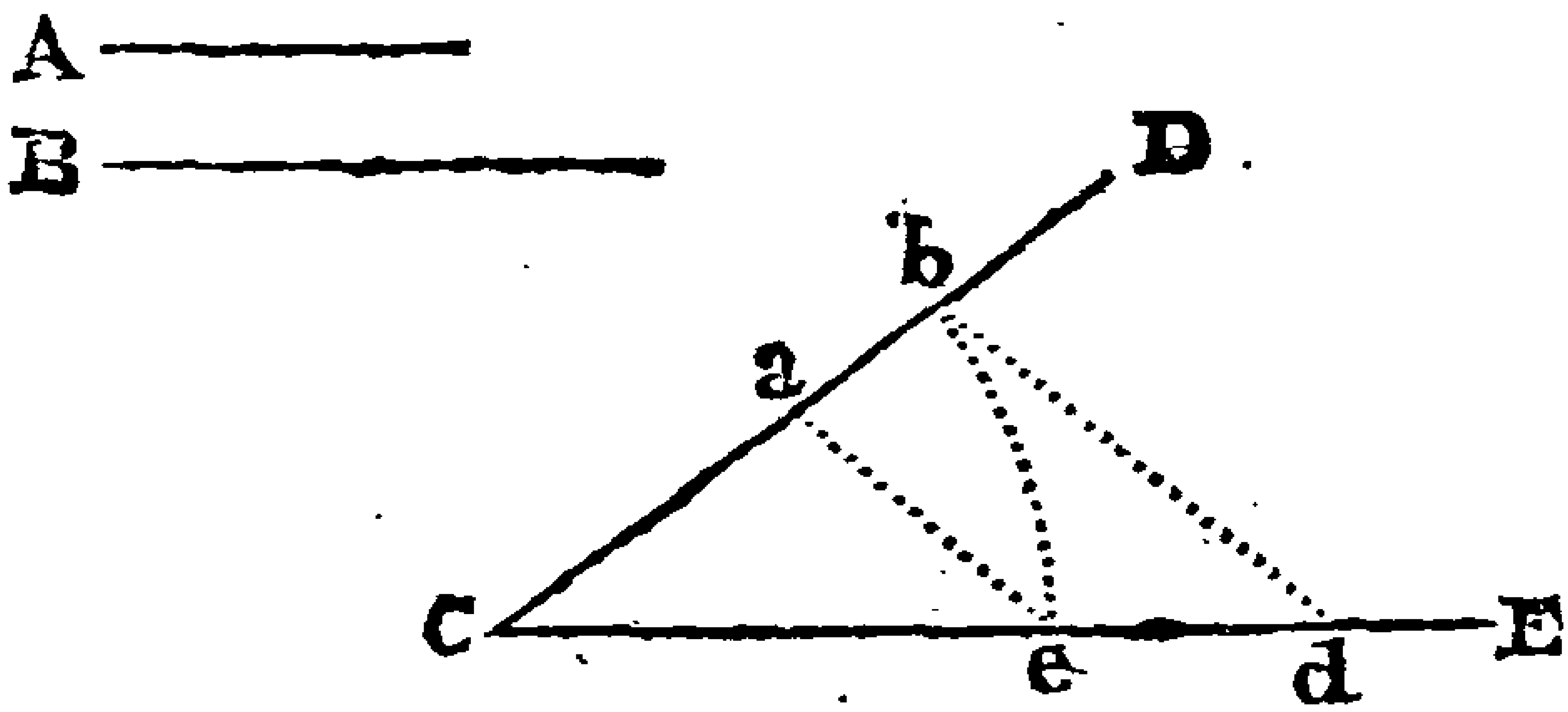
Construction. First, set the two Diameters across each other exactly in the Middle, and divide each Part into two equal Parts, as at *a b c d*. Then through these Divisions draw cross Lines, something longer than the longer Diameter, as in the Figure. Next, setting one Foot of the Compasses in *d*, open the other to the End of the shorter Diameter at C, and describe the Arch *e* C *f*; also, with the same Extent, set one Foot in *b*, and describe the Arch *b* D *g*. Lastly, take in the Compasses the Distance from *a* or *c* to the End of the longer Diameter, and with that Extent, setting one Foot in *a* and *c*, describe the Arches *e* A *b*, and *f* B *g*, which will complete the Oval required.

Pro-

Problem 20.

Two Right Lines being given, to find a third Proportional.

Let the two Lines given be A and B, and let it be required to find a third Line in Proportion thereto.



Construction. First, at the Point C, on the End of the Line C E, make any Angle at Pleasure, as D C E. Then, taking the Line A in the Compasses, set it from C to *a*, on the Line C D. Next, take the Line B, and set it from C to *b*, on the Line C D; and also on the Line C E, from C to *e*. Lastly, draw *a e*, and make *b d* parallel to it; so will C *d*, on the Line C E, be the third Proportional required.

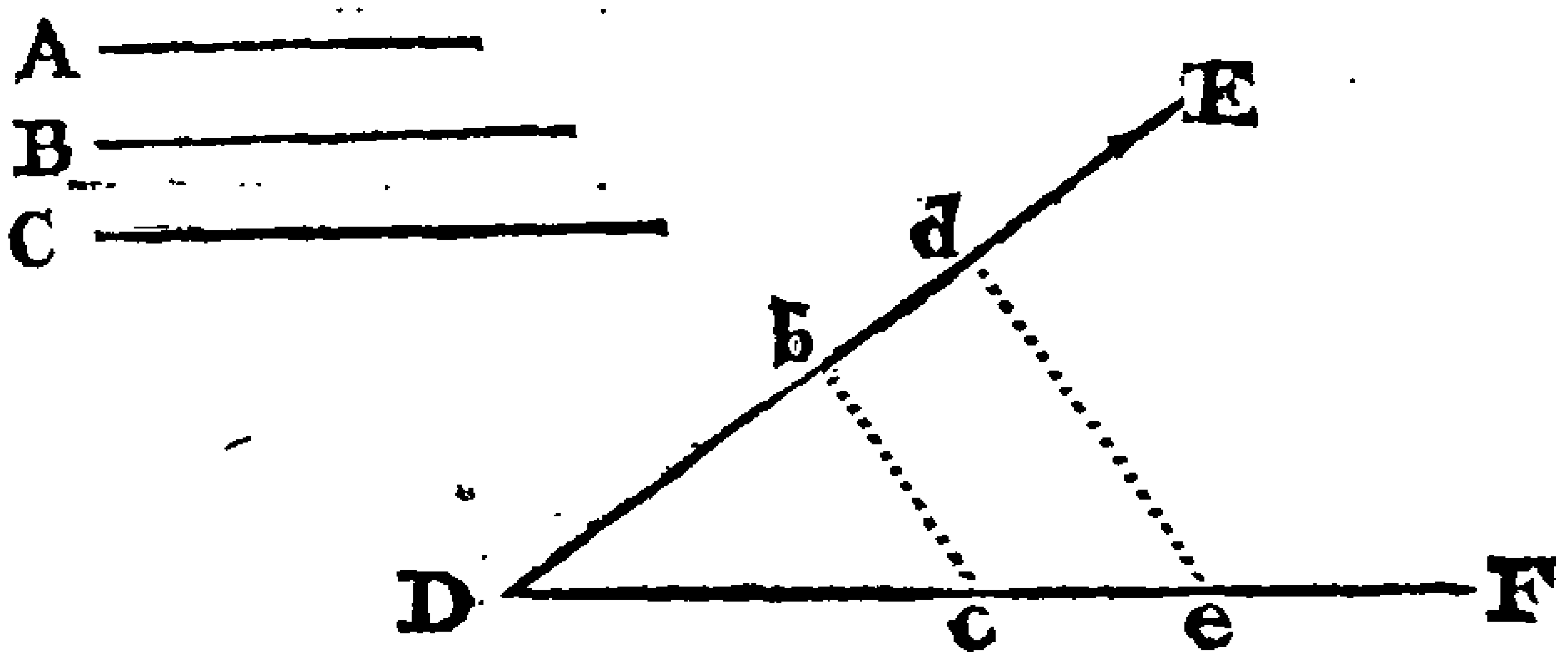
For, as C *a* is to C *b*, so is C *e* to C *d*.

☞ The Construction and Use of the *Sector* is founded on this and the following Problem.

Problem 21.

Three Right Lines being given, to find a fourth Proportional.

Let A B C be the three Lines given, and let it be required to find a fourth Line in Proportion thereto.



Construction. First, at the Point D, at the End of the Line D F, make an Angle of any Quantity at Pleasure, as E D F. Then, taking the Line A in the Compasses, set it from D to *b*, on the Line D E. Next, take the Line B, and set it from D to *c* on the Line D F. Then take the Line C, and set it from D to *d* on the Line D E. Lastly, join *b c*, and draw *d e* parallel to it, so will the Line D *e* be the fourth Proportional required.

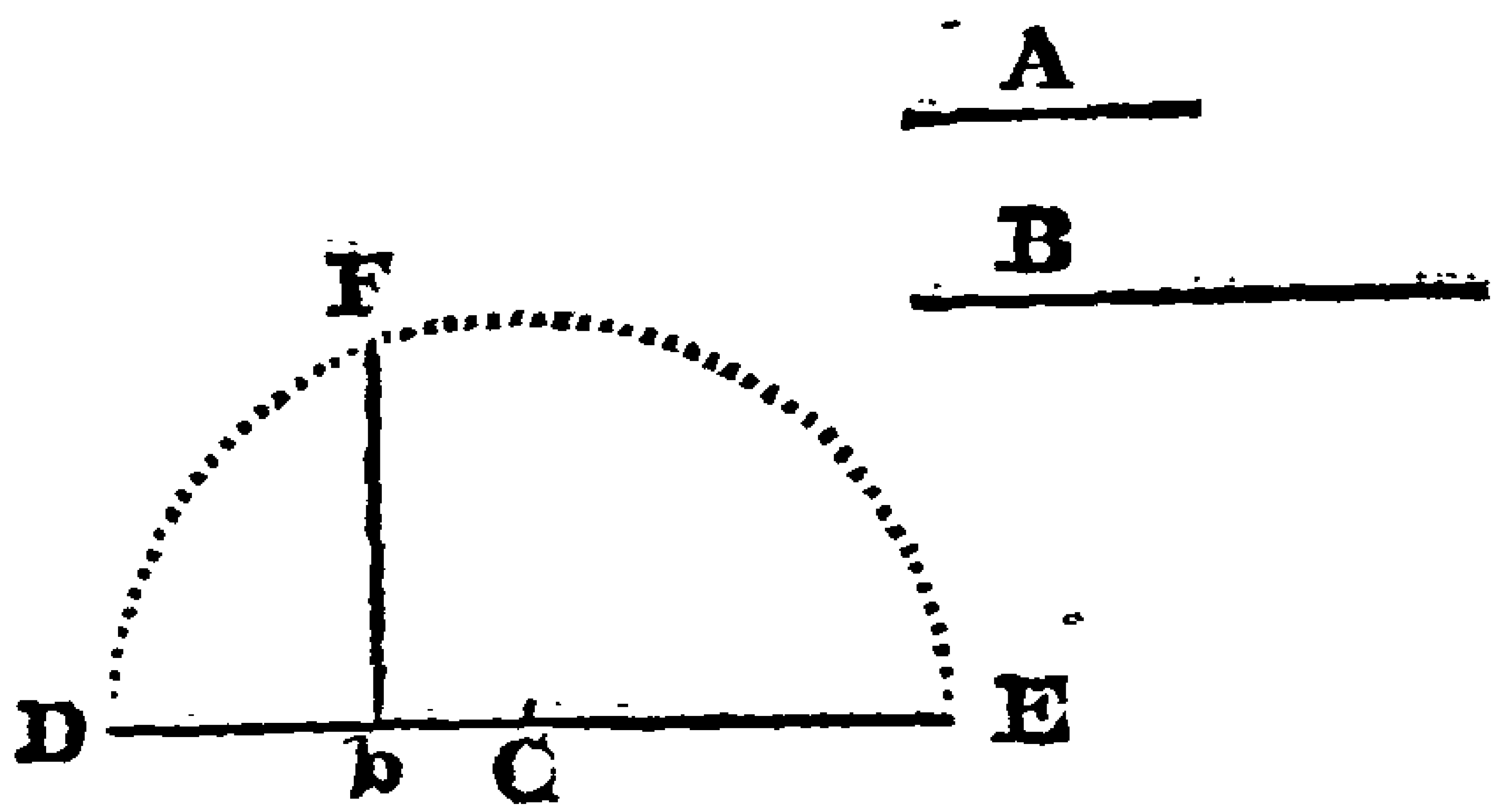
For, as D *b* is to D *c*, so is D *d* to D *e*.

☞ These two Problems do the Work of the *Rule of Three*, without the Use of *Arithmetic*.

Problem 22.

To find a Mean Proportional between two Right Lines given.

Let A and B be the two given Lines, and let it be required to find a Mean Proportional between them.



Conftruction. First, draw the Line D E at Pleafure, upon which fet with the Compaffes the Line A from D to *b*; and alfo the Line B from *b* to E. Next, divide the whole Line D E into two equal Parts in the Point C. Then, upon C, with the Diftance C D or C E, defcribe the Semicircle D F E. Laftly, from the Point *b* draw the Line *b* F perpendicular to the Line D E, and it will be the mean Proportional required.

For, as D *b* is to *b* F, fo is *b* F to *b* E.

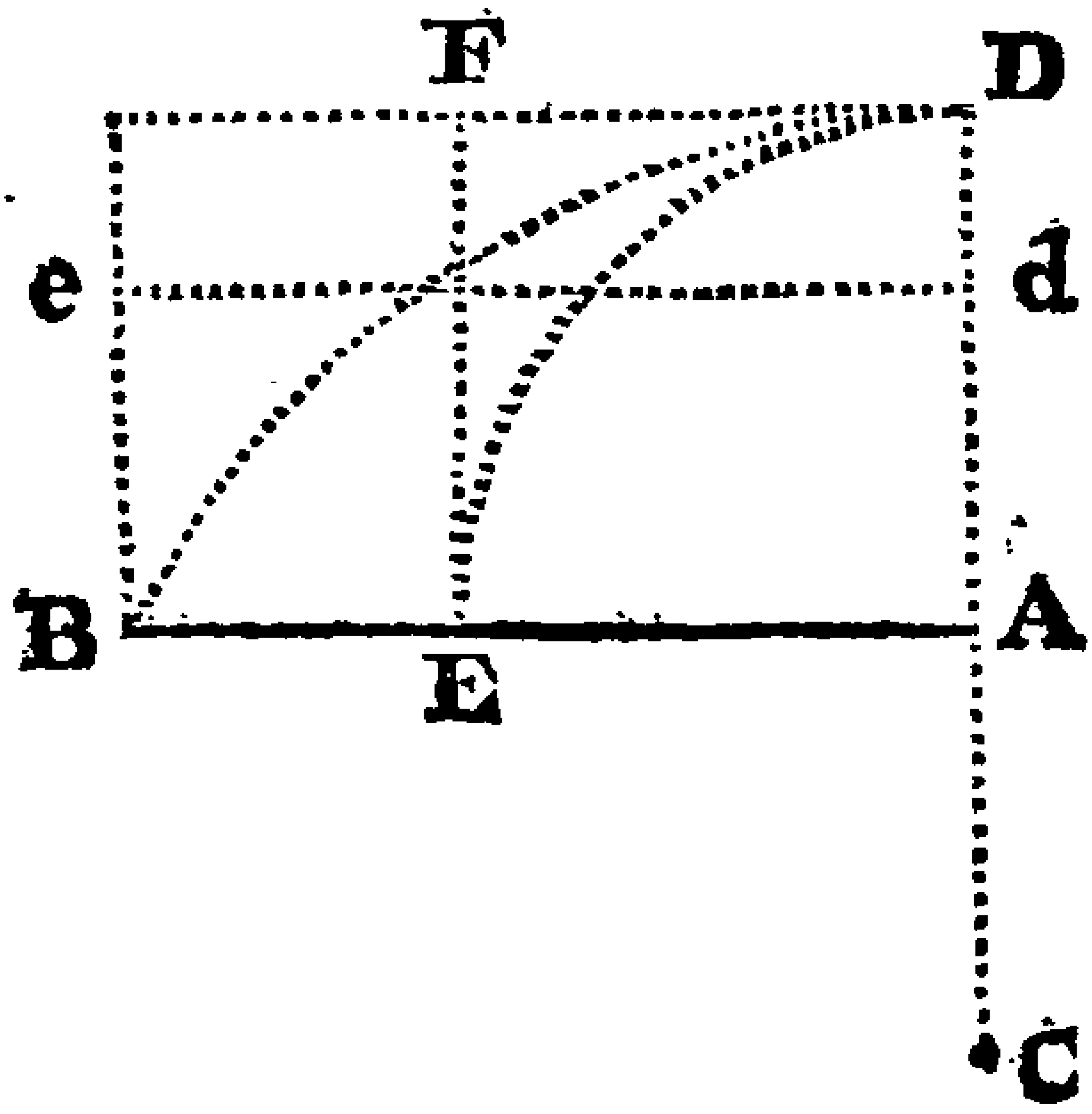
☞ By the Help of this Problem we can find the *Square Root* of any given Number; and readily reduce a *long Square* to a *perfect One*.

Pro-

Problem 23.

To divide a Right Line given into *extreme* and *mean Proportion*; that is, to cut a Line ſo that the Product of the whole Line and one of the Parts ſhall be equal to the Square of the other Part.

Let A B be the Right Line given to be ſo divided.



Conſtruction. Firſt, on the Point A, erect the Perpendicular A D; and produce it downwards alſo toward C. Next, make A C equal to half A B. Then, upon the Point C, with the Diſtance C B, deſcribe the Arch B D; and upon the Point A, with the Diſtance A D, deſcribe the Arch D E, which will cut the Line A B in the Point E in *extreme* and *mean Proportion*, as required. For the Area, Rectangle or Product A B *e d* made of the whole Line A B, and the Part B E, will be equal to the Area or Square A E F D made on the other Part A E.

For, as B E : E A : : E A : A B.

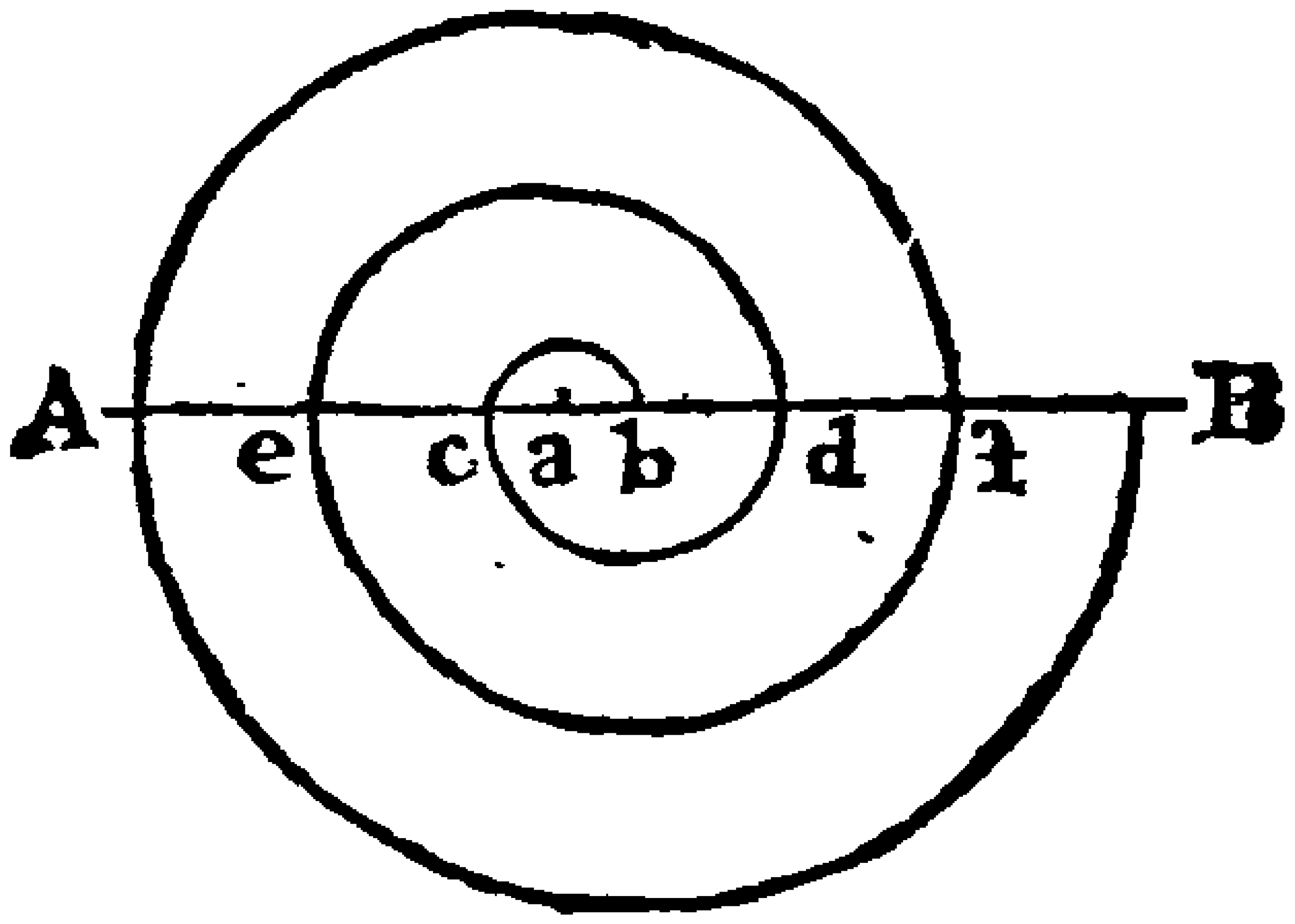
☞ By this Problem we find *(Geometrically)* the Length of the Sides of ſome of the *regular* or *Platonic Solids*.

Pro-

Problem 24.

To deſcribe a ſpiral Line about a given Line.

Let A B be the given Line about which the ſpiral Line is to be deſcribed.



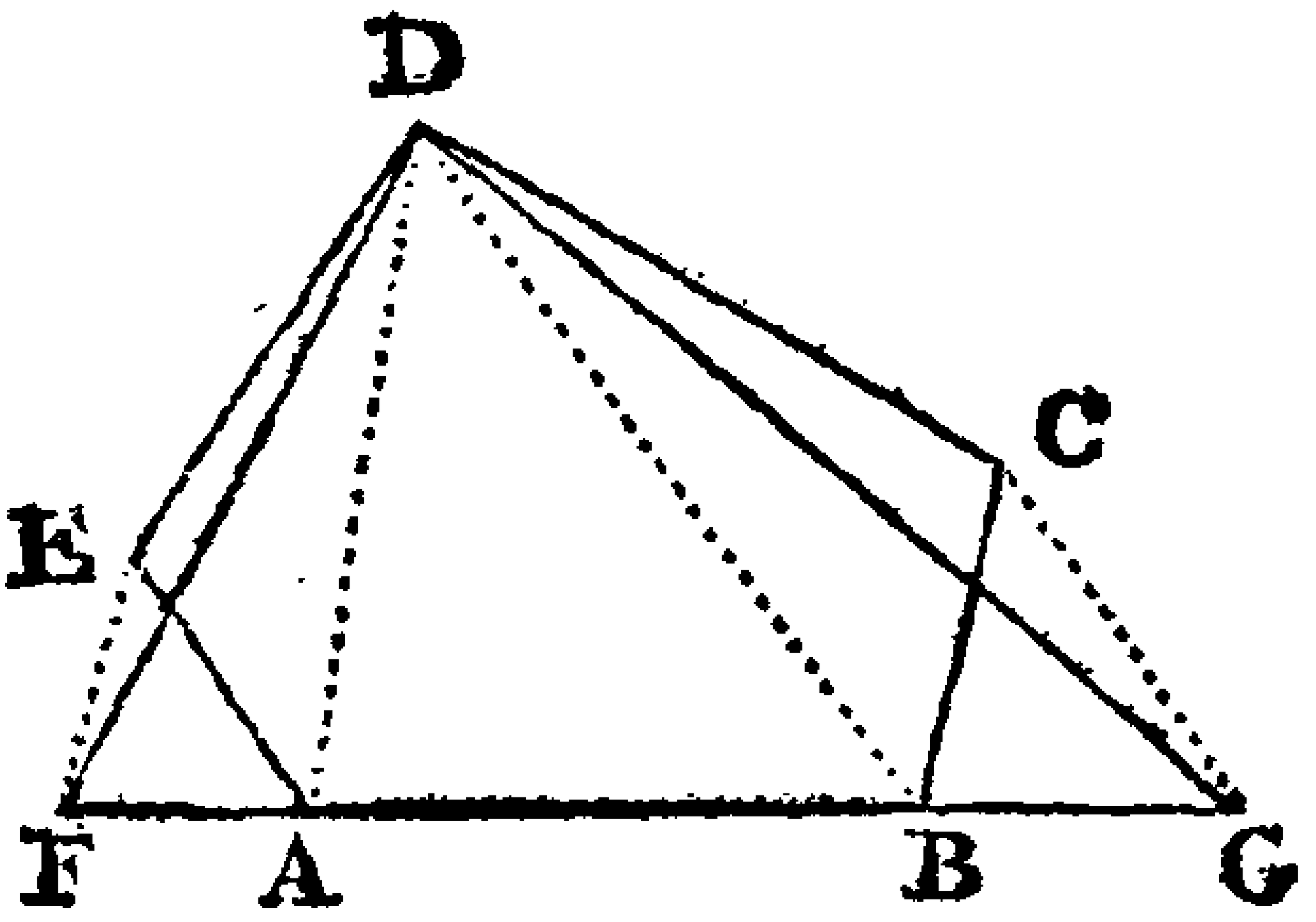
Conſtruction. Firſt, divide Half the Line A *b* or *b* B into as many Parts as there are to be Revolutions of the Spiral, as in *c e* A and *d f* B, which in this Caſe we will ſuppoſe to be *Three*. Next, divide *b c* into two equal Parts in *a*; then, upon the Point *a* deſcribe the upper Semicircles *c b*, *e d*, A *f*. Laſtly, upon the Point *b* deſcribe the under Semicircles *c d*, *e f*, A B, which will complete the Spiral required.

☞ This Problem may be uſeful to *Architects* in drawing the *Capitals* of ſome Orders in Building; particularly the *Ionic*.

Problem 25.

To reduce any Right Lined Figure to a Triangle equal to it.

Let A B C D E be an irregular Figure of five Sides to be reduced to a Triangle equal thereto.



Conſtruction. Firſt, continue the Side A B to F and G at Pleaſure. Next, draw the Diagonals D A and D B; then draw E F and C G parallel to D A and D B. Laſtly, draw the Lines D F and D G; ſo will the Triangle D F G be equal in Area to the irregular Figure A B C D E required.

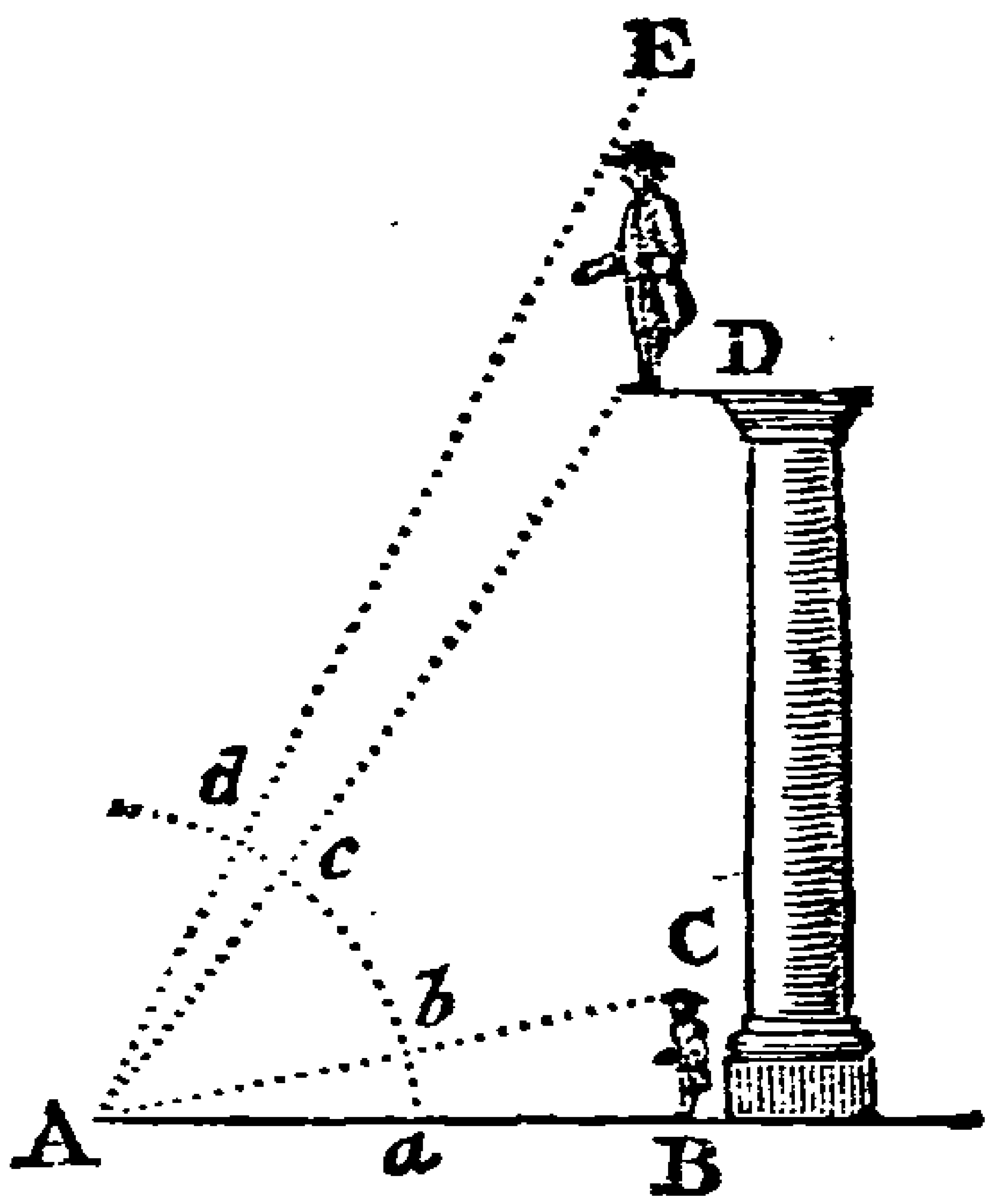
After this Manner may a Trapezium or any other Right Lined Figure be reduced to a Triangle equal to it.

☞ This Problem is uſeful in Surveying, to find the Content of an irregular Piece of Ground at one Operation; without dividing it into many *Triangles*, *Trapeziums*, &c.

Problem 26.

To determine the Height of a Statue, which is to be erected on a Building at any Altitude above the Level of the Eye.

Let C B be a Statue 6 Feet in Height; and let it be required to find what Height a Statue must be to appear of the same Dimension when placed at D, on the Top of the Pillar D B, 50 Feet above it.



Construction. First, draw the Line A B; and let A be the Station from whence the Figures are to be viewed. Then opening the Compasses to any convenient Wideness, set one Foot in A, and describe the Arch *d a*. Next, draw the Lines A D and A C. Now, because all Objects appear under a certain Angle in Proportion to their Distances from the Eye; and because, when those Angles appear equal, the Objects, however placed, will appear so too; take

take therefore the Arch *b a* in the Compaſſes, and ſet it from *c* to *d*, and draw the Line A *d* E, (making the Angles *d* A *c*, and *b* A *a* equal) ſo will the Figure E D appear to the Eye at A of the ſame Height as the Figure C B ſtanding 50 Feet below it.

Note. If E D be meaſured on the ſame Scale on which C B meaſures 6 Feet, you will have the Number of Feet that the Statue E D muſt be made to appear 6 Feet in Height when elevated 50 Feet above the Level of the Eye of the Spectator.

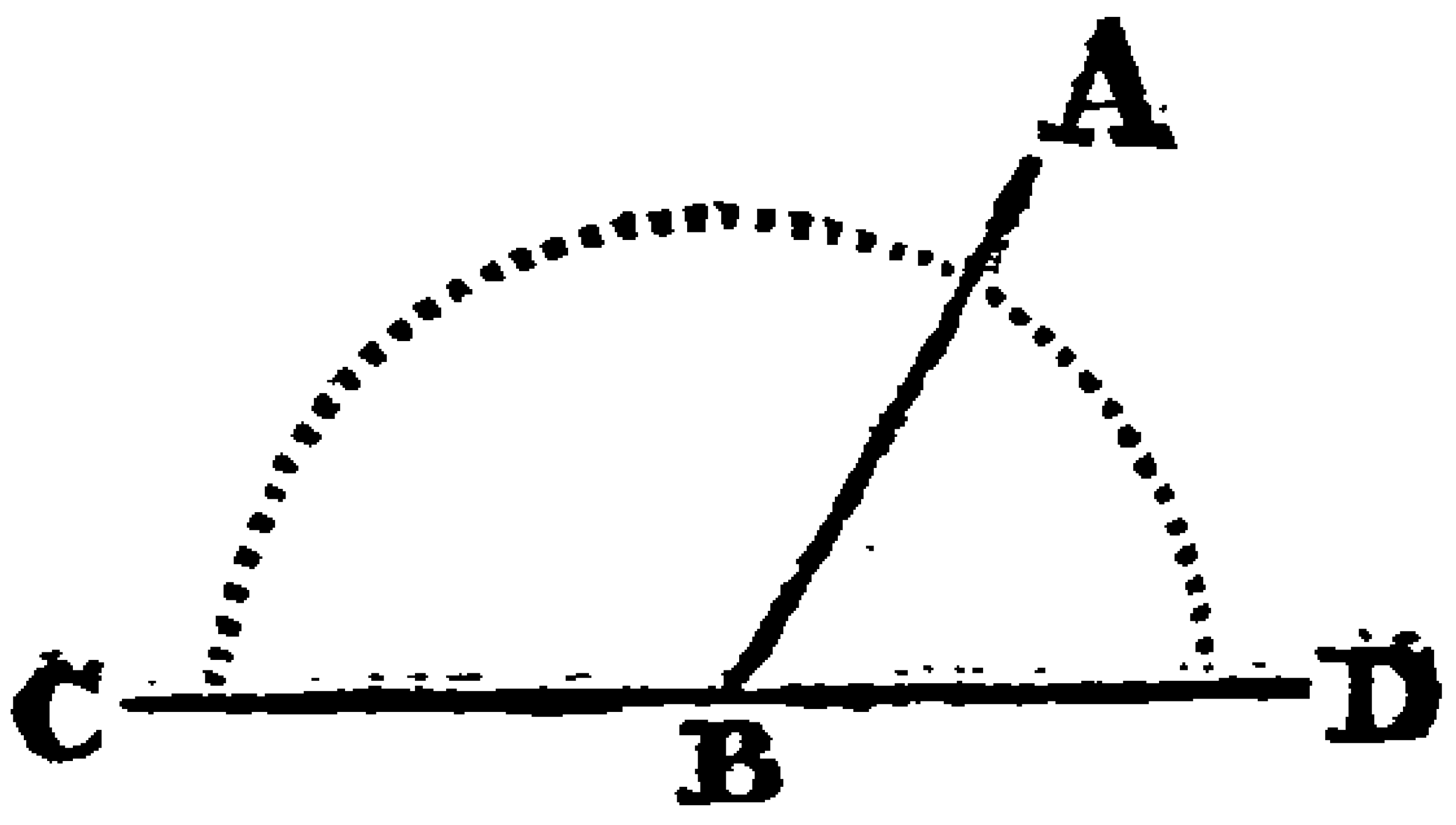
Note. By this *Equality of the Viſual Angles*, we are empowered to write upon a Wall or Building Characters, which, though very unequal, ſhall appear of the ſame Size when ſeen from a certain Point of View.

Geometrical Theorems.

Theorems are Propositions shewing the Nature and Properties of Geometrical Lines, Angles, and Figures.

Theorem 1.

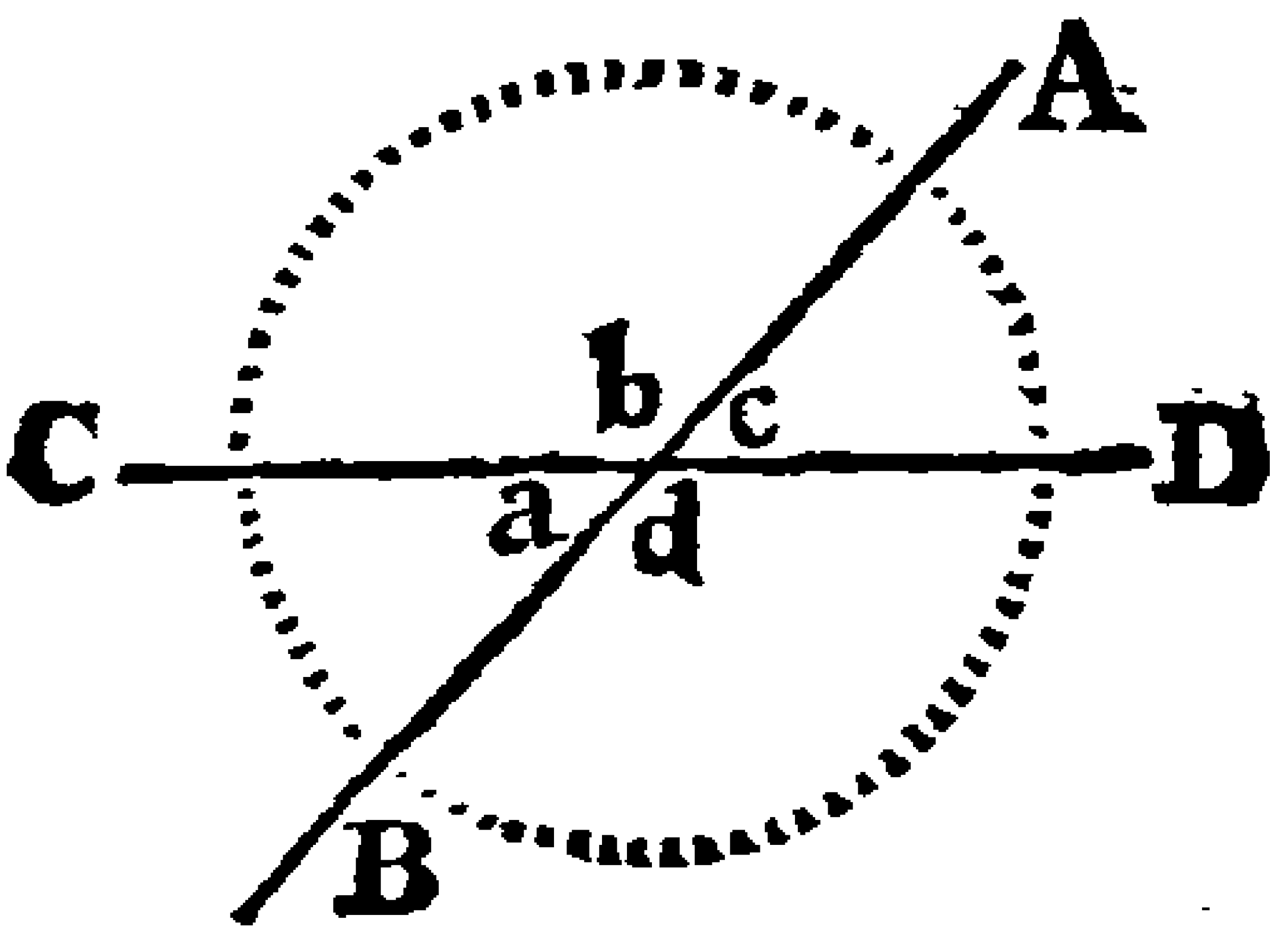
A Right Line A B standing any how upon another Right Line C D, makes the Angles equal to two right Angles.



That is, the Angle A B D and the Angle A B C together are equal to a Semi-circle, or 180 Degrees; equal to two Right Angles.

Theorem 2.

A Right Line A B intersecting another Right Line C D, in the Point *b*, makes the opposite Angles equal.

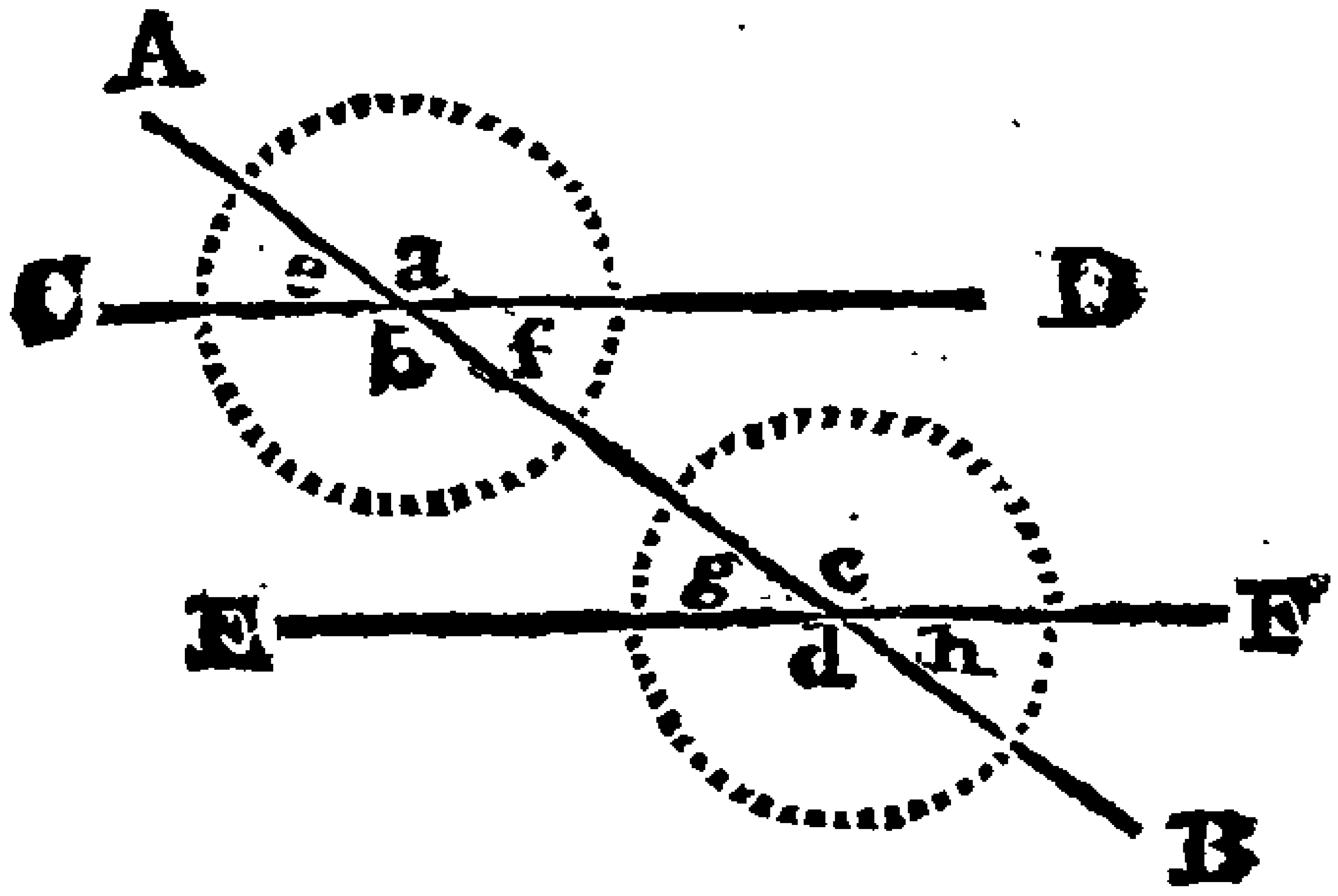


That is, the Angle A *b c* is equal to the Angle B *d* D, and the Angle A *c* D is equal to the Angle C *a* B. The opposite Angles are equal, because the opposite Arches are equal; and all the Angles together make a Circle, or 360 Degrees.

The-

Theorem 3.

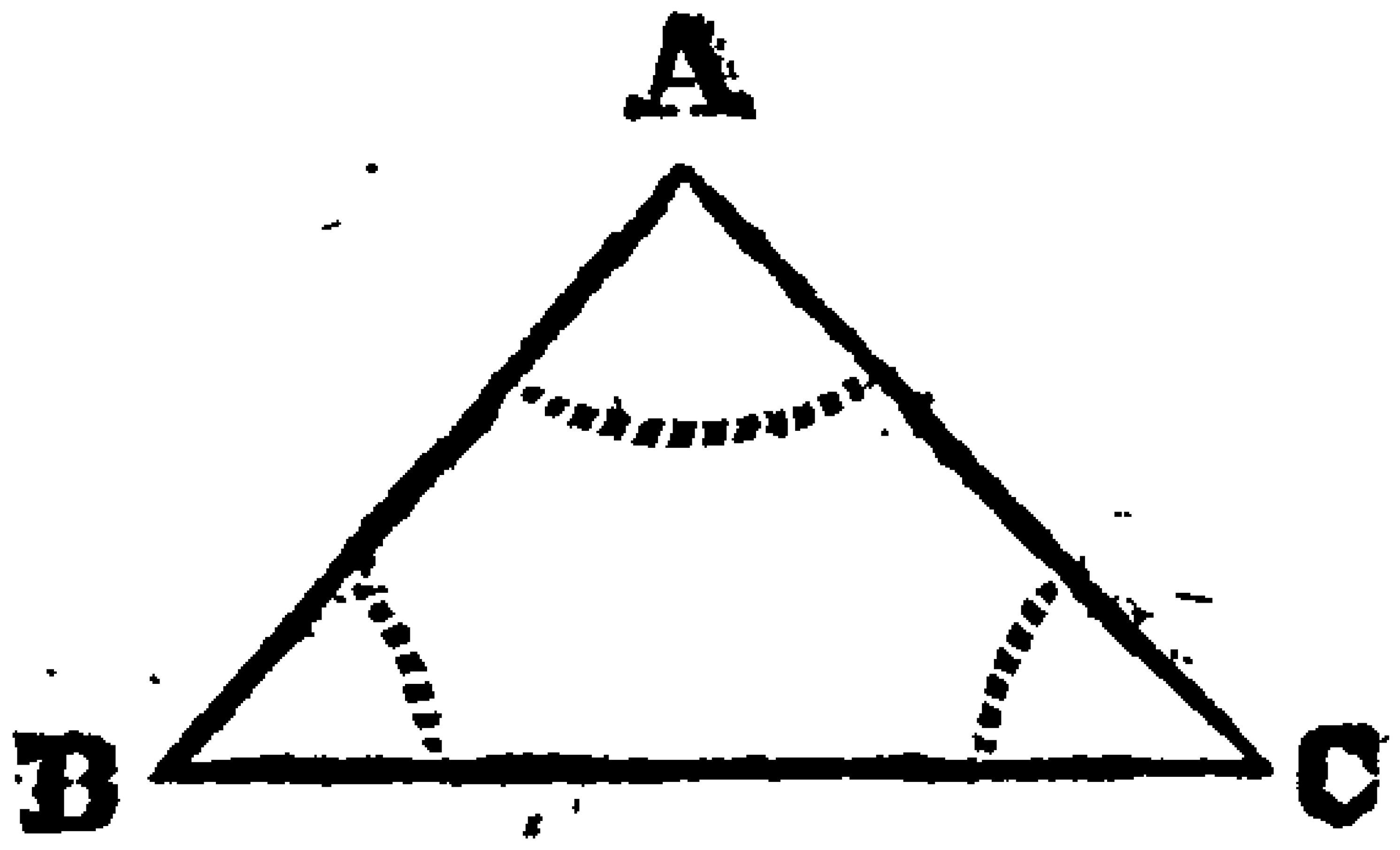
A Right Line A B crossing two Parallel Lines C D and E F, makes the opposite Angles at each Intersection equal.



That is, the Angles *a*, *b*, *c*, *d*, are equal to each other; and the Angles *e*, *f*, *g*, *h*, are also equal; because the Arches intercepted are equal.

Theorem 4.

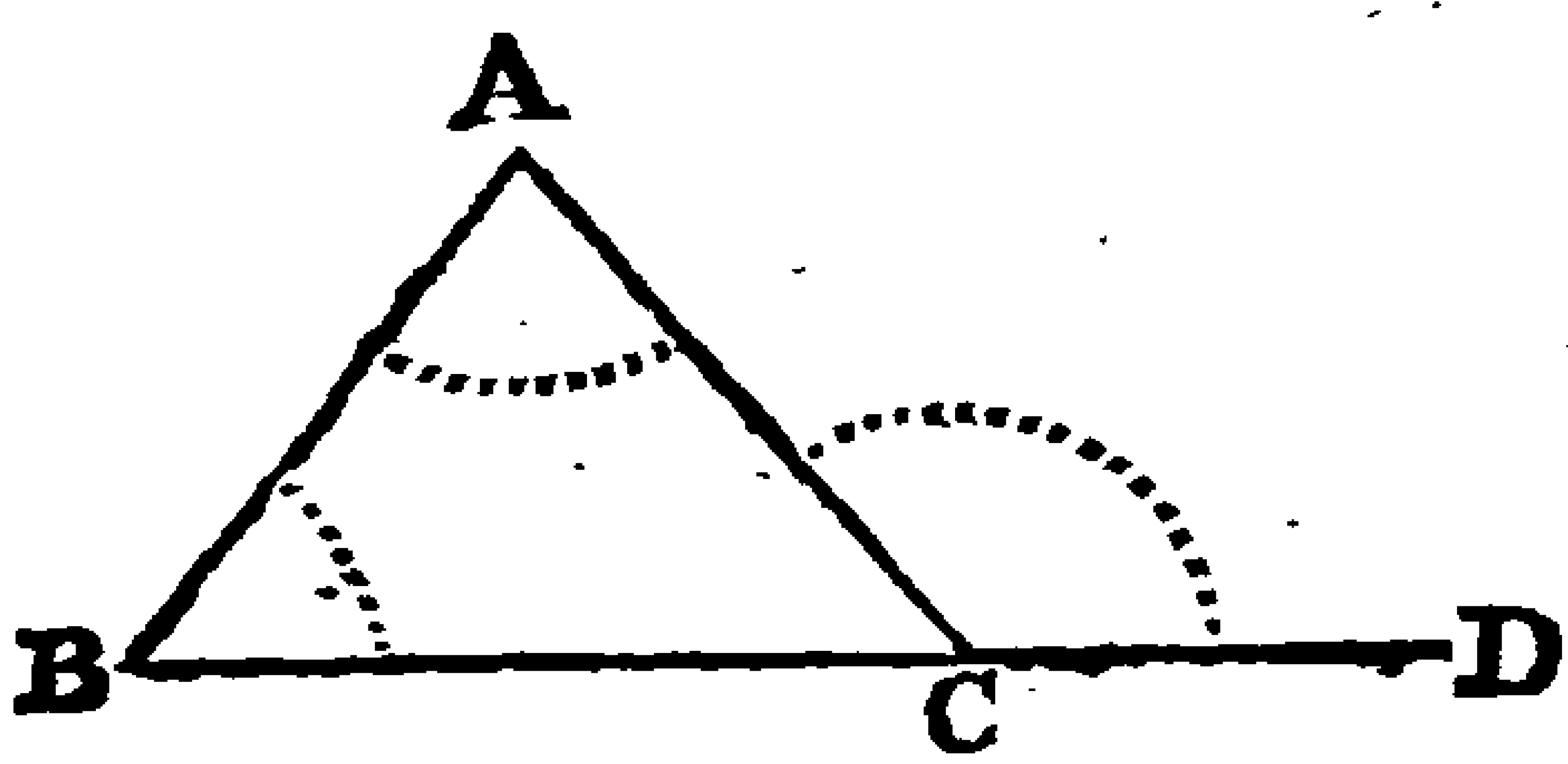
The three Angles of every Right Lined Triangle are equal to two Right Angles.



That is, the Angles A, B, C, taken together, are equal to a Semi-circle, or 180 Degrees, *i. e.* equal to two Right Angles of 90 Degrees each; because the three Arches described on the Angular Points are equal to a Semi-circle.

Theorem 5.

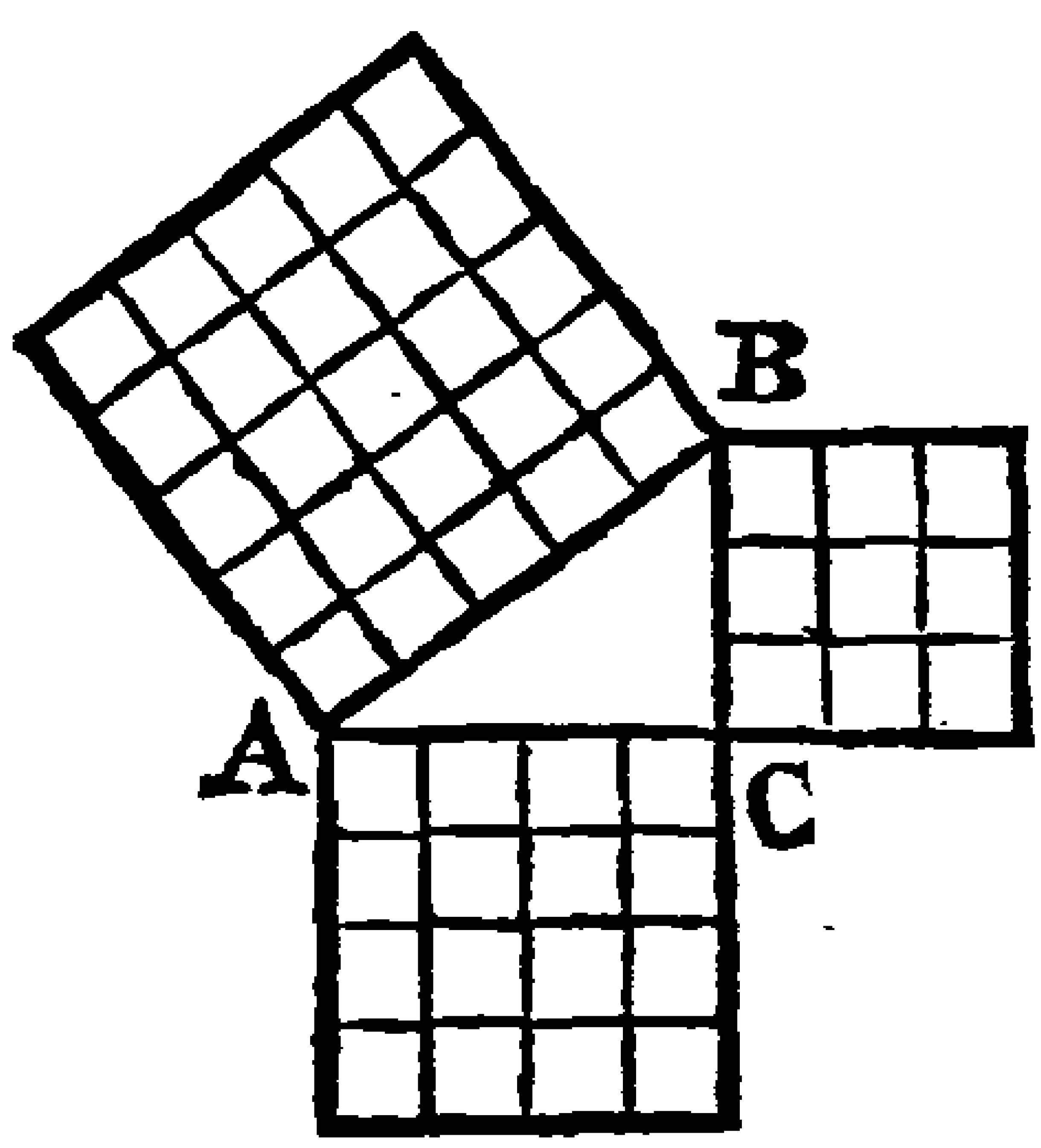
The outward Angle of every Right Lined Triangle is equal to the two inward and oppoſite Angles.



That is, the Angle A C D is equal to the Angles B A C and A B C; becauſe the Arch in the former Angle is equal to both the Arches in the other two Angles.

Theorem 6.

In every Right Angled plain Triangle, the Square of the longeſt Side is equal to the Sum of the Squares of the other two Sides.

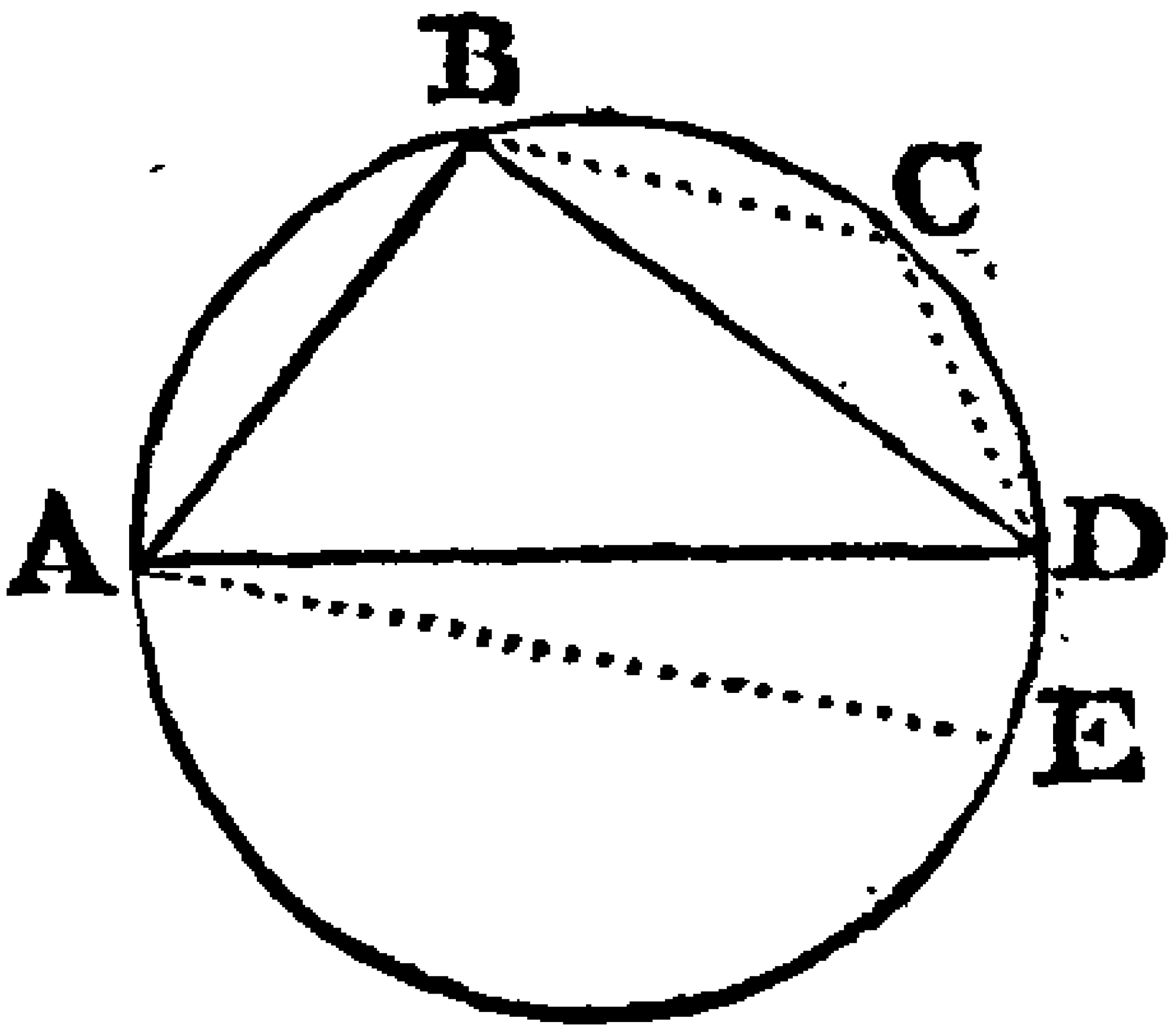


That is, the Square made upon the Line A B is equal to both the Squares made upon the Lines A C and B C, as is evident from an Inſpection of the Figure.

The-

Theorem 7.

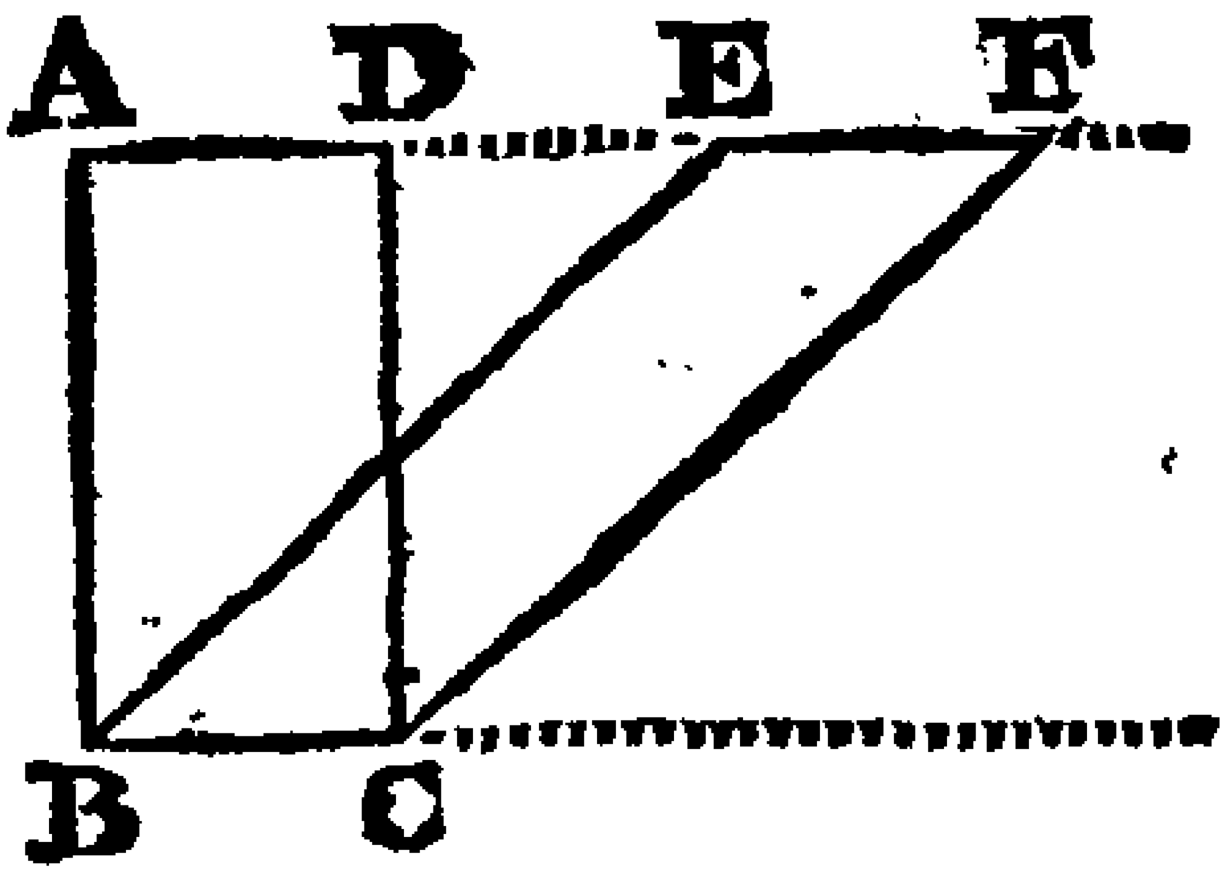
An Angle (or Triangle) made in a Semi-circle, is a Right Angle at the Circumference; if made in a Segment greater than a Semi-circle, the ſaid Angle is Acute; if made in a Segment leſs than a Semi-circle, the Angle is Obtuſe.



That is, the Angle A B D is a Right Angle; but the Angle B A E is leſs than a Right Angle; and the Angle B C D is greater than a Right one.

Theorem 8.

Parallelograms, or long Squares, having the ſame Baſe, and ſtanding between the ſame parallel Lines, are equal to one another.

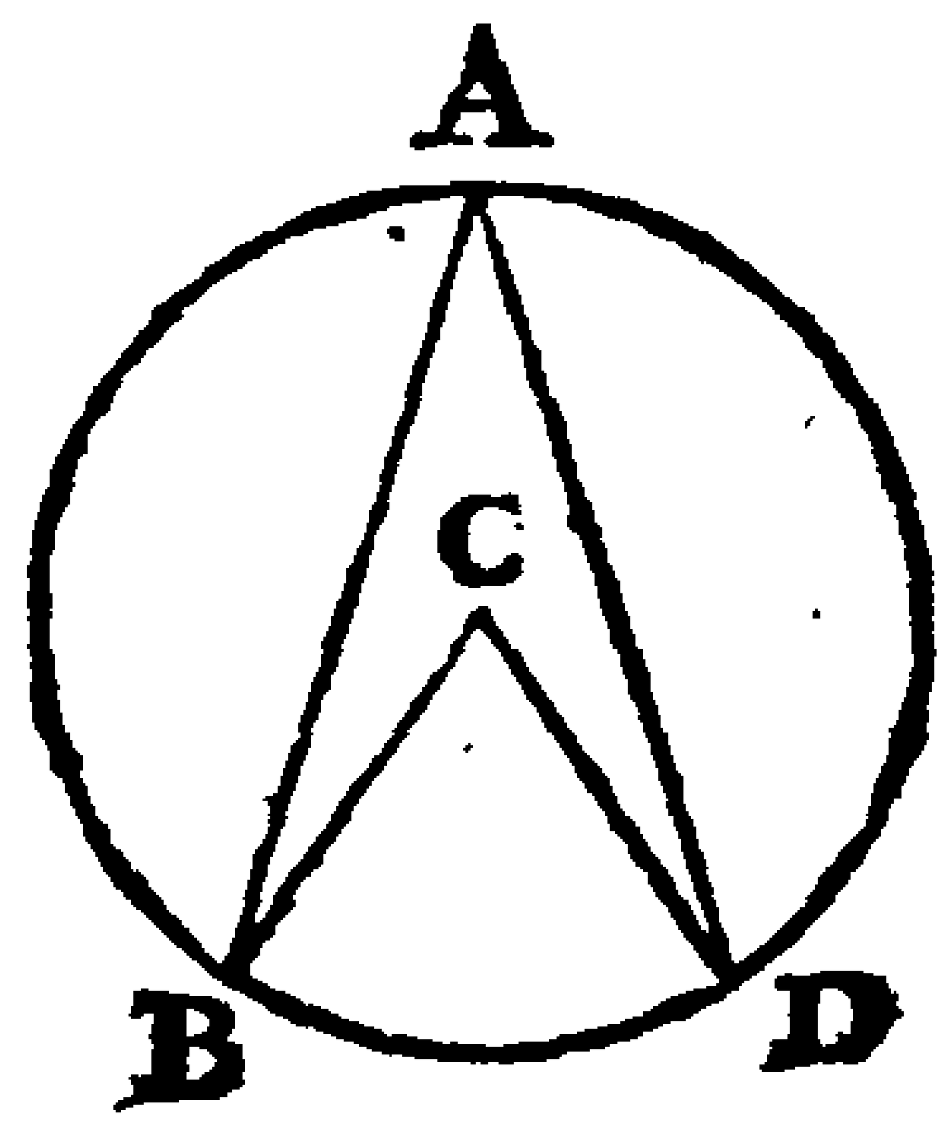


That is, the Parallelogram A B C D is equal to the Parallelogram B E F C.

Note. *Triangles* on the ſame Baſe, and between the ſame Parallels, are equal; becauſe they are Half the circumſcribing Parallelograms. Note alſo, that Triangles and Parallelograms having the ſame Heights are in Proportion to each other as their Baſes.

Theorem 9.

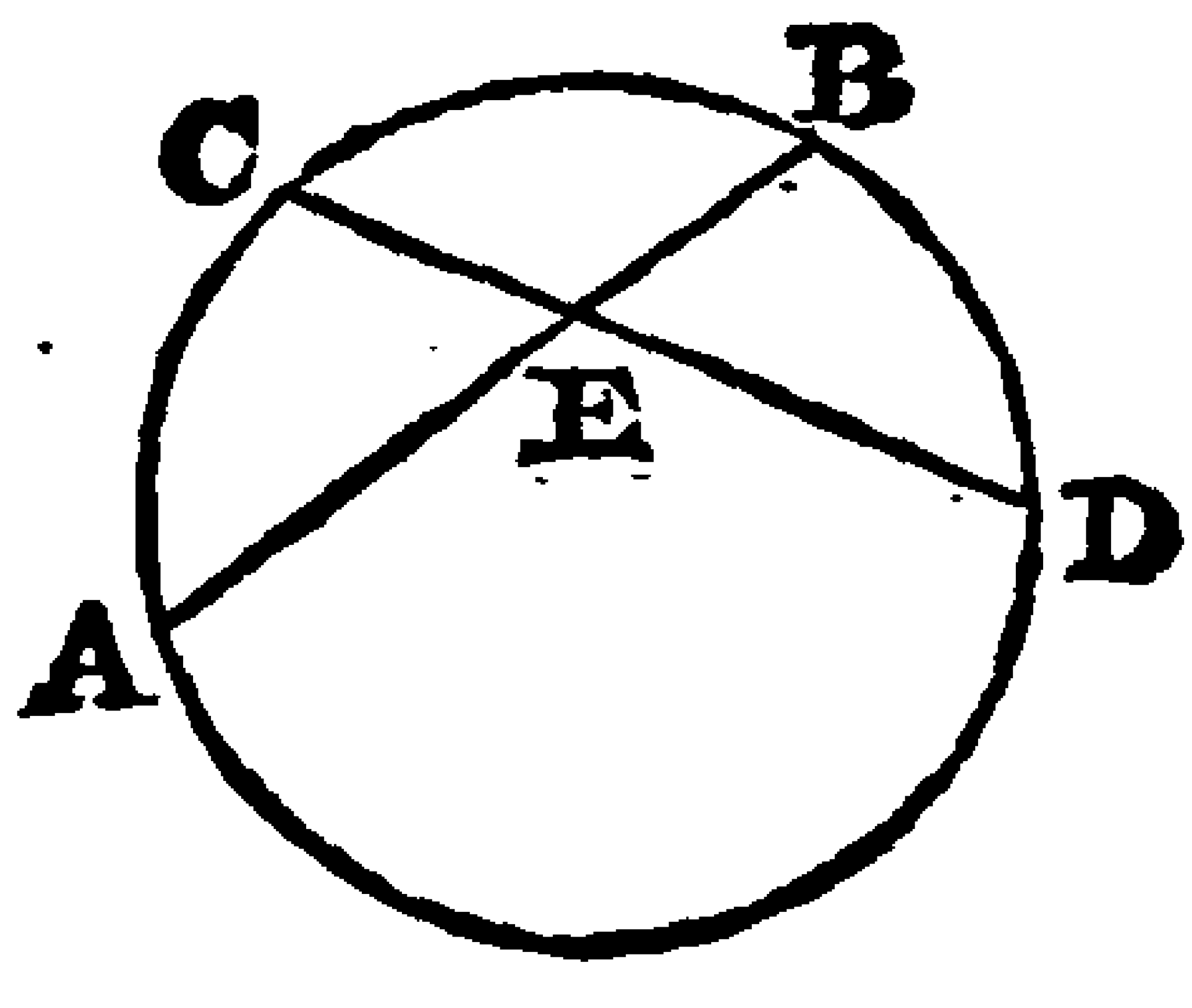
An Angle at the Center of a Circle is double to the Angle at the Circumference, when they ſtand upon, or are bounded by the ſame Arch.



That is, the Angle B C D is double the Angle B A D.

Theorem 10.

If in a Circle two Right Lines be drawn interſecting each other, the Parallelogram, or Product, made of the two Parts of one Line, ſhall be equal to the Parallelogram or Product made of the two Parts of the other Line.

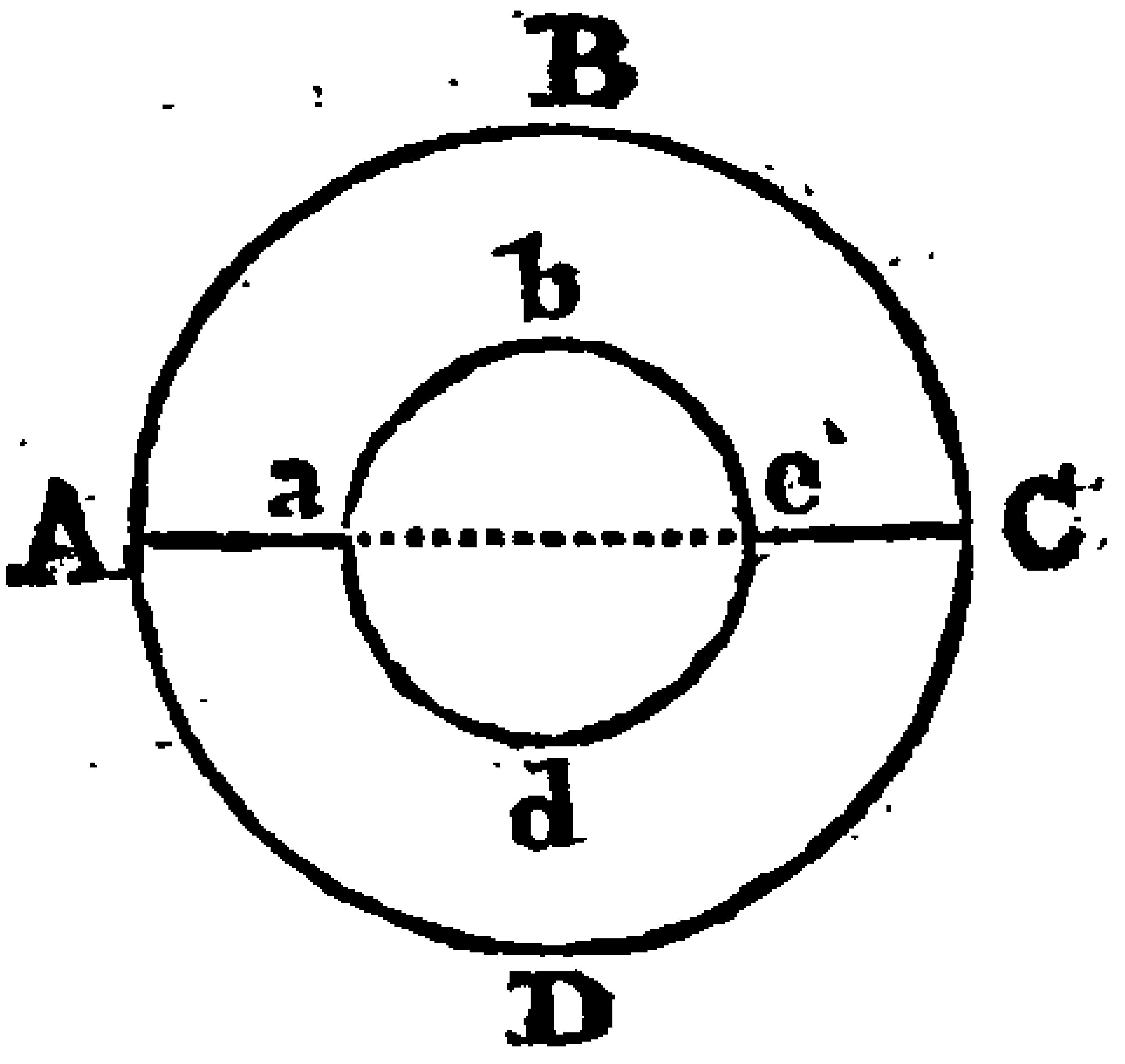


That is, A E multiplied by E B is equal to C E multiplied by E D.

The-

Theorem 11.

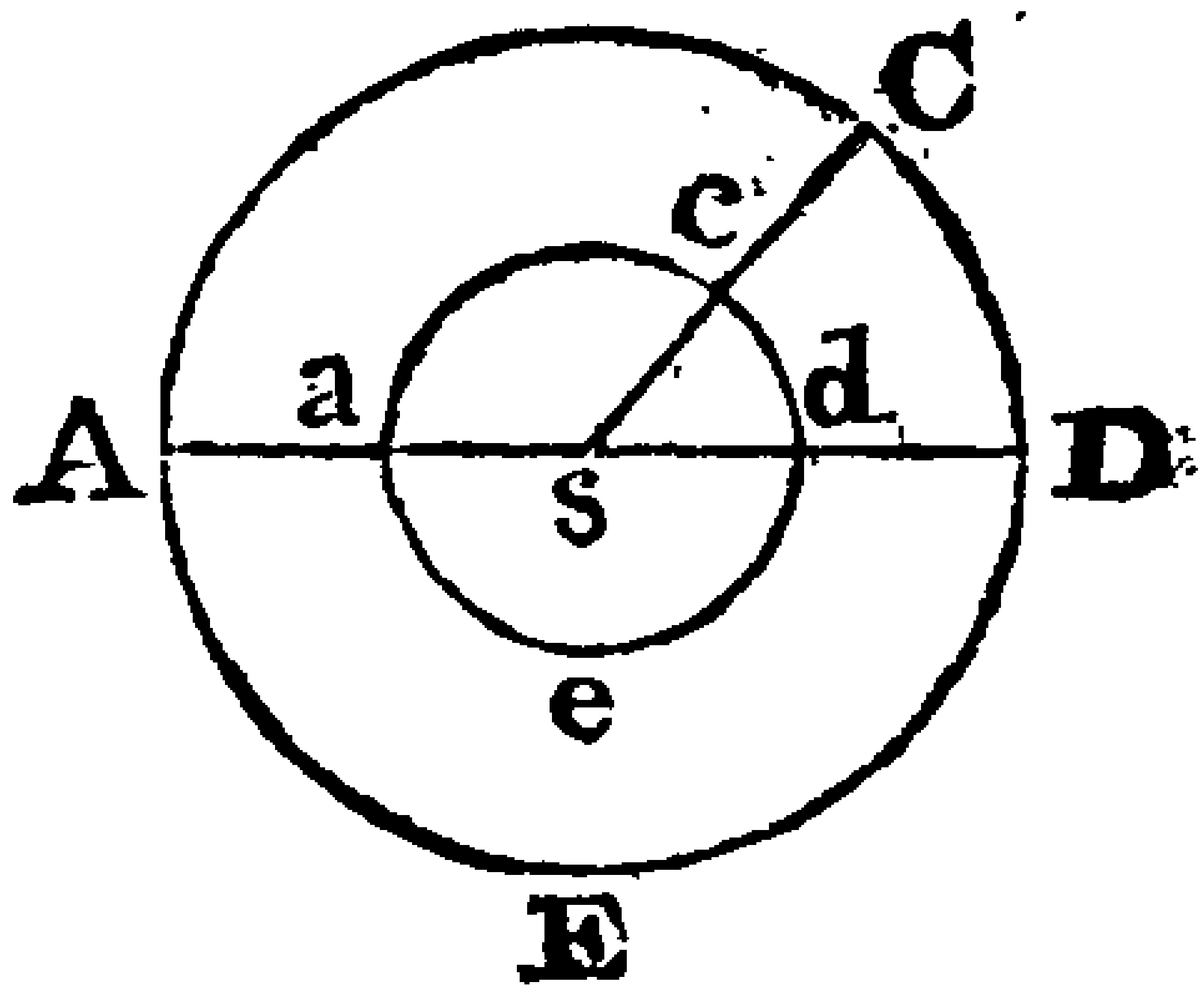
The Circumferences of Circles are in Proportion to their Diameters.



That is, the Diameter of the ſmaller Circle *a c*, is to the Circumference *a b c d*; as the Diameter of the larger Circle A C, is to the Circumference A B C D.

Theorem 12.

In Circles that are concentric, or have the ſame Center, any two Semidiameters will cut off Arches which are proportional and ſimilar to their reſpective Circles.

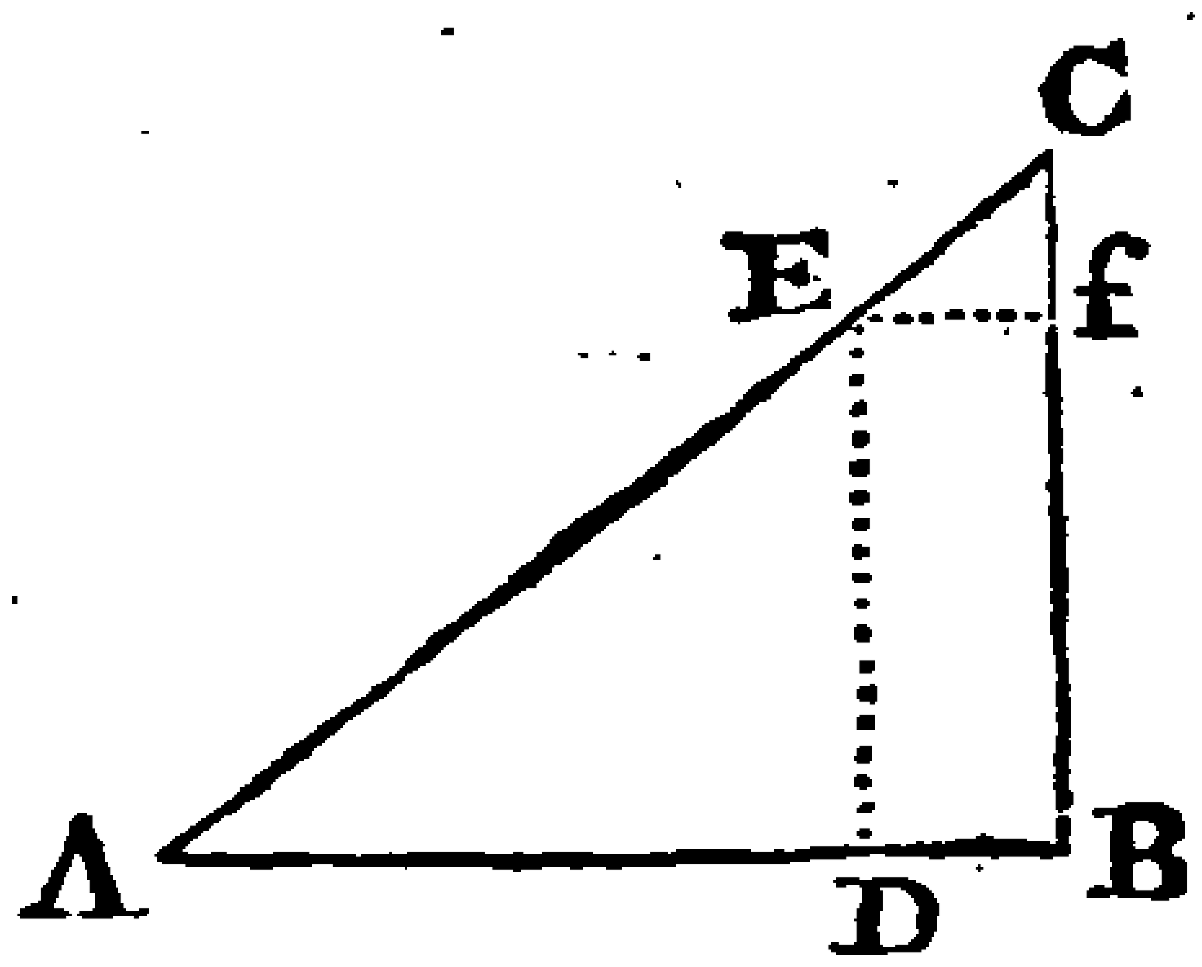


That is, the Circumference of the ſmaller Circle *c a e d*, is to the Arch *c d*; as the Circumference of the larger Circle C A E D, is to the Arch C D.

And it will alſo hold, as *s d* is to S D, ſo is *c d* to C D.

Theorem 13.

If in a Triangle A B C, there be drawn a Line D E parallel to C B, the Angles of the two Triangles being the ſame, the Sides of the one will be proportional to the Sides of the other.

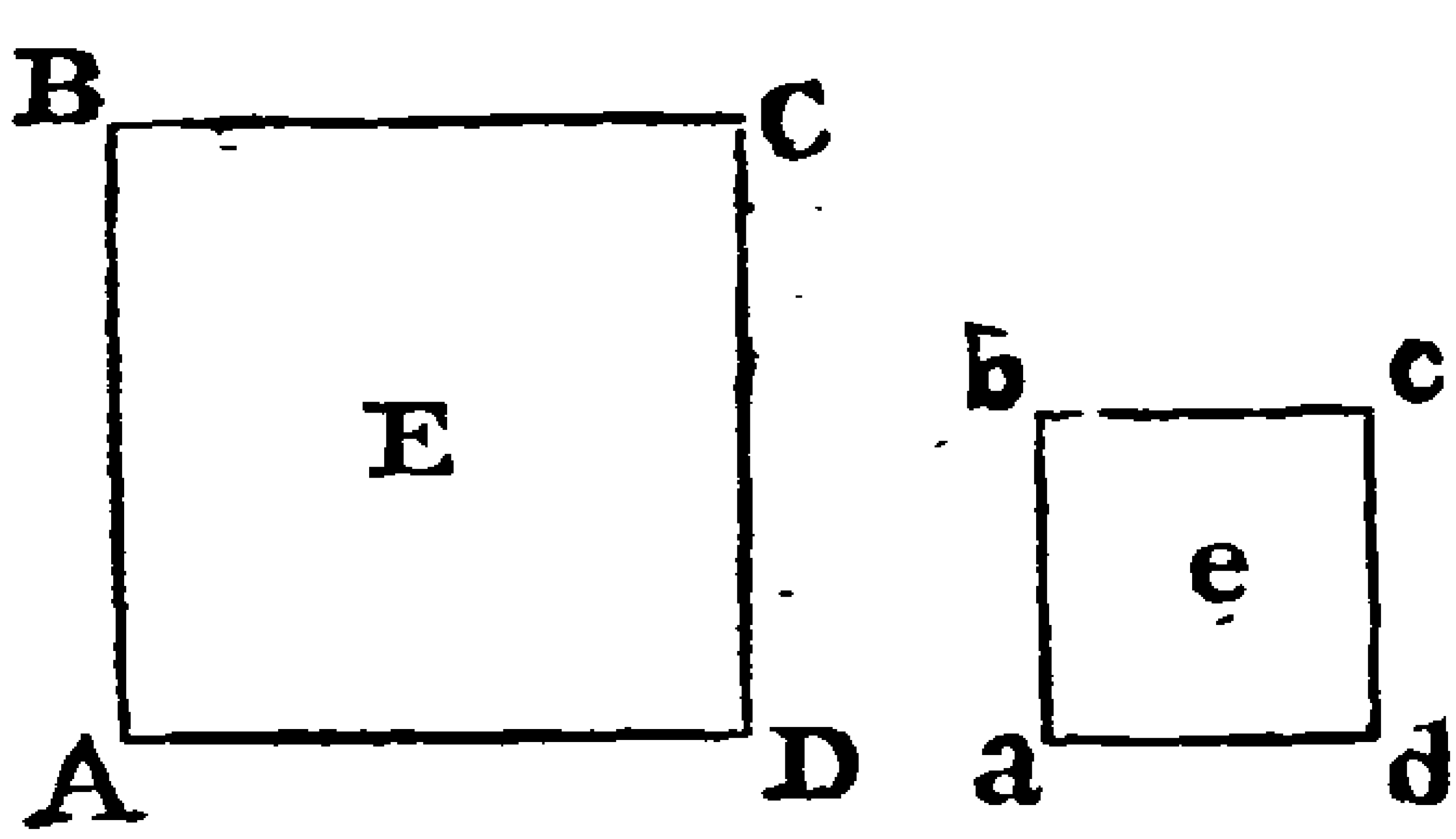


That is, the Side A B, is to the Side A D; as the Side B C, is to the Side D E.

And further, A B : B C :: A D : D E
A D : A E :: A B : A C
A D : D E :: E *f* : *f c*

Theorem 14.

All ſimilar or like Superficies are in a duplicate Proportion, or as the Squares of their like Sides.

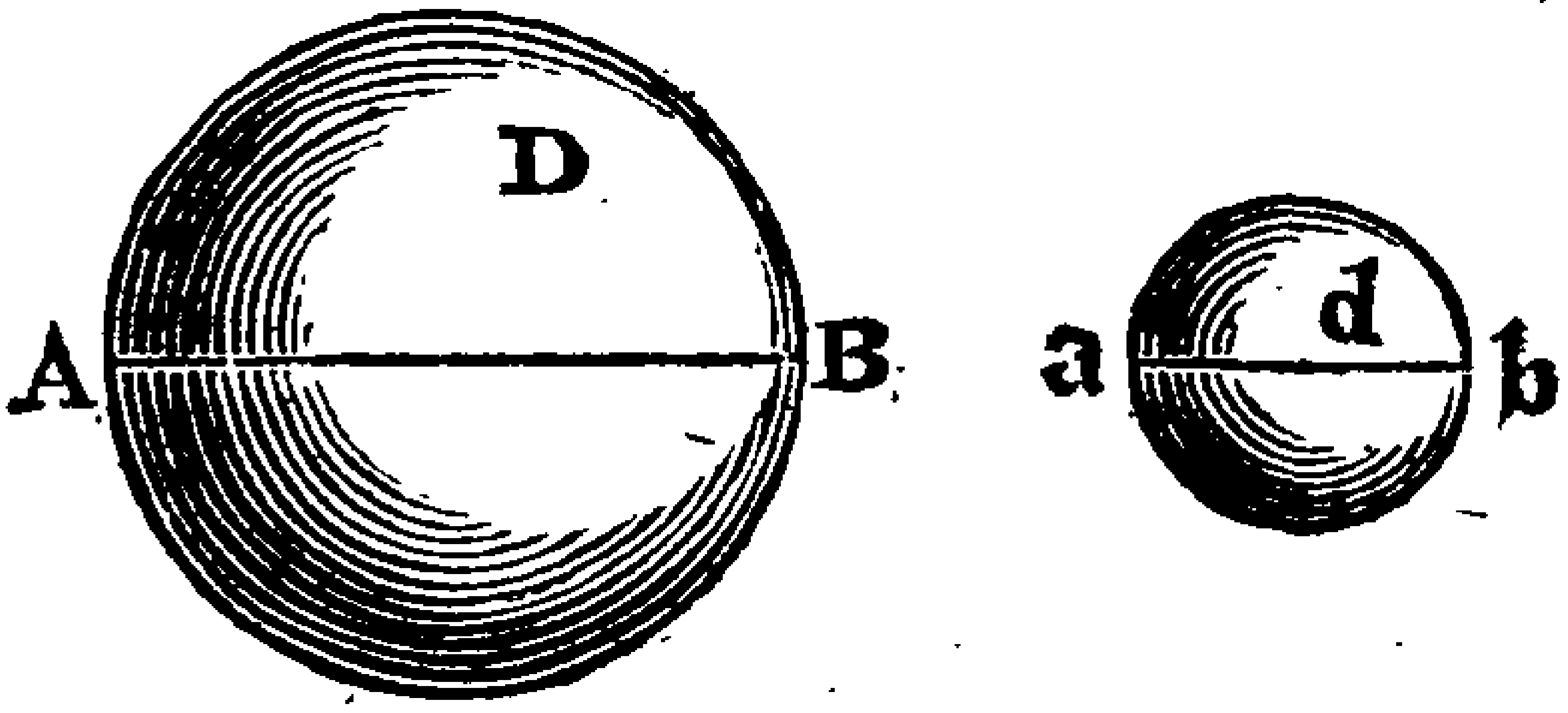


That is, the Area of the Superficies *e*, is to the Superficies E; as the Square of the Side *a d*, is to the Square of the Side A D.

The-

Theorem 15.

Similar, or like Solids are to each other in a triplicate Proportion, or as the Cubes of their like Sides.



That is, the Solidity of the ſmaller Body *d*, is to the Solidity of the greater Body D; as the Diameter *a b* cubed, is to the Diameter A B cubed.

Many other Theorems might be added, but theſe will be ſufficient for the young Geometer at preſent. The Reſt, as they would better appear in a Treatiſe of Trigonometry, we have reſerved for a Place in that Work.

Note. That the *Side* upon which any Figure ſtands is called its *Baſe*.—That a Line drawn from its *Top*, perpendicular to the *Baſe*, is called its *Altitude* or *Height*.—That two *Right Lines* cannot include any *Space*, or form any *Superficies* whatever.

The Explanation of ſuch Characters as are generally uſed in the Solution of the following Geometrical Problems.

SIGNS & MARKS.	EXPLANATIONS.
$+$ plus, or more.	The Sign of *Addition*; as $5+2=7$; that is, 5 added to 2 is equal to 7.
$-$ minus, or leſs.	The Sign of *Subtraction*; as $9-4=5$; that is, 9 leſſened by 4 is equal to 5.
$\times$ multiply by	The Sign of *Multiplication*; as $6\times 8=48$; that is, 6 multiplied by 8 is equal to 48.
$\div$ divided by	The Sign of *Diviſion*; as $12\div 4=3$; that is, 12 divided by 4 is equal to 3.
$=$ equal to	The Sign of *Equality*; as 16 ozs. $=$ 1 lb. that is, 16 Ounces are equal to 1 Pound.
$::$ Proportion.	The Sign of *Proportion*; as $3 : 6 :: 8 : 16$; that is, as 3 is to 6, ſo is 8 to 16.
$\frac{365}{52}$ Fraction.	Numbers placed like a Fraction do alſo denote Diviſion; the upper Number being the Dividend, and the lower the Diviſor.
$12 - 4 + 2 = 10$	Shews that the Difference between 12 and 4 added to 2, is equal to 10.

$10 - \overline{2+3} = 5$	Signifies, that the Sum of 2 and 3 taken from 10 is equal to 5.
$\sqrt{}^2$ Square Root.	This Mark being prefixed to any Number, ſignifies that the Square Root of that Number is to be extracted.
$\sqrt{}^3$ Cube Root.	Signifies, that the Cube Root of that Number is to be extracted.
$\square$ Square.	This Sign ſignifies, that the Number to which it is prefixed muſt be ſquared, or raiſed to the ſecond Power.
Cube.	Shews the Number following muſt be cubed, or raiſed to the third Power.
Q. E. D.	Signifies,—which was to be demonſtrated.
Q. E. I.	——— which was to be found.
Q. E. F.	——— which was to be done.

☞ The Application of Algebra and Fluxions to the Solution of Problems in Geometry muſt be deferred till the Learner is acquainted with the firſt Principles of that Science.

PLANO-

PLANOMETRY;

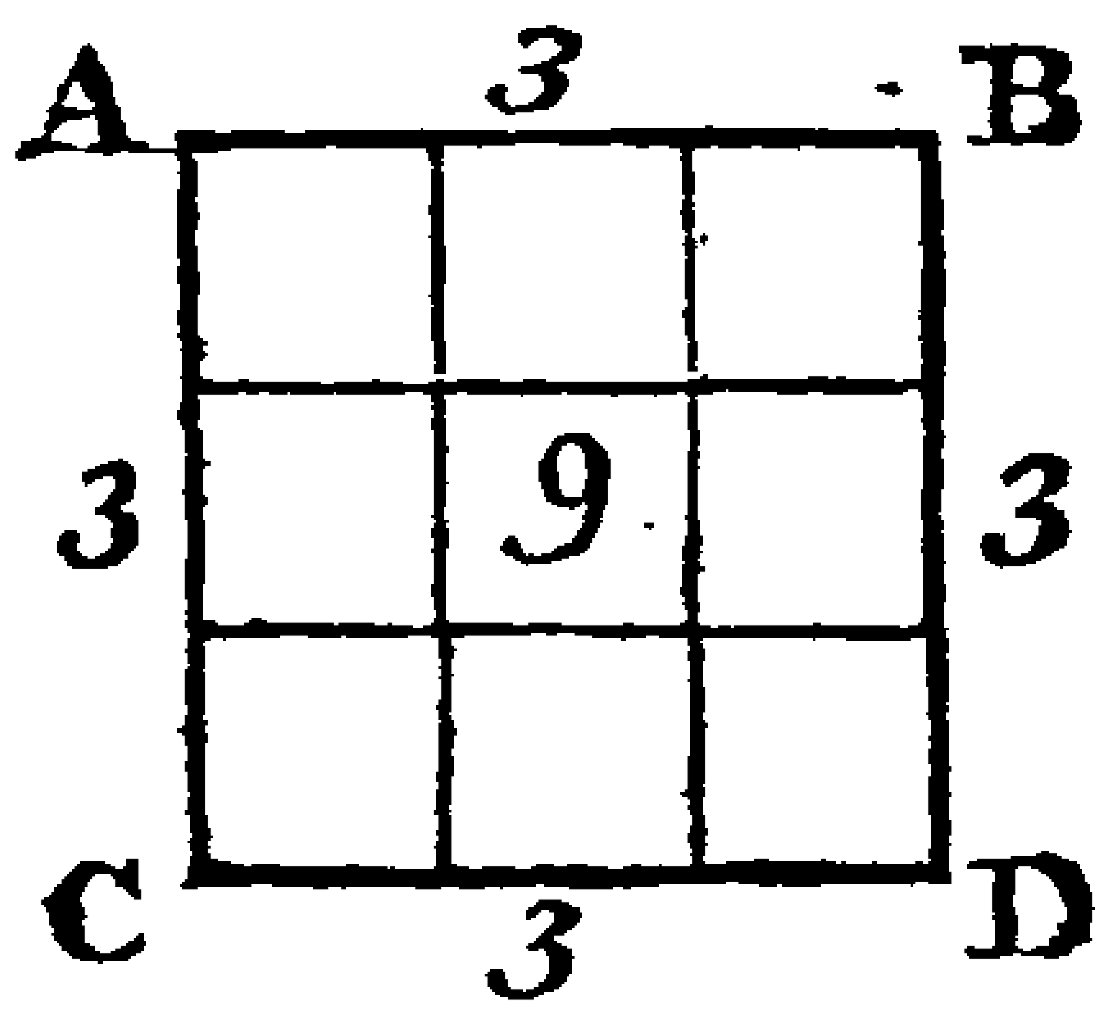
Or the Meaſuring of

PLAIN SURFACES.

A Surface or Superficies is that which hath Length and Breadth, without Thickneſs. *

The *Meaſure*, the *Area*, or the *Content* of any ſuperficial Figure is ſaid to be known, when we know how many leſs Squares, as ſuppoſe of *Feet*, *Inches*, *Yards*, *Perches*, &c. are contained within it.

Thus, ſuppoſe in the Figure A B C D each of its Sides is found to be 3 equal Parts, then it is evident the Number of little Squares of the ſame Kind contained in it will be 9. For, dividing the Sides into the Number of Parts they contain, and drawing Lines through from Side to Side, the Number of little Squares formed within, will be the Area of the larger Figure.



The *Area* or *Content* is always of the ſame Denomination with the Dimenſions; that is, if the Meaſure be taken in Feet, the Content will be Feet; if taken in Inches, the Content will be Inches; and ſo of any other.

* If a Superficies be raiſed up, it is ſaid to be *convex*; if it be hollow, it is called *concave*; and if it be flat and even, it is called a *Plane*, or *plain Superficies*.

Problem 1.

To Meaſure a *Square*; that is, to find the Superficial Content or Area of a Square.

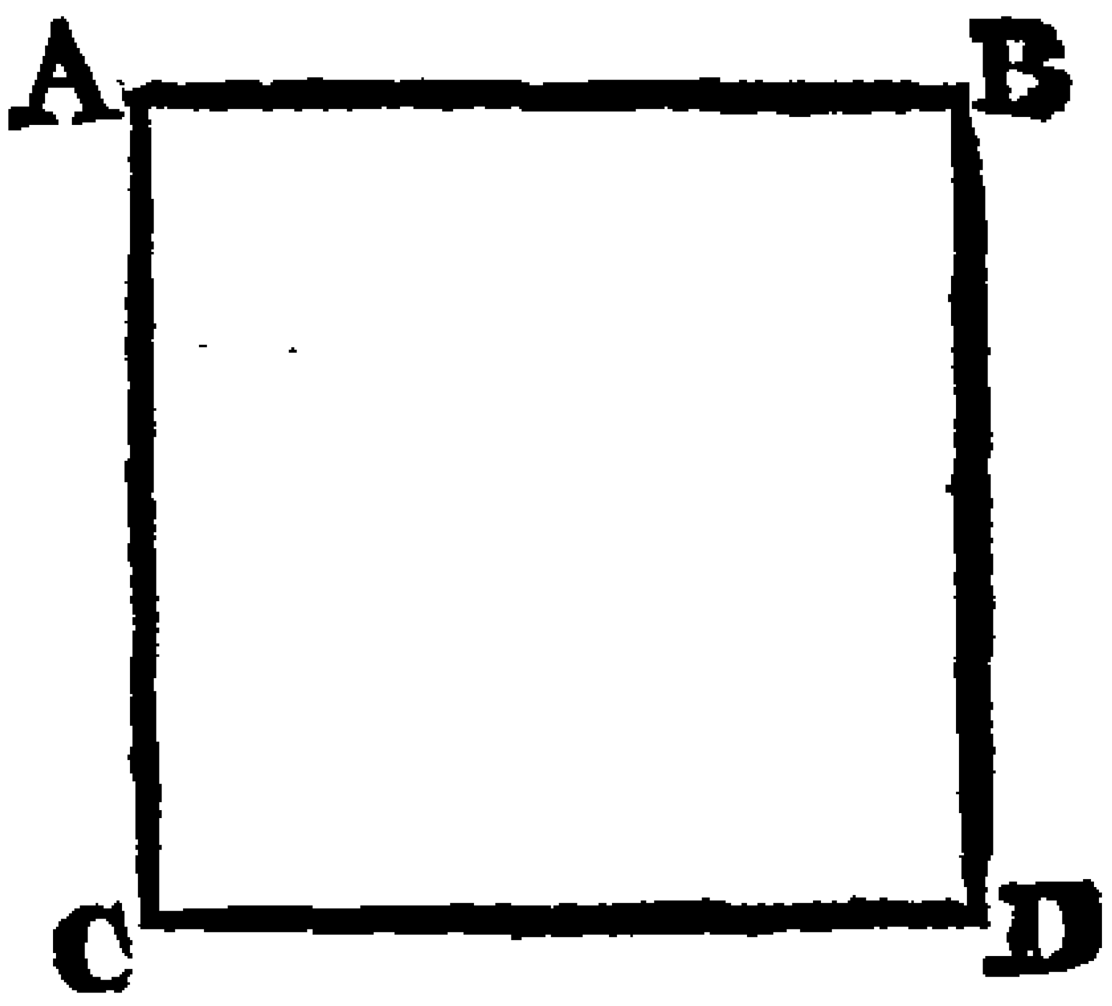
Definition. A *Square* hath four equal Sides and four Right Angles; as A B C D, in the Figure below.

Rule. *

Multiply the given Side by itſelf, and the Product is the *Area* required.

Example.

Suppoſe the Side A B of the Square A B C D to be 16.2 Inches, what is the Area?



Operation. $16.2 \times 16.2 = 262.44$, the Area in Inches.

* The Reaſon of this Rule is evident from a Sight of the laſt Figure divided into little Squares.

Pro-

Problem 2.

To find the Area of a *Parallelogram*, or Oblong Square.

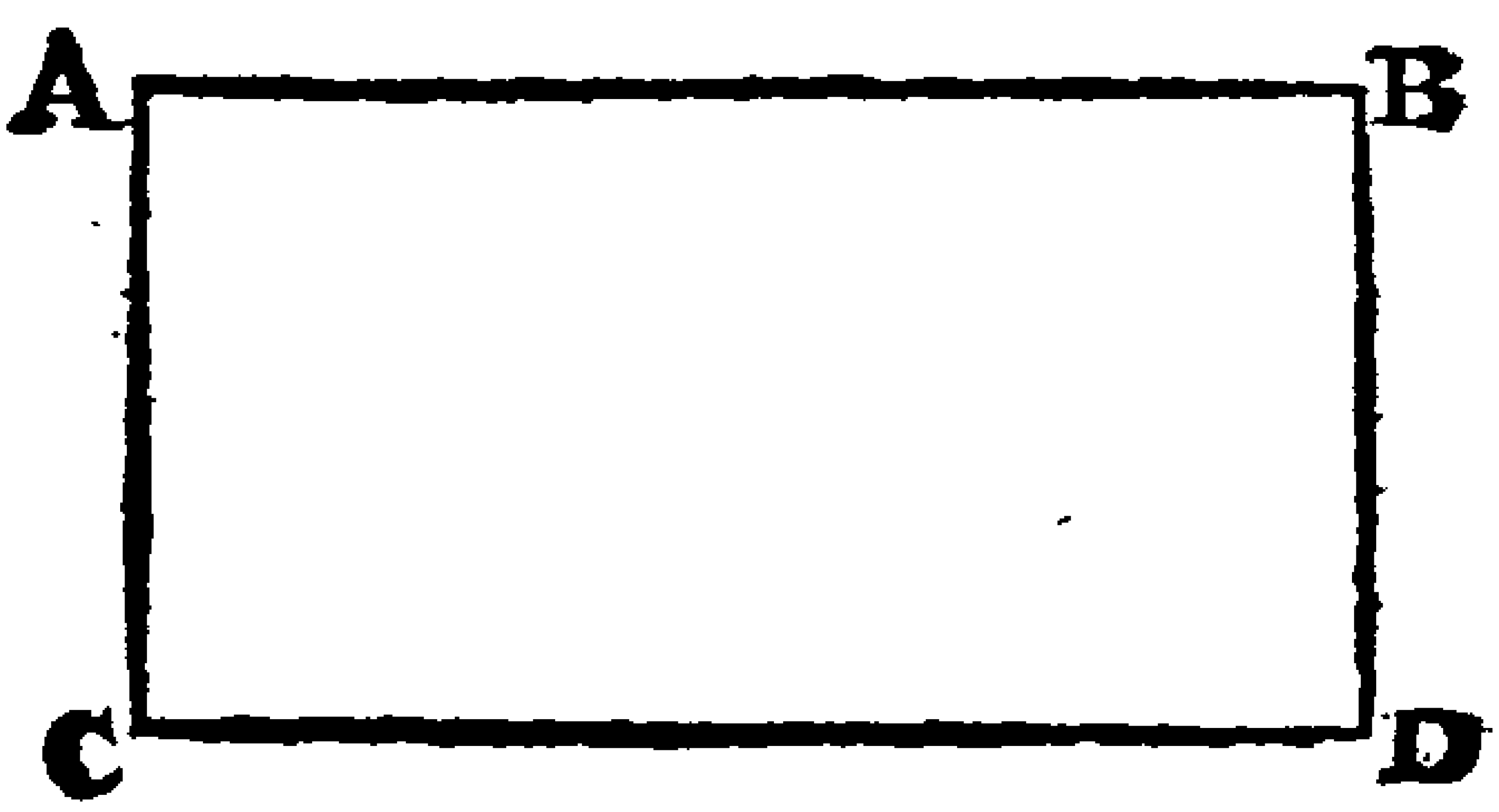
Def. A *Parallelogram* hath four Right Angles; and its opposite Sides equal and parallel.

Rule. *

Multiply the Length by the Breadth, and the Product gives the *Area*.

Example.

Let A B C D be a *Parallelogram*, whose Length A B is 25.5 Inches, and its Breadth B D 12.75 Inches; what is the Area?



Operation. $25.5 \times 12.75 = 325.125$, the Area in Inches.

* The Reason of this Rule appears from the 1st Figure, because this is only a long Square.

Pro-

Problem 3.

To find the Area of a *Rhombus*.

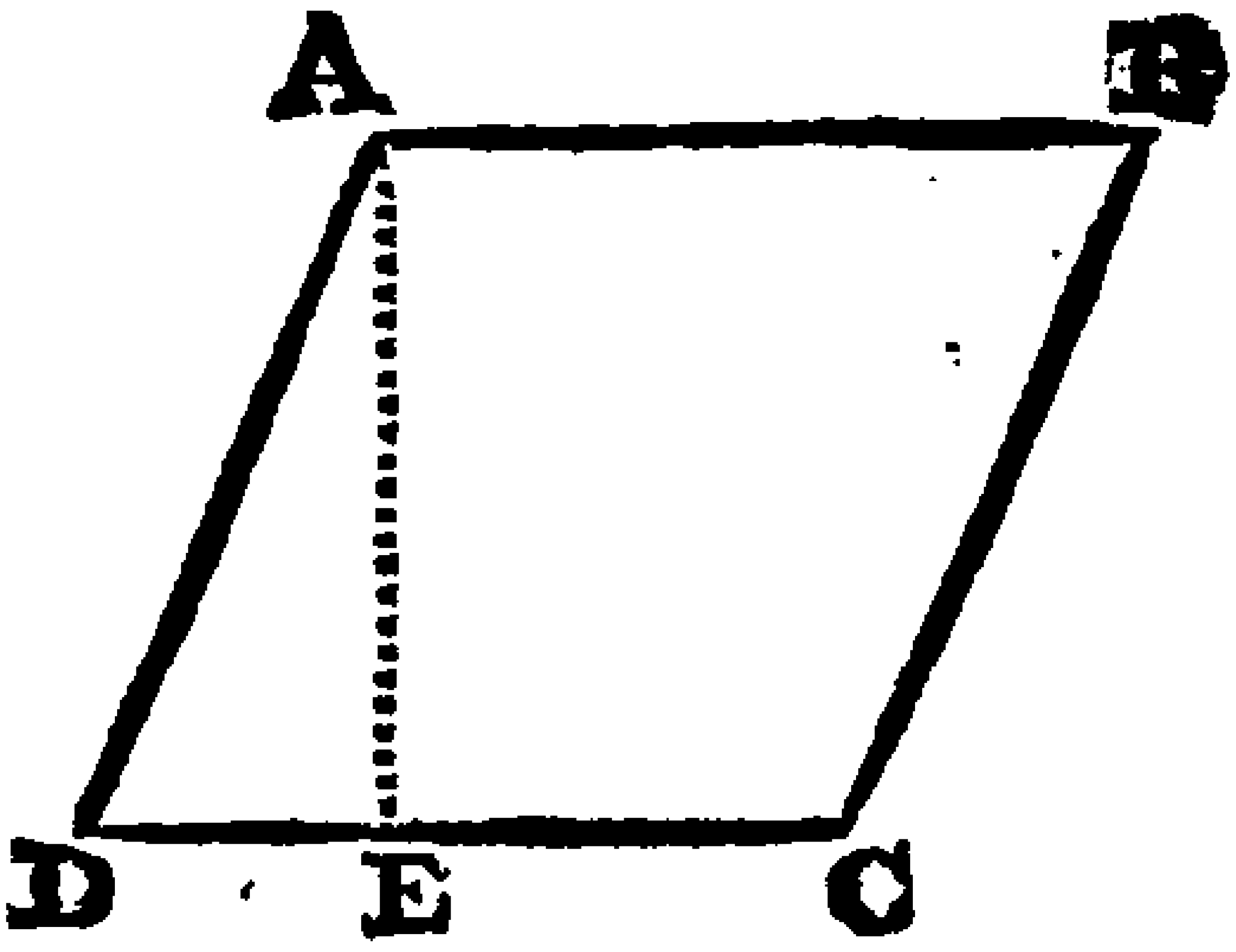
Def. A *Rhombus* hath four Sides all equal, but no Right Angle; it is in Shape like a Diamond Pane of Glaſs.

Rule. *

Multiply the Baſe by the Perpendicular Height, and the Product is the *Area*.

Example.

Let A B C D be a Quarry of Glaſs, or Marble; and ſuppoſe the Baſe D C be 12.5 Inches; and the Perpendicular A E 9.25 Inches; what is the *Area*?



Operation. $12.5 \times 9.25 = 115.625$ Inches; the Area required.

* Every *Rhombus* is equal to a *Parallelogram* of the ſame Baſe and Altitude.

Pro-

Problem 4.

To find the Area of a *Rhomboides*.

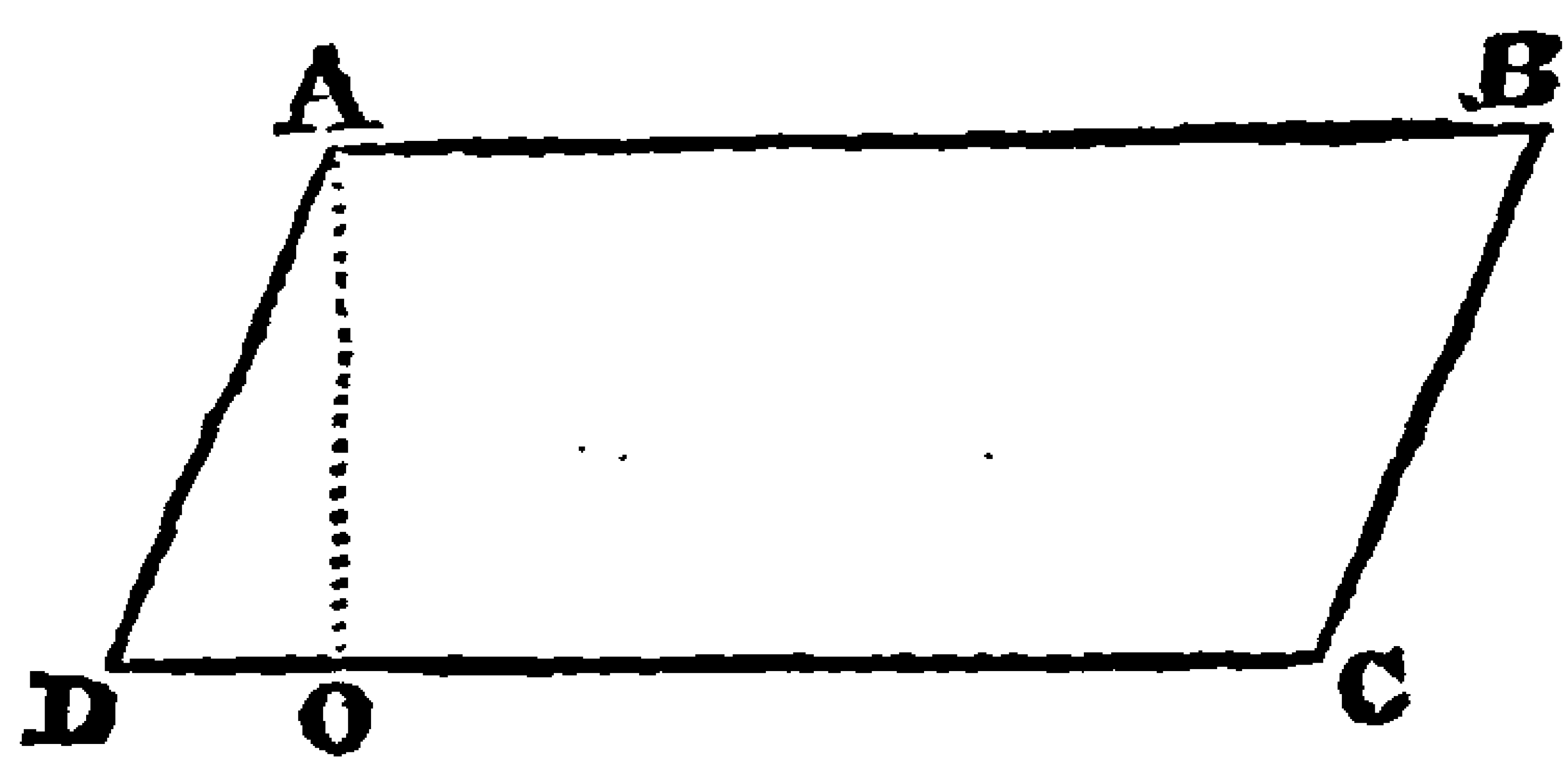
Def. A *Rhomboides* hath its two Sides equal and parallel, but no Right Angle. It is a long Square pushed aside.

Rule. *

Multiply the longer Side by the Perpendicular Height (or Breadth), and the Product is the Area.

Example.

Let A B C D be a Rhomboides, whose longer Side A B, or C D, is 20.5 Inches, and the Breadth A O equal to 13.5 Inches, what is the Area?



Operation. $20.5 \times 13.5 = 276.75$, the Area required.

* Every *Rhomboides* is equal to a *Parallelogram* of the same Base and Altitude.

Pro-

Problem 5.

To find the Area of a *Triangle*.

Def. A *Triangle* is a Figure bounded by three Right Lines.

Note. If a *Triangle* hath one of its Angles a true Square, or just 90 Degrees, it is called a *Right Angled Triangle*; if it hath no Right Angle, it is called an *Oblique Triangle*.

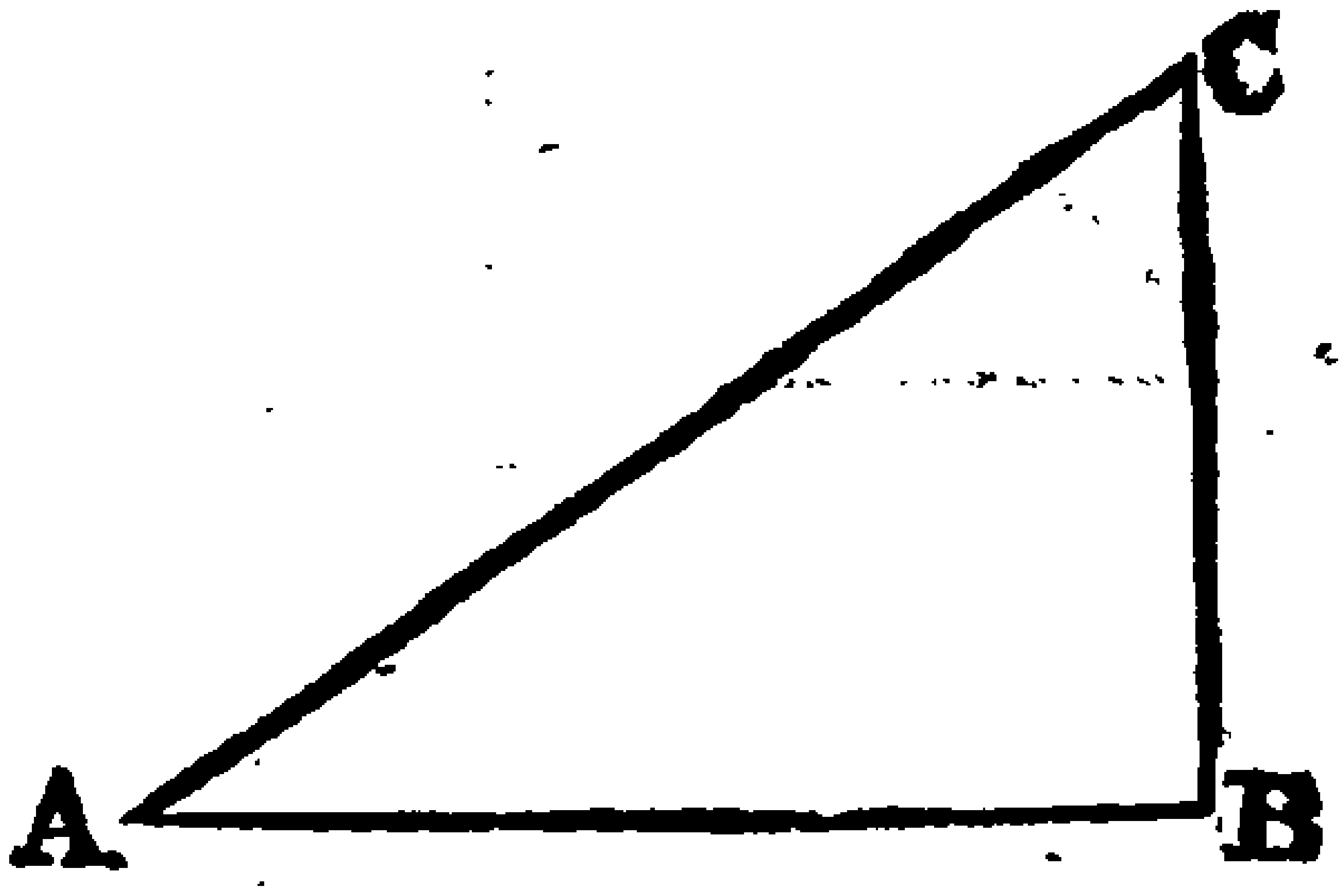
To Measure a *Right Angled Triangle*.

Rule. *

Multiply one of the Legs forming the Right Angle by Half the other; and the Product is the Area.

Example.

Let A B C be a Right Angled Triangle, whose Base A B is 14.1 Inches, and the Perpendicular B C is 12 Inches; what is the Area?



Operation. $14.1 \times 6 = 84.6$ Inches, the Area.

* Every Right Angled Triangle is equal to Half the Parallelogram of the same Base and Altitude.

Pro-

Problem 6.

To find the Area of an *Oblique Triangle*.

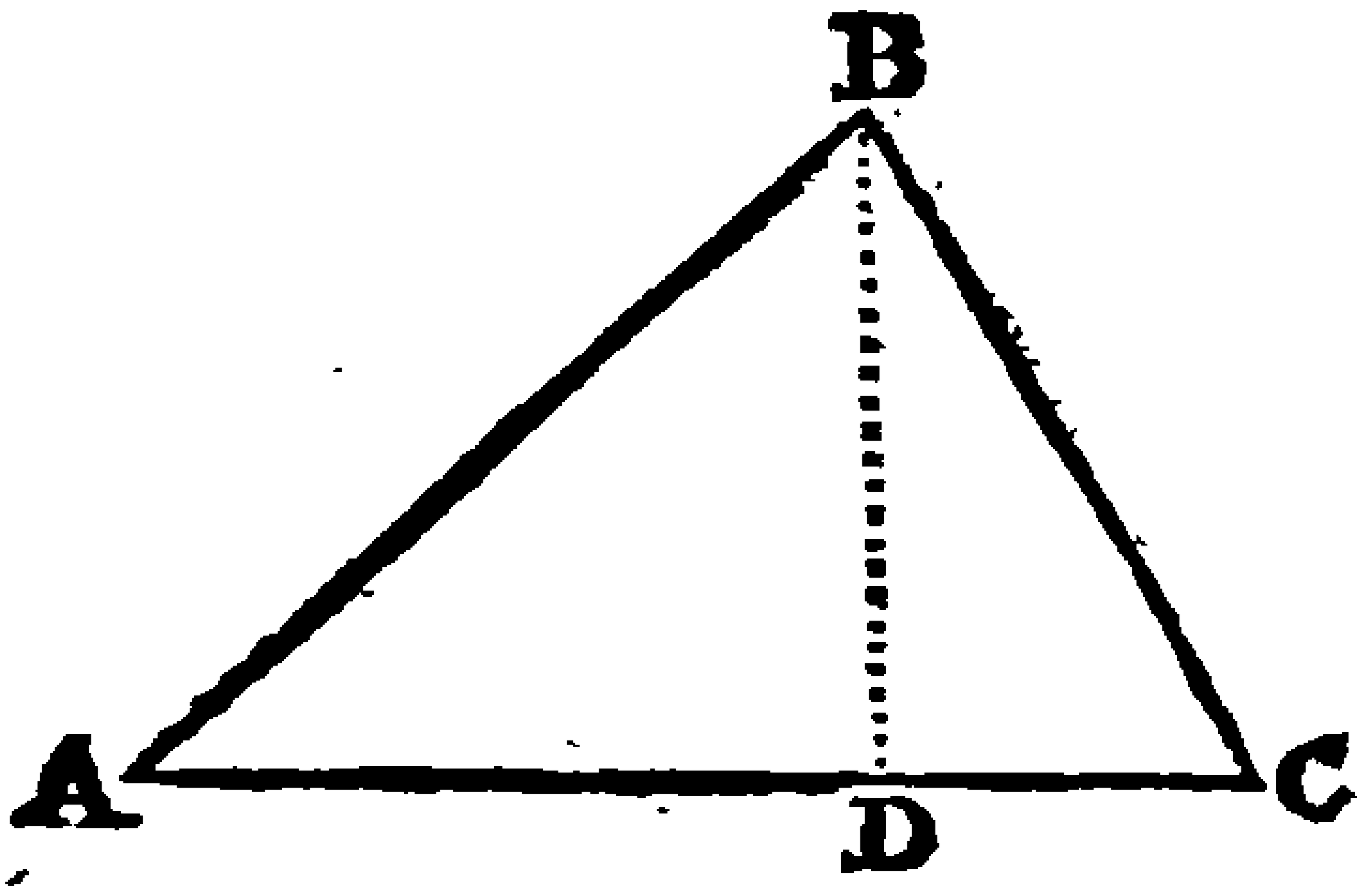
Def. If a Triangle hath no Right Angle, it is called an *Oblique Triangle*.

Rule.*

Multiply the longeſt Side by Half the Perpendicular let fall from the Angle oppoſite the ſame, and the Product is the Content.

Example.

Suppoſe A B C be a Triangle, whoſe Baſe A C is 38.6 Inches, and the Perpendicular Height B D is 30.2 Inches; what is the Area?



Operation. $38.6 \times 15.1 = 582.86$ Inches, the Area.

* The Reaſon is becauſe every oblique Triangle is equal to *Half* its Circumſcribing Parallelogram, as in the laſt Problem.

Pro-

Problem 7.

To find the Area of a *Trapezium*.

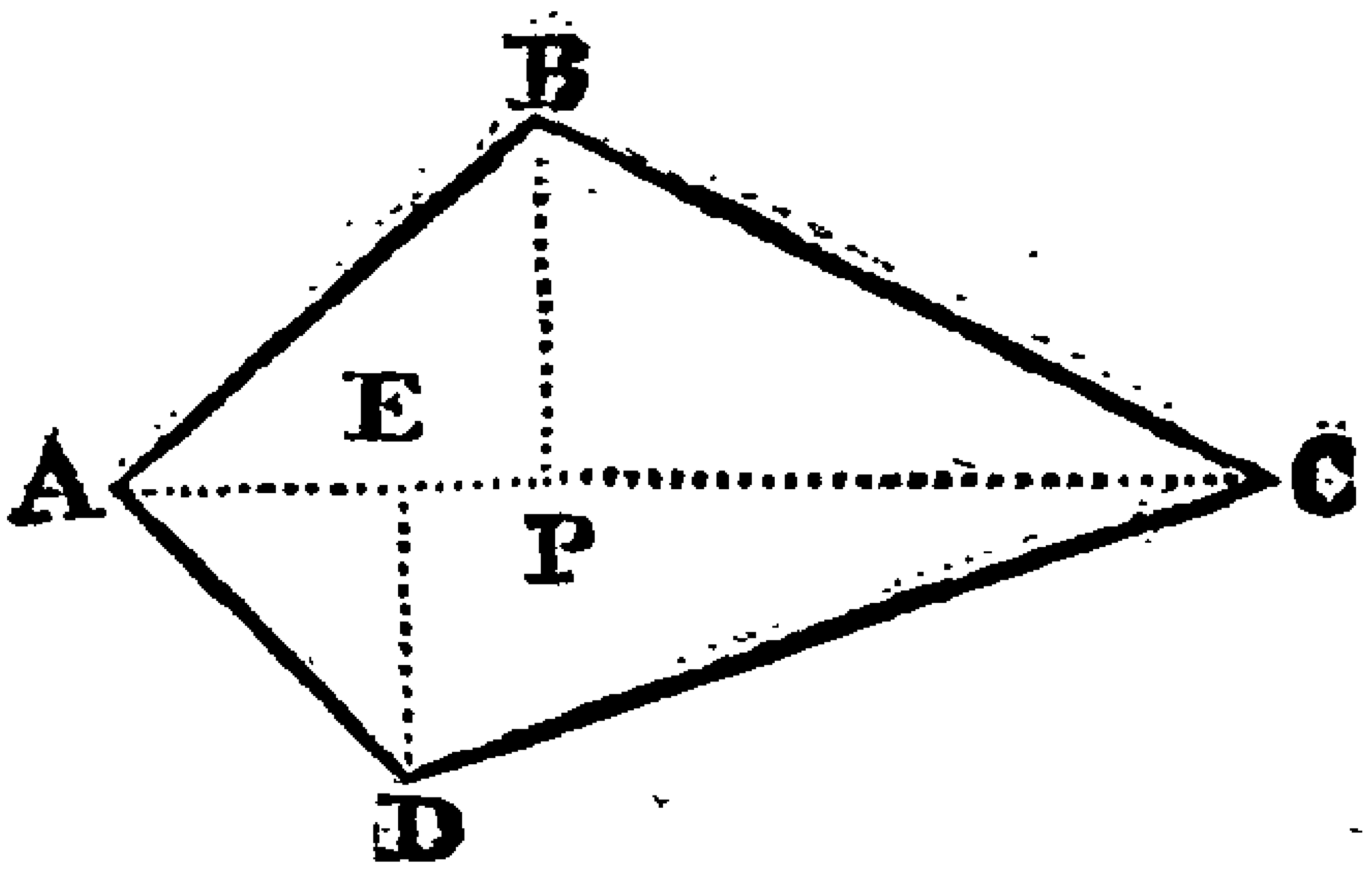
Def. A *Trapezium* consists of four unequal Sides, and four unequal Angles.

Rule.*

Add the two Perpendiculars together, and multiply that Sum by Half the Diagonal; (or multiply the Diagonal by Half the Sum of the Perpendiculars) and the Product will be the Area.

Example.

Suppose A B C D be a Trapezium, whose Diagonal A C is 108 Inches; the Perpendicular B P 38 Inches; and the Perpendicular D E 34.5 Inches; what is the Area?



Operation. $72.5 \times 54 = 3915.0$ Inches, the Area.

* Every *Trapezium* being a double Triangle, is consequently equal to *Half* its circumscribing Parallelogram.

Pro-

Problem 8.

To find the Area of a Regular Polygon.

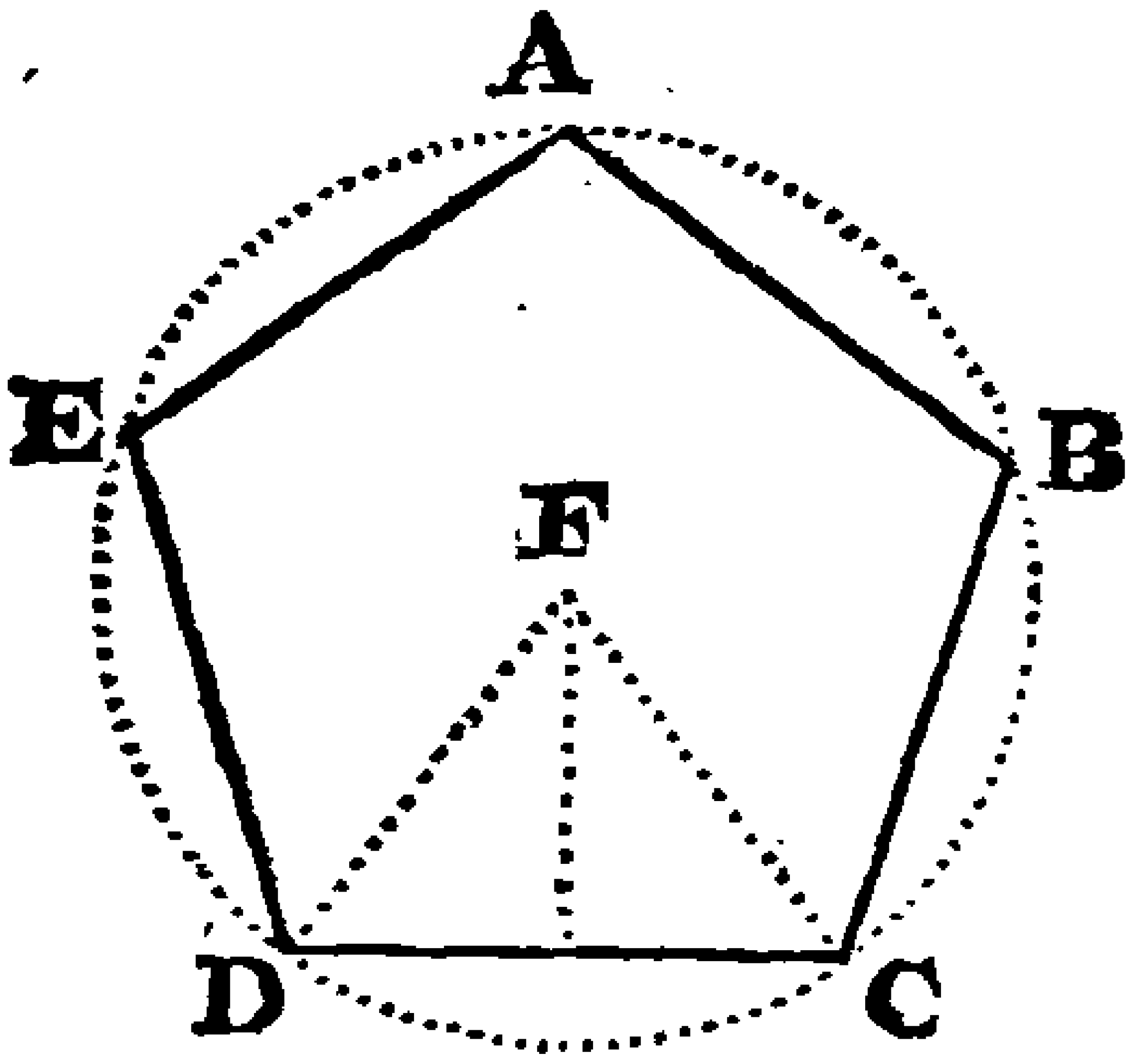
Def. All Figures that have more than four Sides, and those all equal, are called *Regular Polygons*. Of such Figures, those of five Sides are called *Pentagons*; those of six Sides, *Hexagons*; of seven Sides, *Heptagons*; of eight Sides, *Octagons*; of nine Sides, *Nonagons*; of ten Sides, *Decagons*, &c. whence it is evident, that they take their Names from the Number of Angles within them.

Rule. *

Multiply Half the Sum of the Sides by the nearest Distance of any Side from the Center.

Example.

Let A B C D E be a *Pentagon*, each of whose Sides is 25 Inches, and the nearest Distance from the Center to the Middle of any Side is 17.2 Inches; what is its Area?



Operation. $62.5 \times 17.5 = 1075.00$ Inches, the Area sought.

* Every *Regular Polygon* is equal to a Triangle, whose Base is equal to the Sum of the Sides, and whose Height is the Distance of any Side from the Center. ☞ To find the Center, only bisect 2 of its Angles, as D and C; and the Point of Meeting of these Lines will be the Center required.

Pro-

Problem 9.

To find the Area of a Circle.

Def. A *Circle* is a plain Figure, whoſe Area is bounded by one continued Line, called the *Circumference*, or *Periphery*; and it is every where equally diſtant from a Point within, called its *Center*.

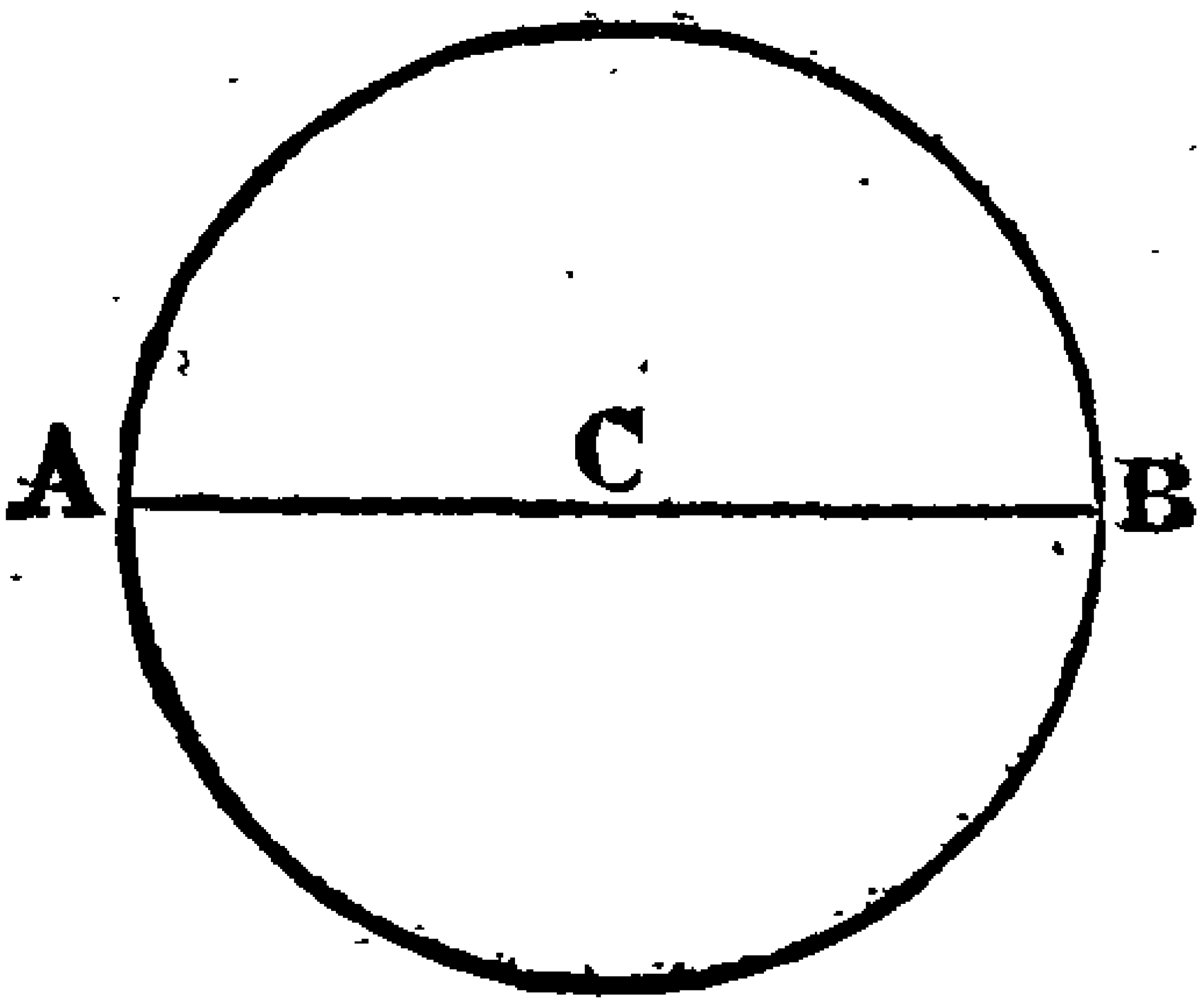
Note. The Line going through the Center of the *Circle*, dividing it into two equal Parts, is called the *Diameter*; Half of which is called the *Semi-diameter*, or *Radius*.

Rule.*

Square the Diameter, and multiply it by .7854 (a Decimal) and that Product will be the Content.

Example.

Suppoſe the Diameter of the Circle A B be 41,2 Inches, what is its Area?



Operation. $41.2 \times 41.2 = 1697.44 \times .7854 = 1333.169376$ Inches, the Area.

* The *Area* of a Circle whoſe Diameter is 1, is .7854; and the Areas of all Circles are to each other, as the Square of their Diameters; whence the above Rule.—☞ The *Square* of a Number is the Product ariſing from multiplying that Number by itſelf.

Pro-

Problem 10.

To find the Area of a *Circle* by another Method.

Rule.*

Multiply half the Circumference by half the Diameter, and the Product is the Area.

☞ Note. If the Diameter of a Circle be given, the Circumference of it may be easily found by the following Proportions:

As 7 is to 22, so is the Diameter to the Circumference.

Or, as 113 is to 355, so is the Diameter to the Circumference.

Or, as 1 is to 3.14159, so is the Diameter to the Circumference.

The *Squaring of a Circle*, as it is usually called, or finding a Square exactly equal to a Circle given, is what many have endeavoured to perform; but none as yet have absolutely compleated it; because none have found out the exact Proportion of the *Diameter* to the *Circumference*. † *Archimedes*, a very famous Mathematician, first discovered that the Diameter was in Proportion to the Circumference, as 7 is to 22 nearly; which serves very well for common Use. After him, one *Metius* found that the Diameter was (more nearly) to the Circumference, as 113 to 355.

* Every Circle, as a Polygon of an infinite Number of Sides, is equal to a Triangle, whose Base is the Circumference and Height = to half the Diameter.

† The Emperor Charles V. offered a Reward of One Hundred Thousand Crowns to the Person who should solve this celebrated Problem; and the States of Holland have proposed a Reward for the same Purpose.

But

But the Moderns have computed the Proportion of the Diameter to the Circumference to greater Exactneſs. For, ſuppoſing the Diameter to be 100000, the Periphery they find will be more than 31415, and leſs than 31416; but *Ludolphus Van Ceulen* has exceeded the Labours of all who went before him; for by immenſe Application he found, that ſuppoſing the Diameter to be

1.00000.000.000.000.000.000.000.000.000.000

the Periphery or Circumference will be leſs than

3.14159.265.358.979.323.846.264.338.327.951

but greater than

3.14159.265.358.979.323.846.264.338.327.950

This able Mathematician has carried on his Calculation to 36 Places of Decimals; and to eternalize the vaſt Work, they are engraven upon his Tomb-ſtone in St. *Peter's Church* at *Leyden*, in *Holland*, of which large Number the firſt ſix Places 3.14159, anſwering to the Diameter of 1.00000, are ſufficient in any Calculation.

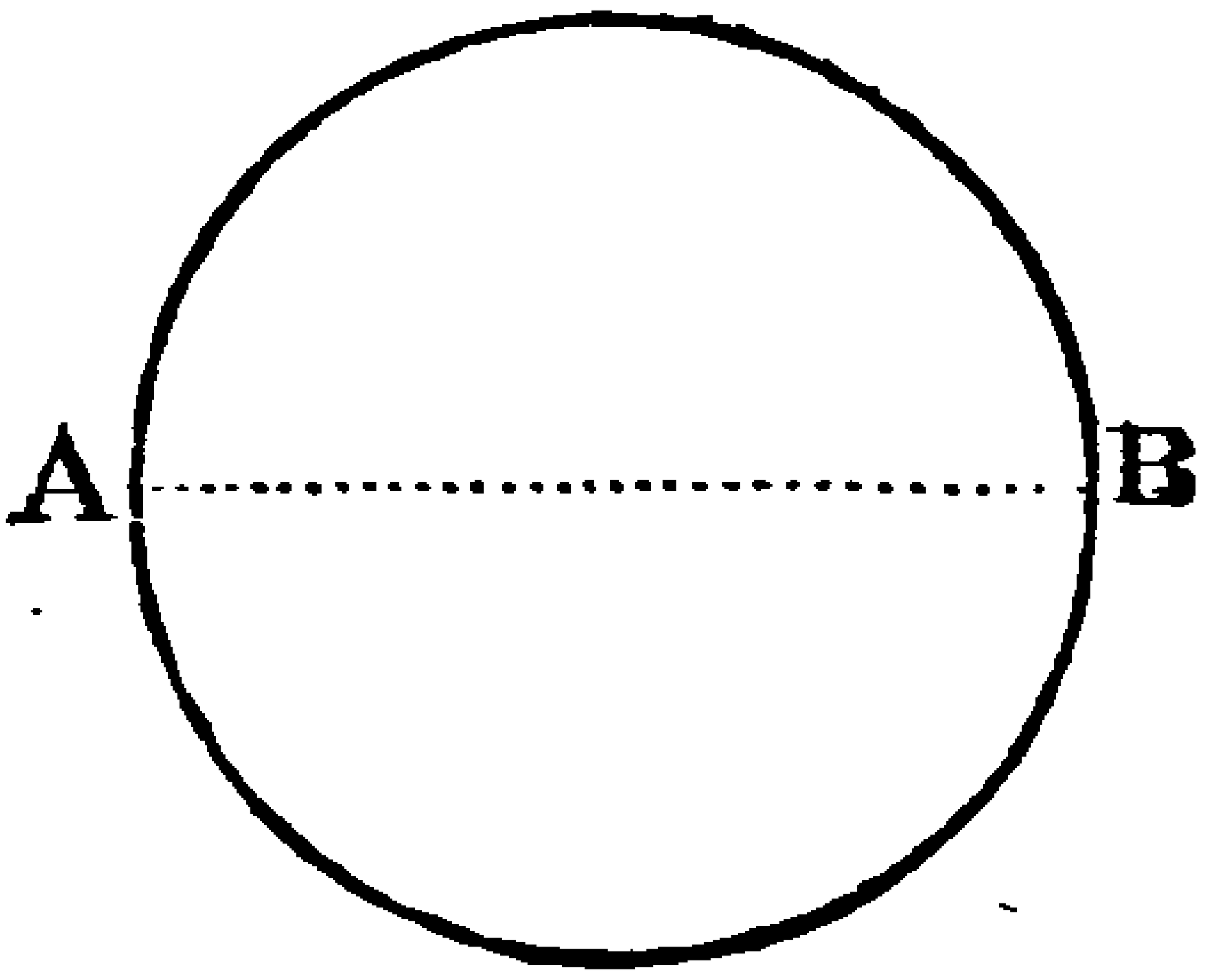
But ſince *Van Ceulen*'s Time, our Countryman Mr. *Machin* has carried the Proportion on to 100 Places. He finds that if the Diameter be 1, the Circumference will be

3.14159.26535.89793.23846.26433.83279.50288.47971.69399.37510.58209.74944.59230.78164.05286.20899.86280.34825.34211.70679 of the ſame Parts.

But the *Ratios* generally uſed are thoſe of *Archimedes*, *Metius*, or four or five leading Figures of *Van Ceulen*.

Example.

Suppose the Diameter of a Circle A B to be 22.6 Inches, what is its Circumference, and also its Area?



Dia. Circum.
First. As 7 : 22 : : 22.6 : 71.028, according to *Archimedes.*

Or, as 113 : 355 : : 22.6 : 71, according to *Metius.*

Or, as 1 : 3.141593 : : 22.6 : 71.0000018, according to *Van Ceulen.*

Then half the Circumference multiplied by half the Diameter gives the Area = 401.15 Inches.

Note. If the Circumference of a Circle be given, the Diameter may be found by reverting the above Proportion, thus,

As 22 : 7 : : Circumference to the Diameter;

Or, as 355 : 113 : : Circumference to the Diameter;

Or, as 3.14159 : 1 : : Circumference to the Diameter.

$$\left.\begin{array}{r}\text{Other Proportions are as } 106 : 333 \\ \text{or as } 1702 : 5347 \\ \text{or as } 1815 : 5702\end{array}\right\} : : \text{Diam. to Circumference.}$$

Pro-

Problem II.

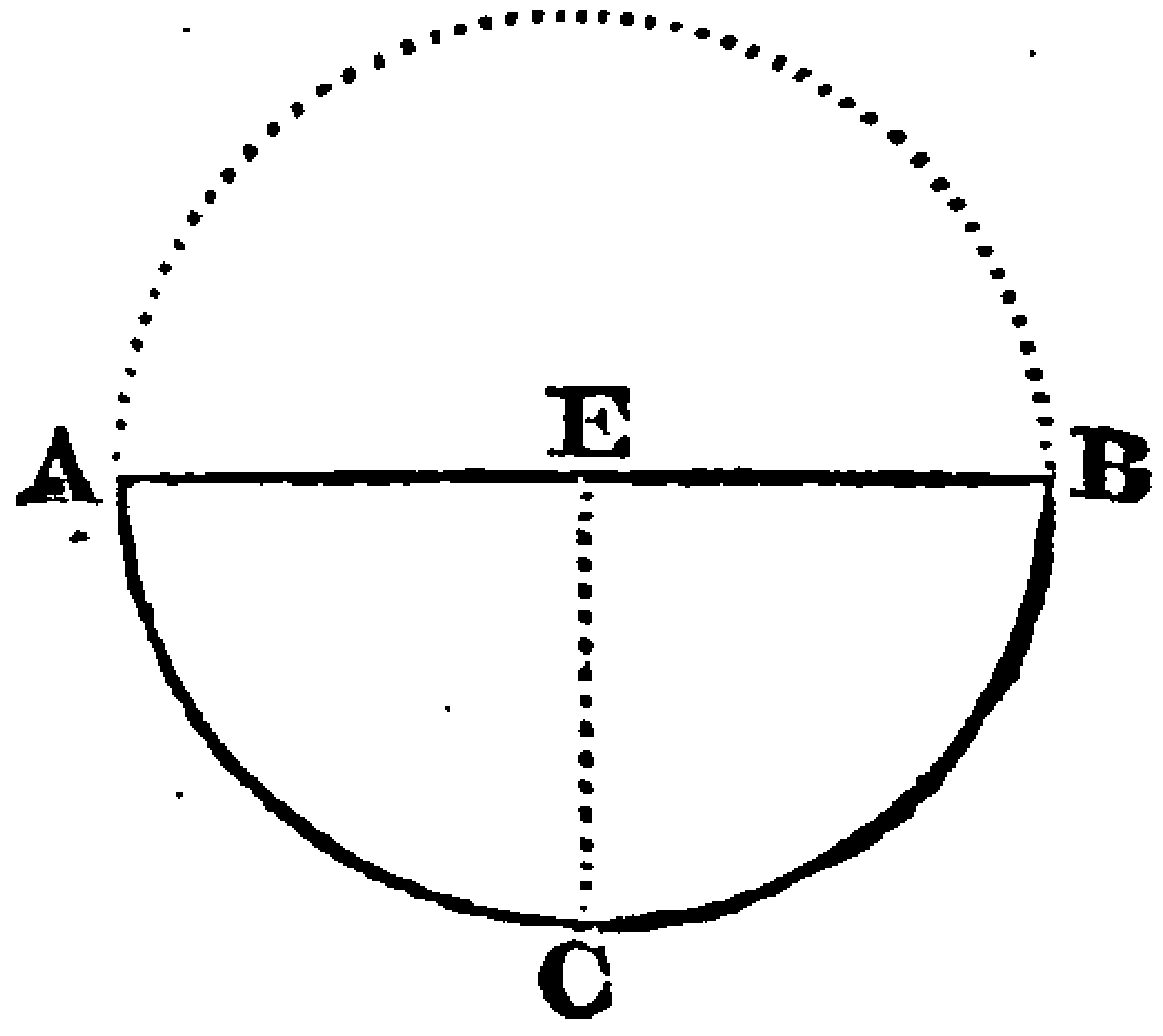
To find the Area of a Circle; of a *Semi-circle*; or of a *Quadrant*.

Rule.

Square the *Diameter*, and multiply the Product by .7854 (a Decimal); this last Product will be the Area of the *Whole Circle*; which divided by 2 gives the Area of the *Semi-circle*; or by 4 gives the Area of the *Quadrant*.

Example.

Suppose the Diameter A B of the following *Semi-circle* A B C is 41.2 Inches, what is its Area?



Operation. $41.2 \times 41.2 = 1697.44 \times .7854 = 1333.169376$ Inches, Area of the *Whole Circle*.

Then, $1333.169376 \div 2 = 666.584688$, Area of the *Semi-circle* A B C.

And, $1333.169376 \div 4 = 333.292344$, Area of the *Quadrant* A E C, or B E C.

Problem 12.

To find the Area of the *Sector* of a Circle.

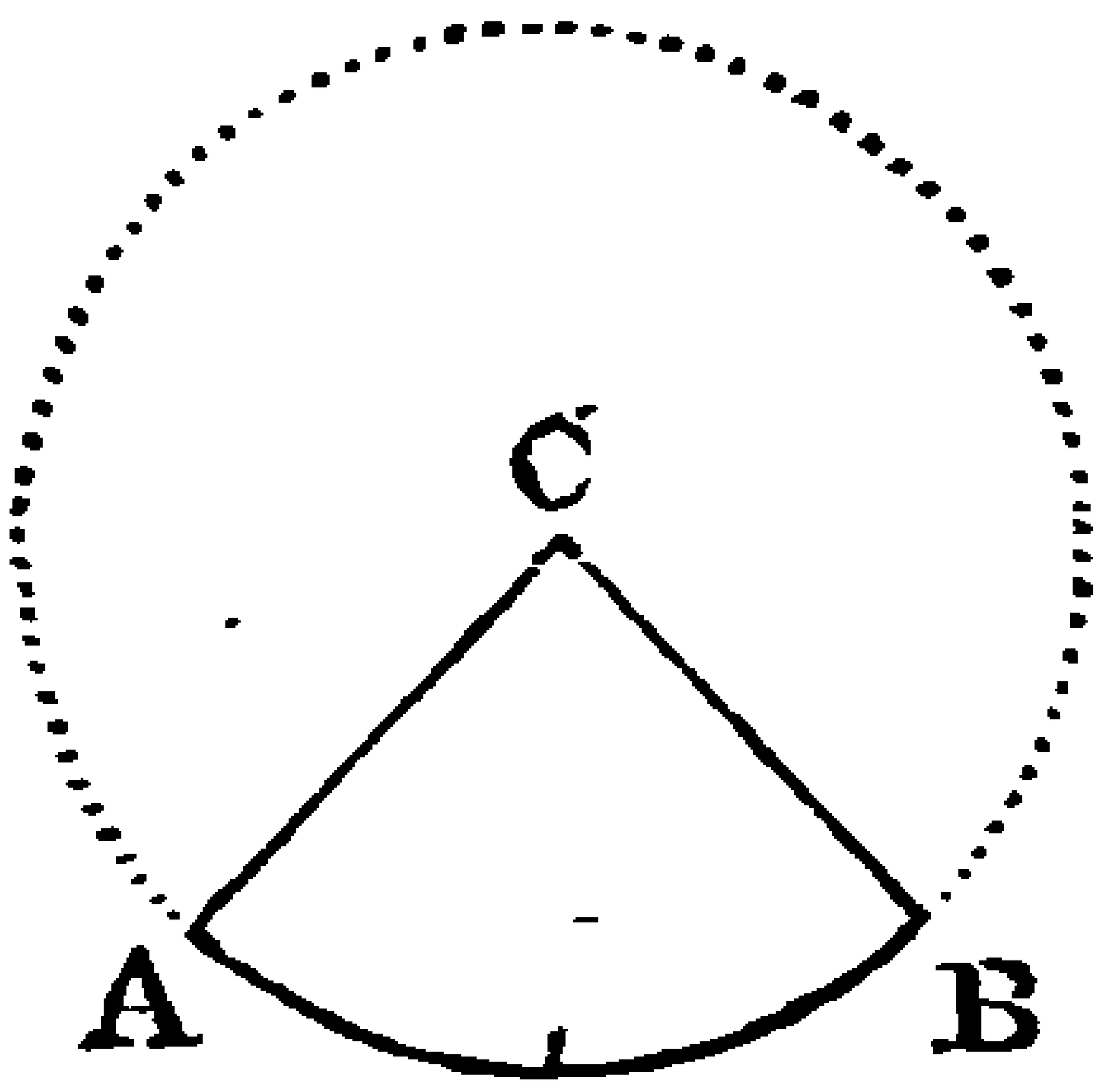
Def. The *Sector* of a Circle is a Figure bounded by two Semi-diameters, or Radius's, having ſome Part of the Periphery or Circumference for its Baſe.

Rule. *

Multiply the Semi-diameter or Radius by half the Arch of the Sector, and the Product will be the Area.

Example.

Let A B C be the *Sector* of a Circle, whoſe Radius A C, or B C, is 20.2 Inches; and *half* the Arch A B, 15 Inches, what is its Area?



Operation. $20.2 \times 15 = 303.0$ Inches, the Area ſought.

* A *Sector* of a Circle is equal to a Triangle whoſe Baſe is equal to the Arch, and Height equal to the Radius or Semi-diameter.

Pro

Problem 13.

To find the Area of a *Segment* of a Circle.

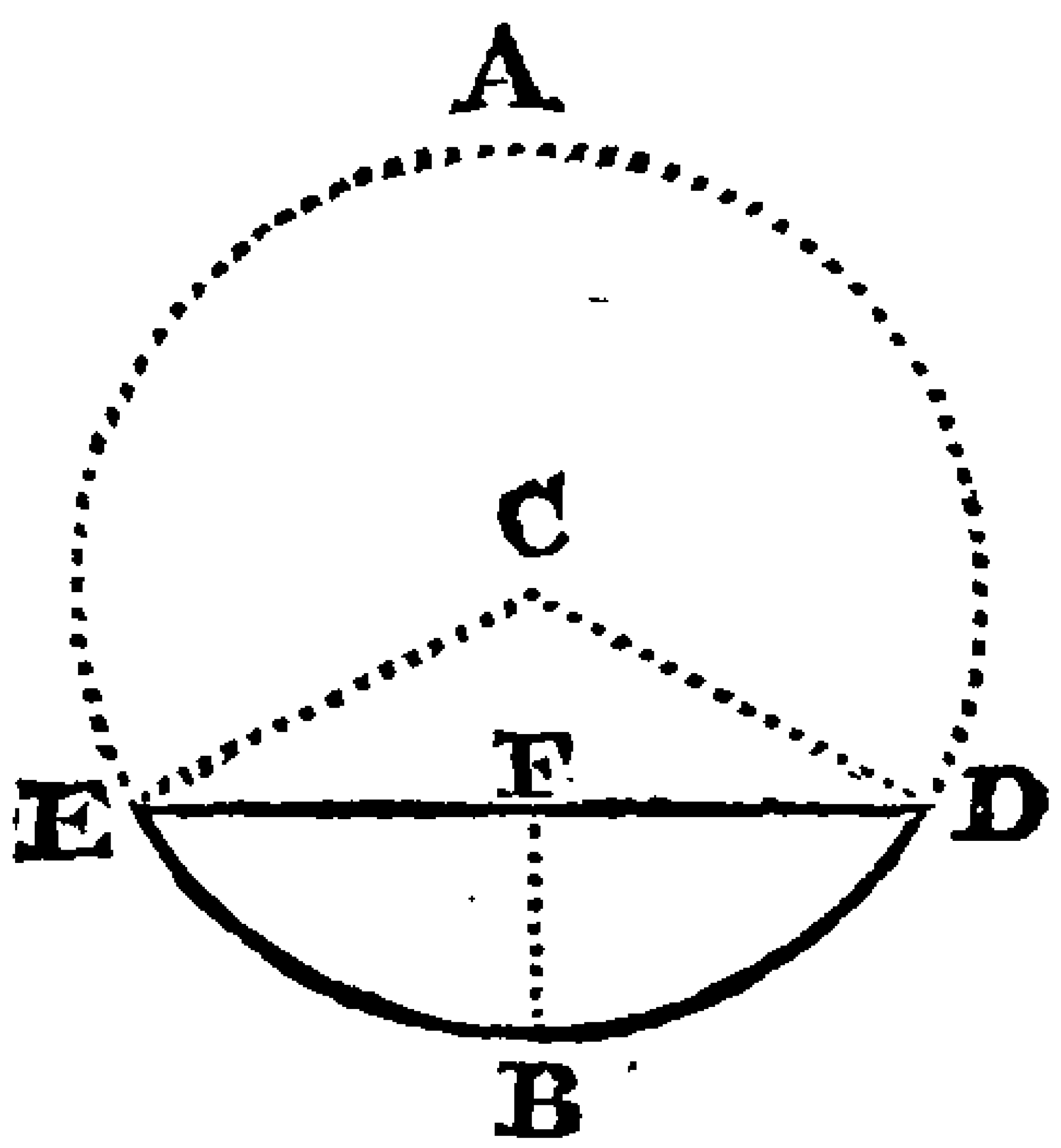
Def. A *Segment* of a Circle is a Part cut off by a right Line, less than the Diameter, drawn within the Circle, as E D; so that E B D is a Segment less than a *Semi-circle*; and E A D is greater than a Semi-circle.

Rule.

To the Square of half the Chord add $\frac{2}{5}$ of the Square of the Depth; the Square Root of this Sum multiplied by $\frac{4}{3}$ of the Depth will give the Area of the Segment. *

Example.

Suppose the Chord E D of the Segment E D B be 40 Inches, and the Depth F B 10 Inches, what is its Area?



Operation. $20 \times 20 = 400 + 40 = 440$, whose Square Root is $20.973 \times 13.33 = 279.57$, the Area sought.

* Or find the Area of the Sector C E B D, and also the Area of the Triangle C E D, which *subtracted* from the Area of the Sector, will leave the Area of the Segment E B D required. If the Segment be greater than half the Circle, the Triangle must be added to the Sector to give the Area.

Problem 14.

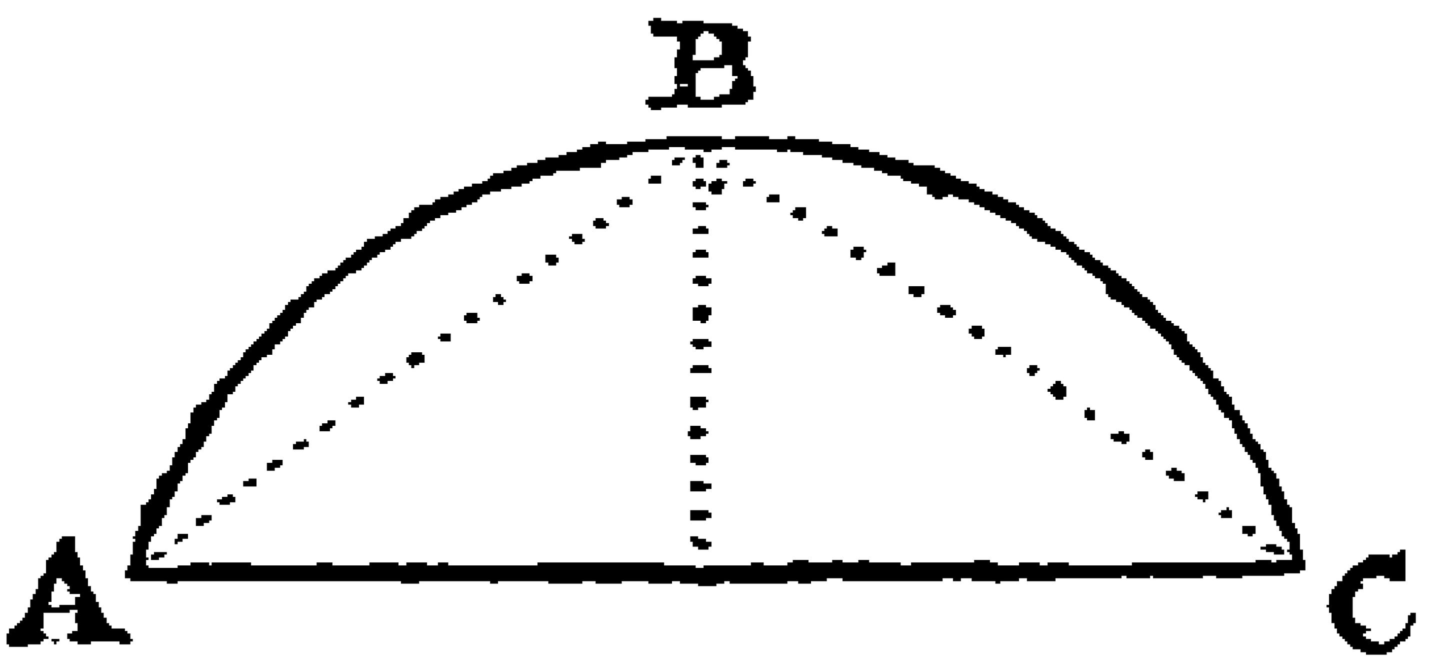
To find the Length of an *Arch* of any Circle?

Rule.

Multiply the Chord of $\frac{1}{2}$ the Arch by 8; from the Product subtract the whole Chord; divide the Remainder by 3; and the Quotient will be equal to the Length of the Arch required. *

Example.

Suppose the Chord A C of the Arch A B C be 50.8 Inches, and the Chord A B of half the Arch be 30.6, what is the Length of the Arch A B C?



Operation. $30.6 \times 8 = 244.8 - 50.8 = 194.0 \div 3 = 64.6$, the Length of the Arch required.

* Or, from *twice* the Chord of $\frac{1}{2}$ the Arch, subtract the whole Chord; divide the Remainder by 3; the Quotient added to *twice* the Chord of $\frac{1}{2}$ the Arch, will give the Length of the Arch (nearly) as before.

Pro-

Problem 15.

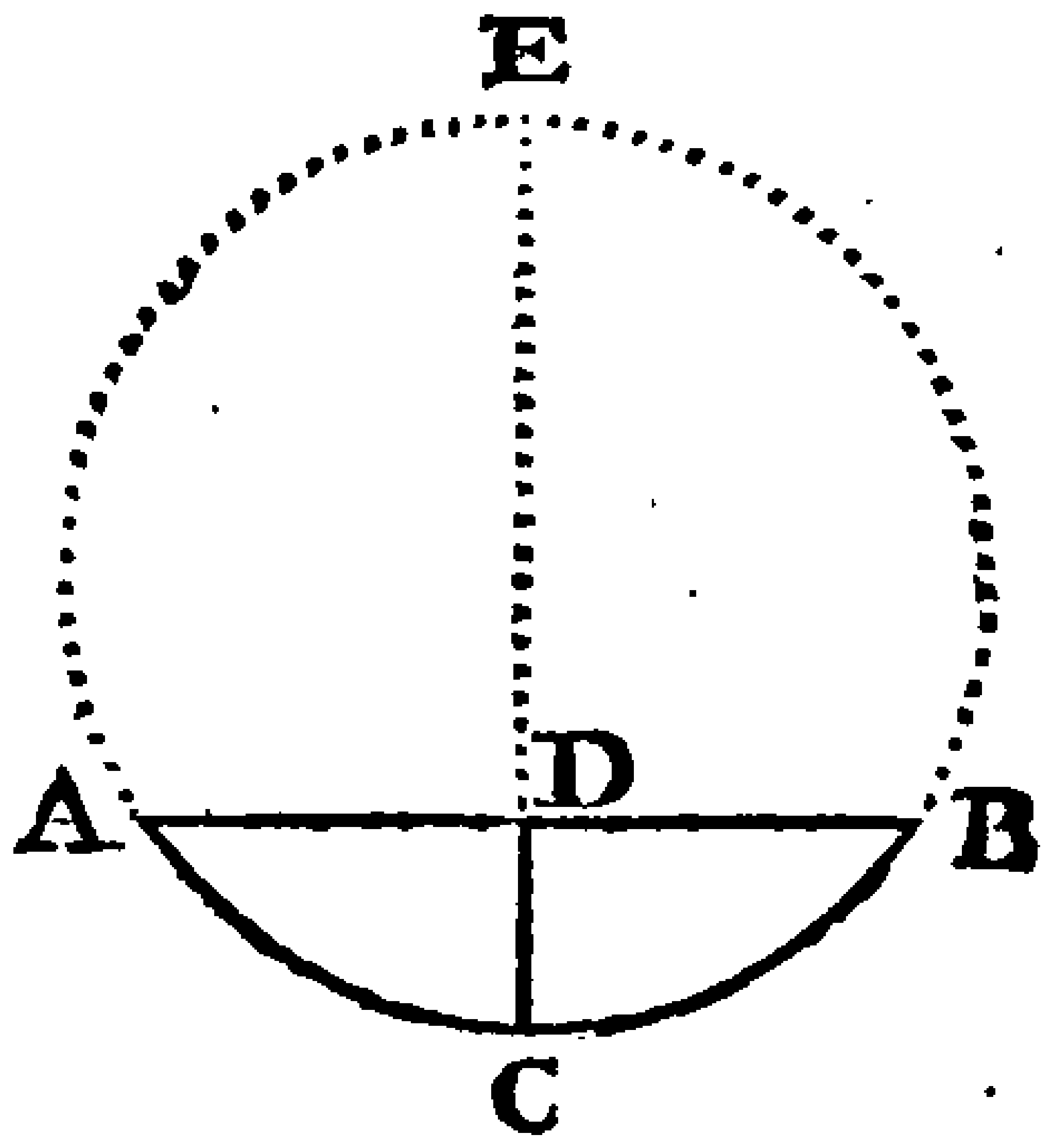
The Chord and versed Sine (or Depth) of a *Segment* of a Circle being given, to find the (whole) Diameter of the Circle.

Rule.

Divide the Square of half the Chord by the versed Sine (or Depth), and the Quotient will be the other Part of the Diameter, which added to the versed Sine, the Sum will be the whole Diameter sought. *

Example.

Suppose A C B be a *Segment* given, whose Chord A B is 36 Inches, and the versed Sine (or Depth) C D 6 Inches, what is the Diameter of the whole Circle C E?



Operation. A D $18 \times 18 = 324$, which $\div$ D C $6 = 54$, the Part D E. Then D E $54 +$ D C $6 = 60$ Inches; the whole Diameter E C required.

* Half the Chord of the Segment is always a mean Proportion between the two Parts of the Diameter; it will therefore ever hold,—as the versed Sine : is to half the Chord : : so is half the Chord : to the remaining Part of the Diameter.

Problem 16.

To find the Area of a *Circular Ring.*

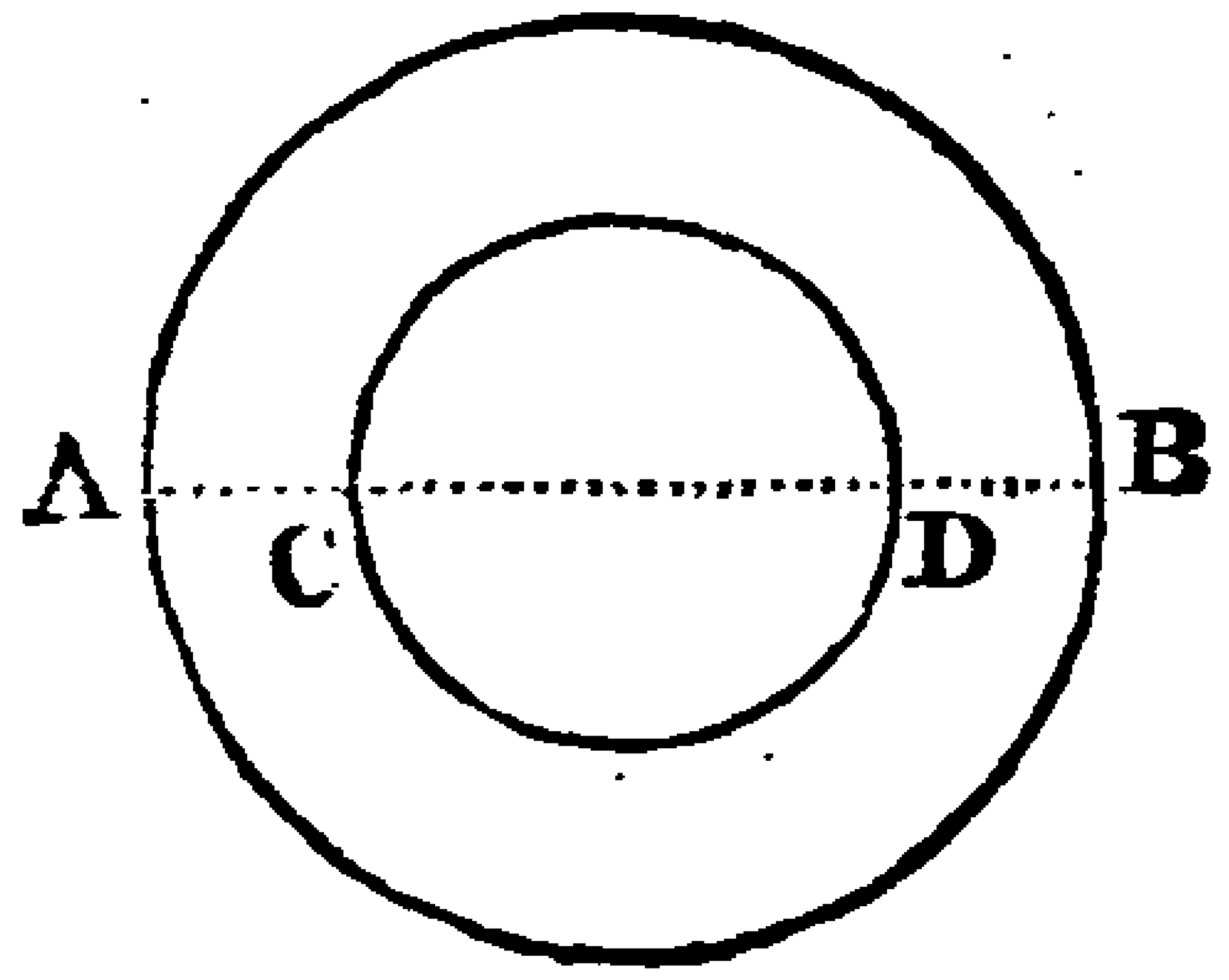
Def. The Space included between two concentric Circles of different Diameters is called a *Circular Ring.*

Rule.

Multiply the Sum of the two Diameters by their Difference, and multiply this Product again by .7854; this last Product will be the Area required. *

Example.

Suppose the Diameter of the larger Circle AB be 30 Inches, and the Diameter of the smaller CD be 18 Inches, what is the Area of the *Ring* formed by these two Circles?



Operation. $30+18=48\times12=576\times.7854=452.3904$ Inches, the Area of the Ring required.

* Or, find the Area of each Circle, and subtract the less from the bigger, the Remainder will be the Area of the *Ring.*

Pro-

Problem 17.

To find the Area of a *Crescent or Lune.*

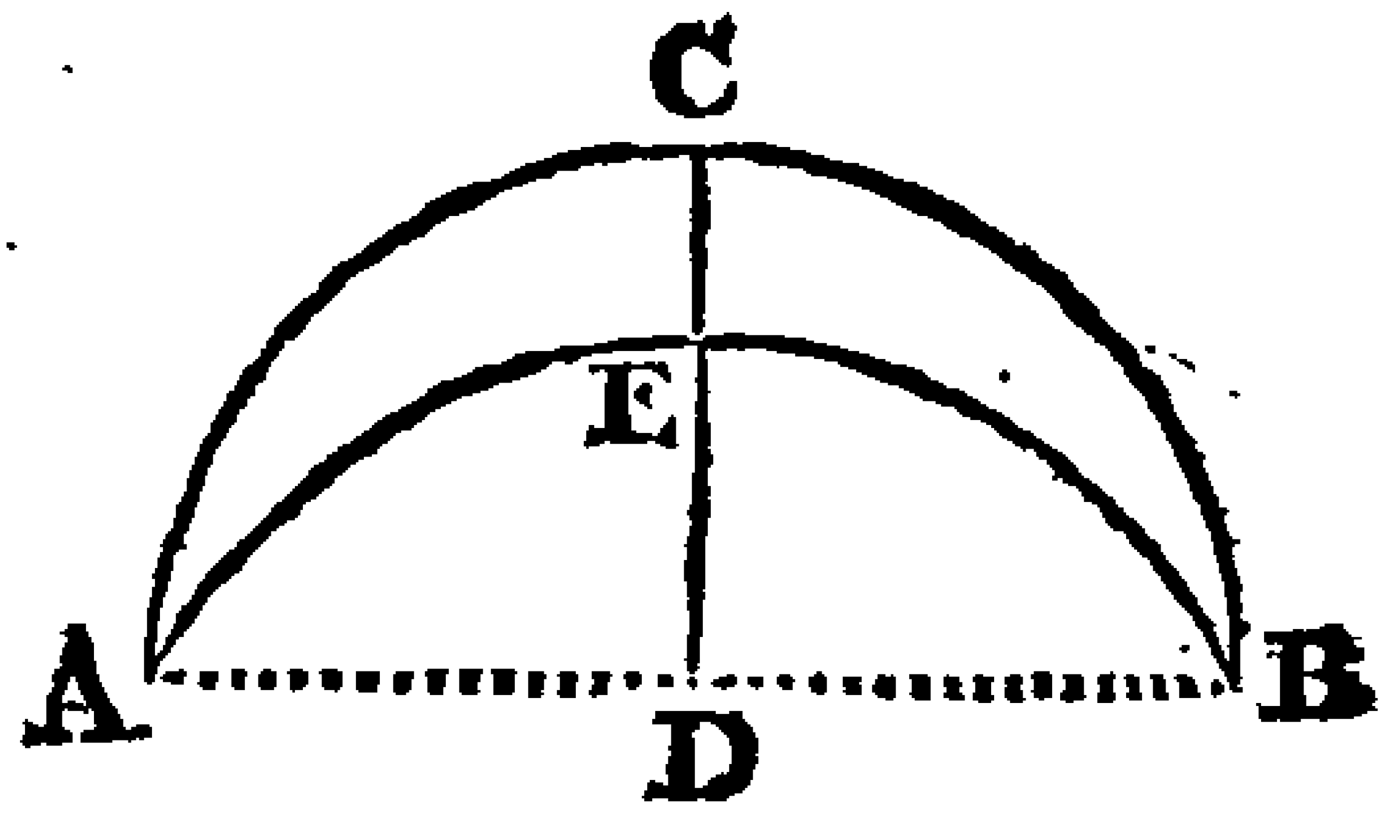
Def. A *Lune*, &c. is the Space included between the Arches of two eccentric Circles interfecting or cutting each other.

Rule.

Find the Areas of the two Segments from which the Crescent or Lune is formed; and their Difference will be the Area required.

Example.

Let the Chord A B in the following Figure be 40 Inches, and the Height D C 10 Inches, and D E 4 Inches, what is the Area of the *Lune* or *Crescent* A C B D E?



Operation. By Problem 13th.

The Area of the bigger Segment is	279.57
Area of the lesser Segment is	107.50
The Area of the Lune or Crescent required	172.07 In.

Problem 18.

To find the Area of an *Oval*.

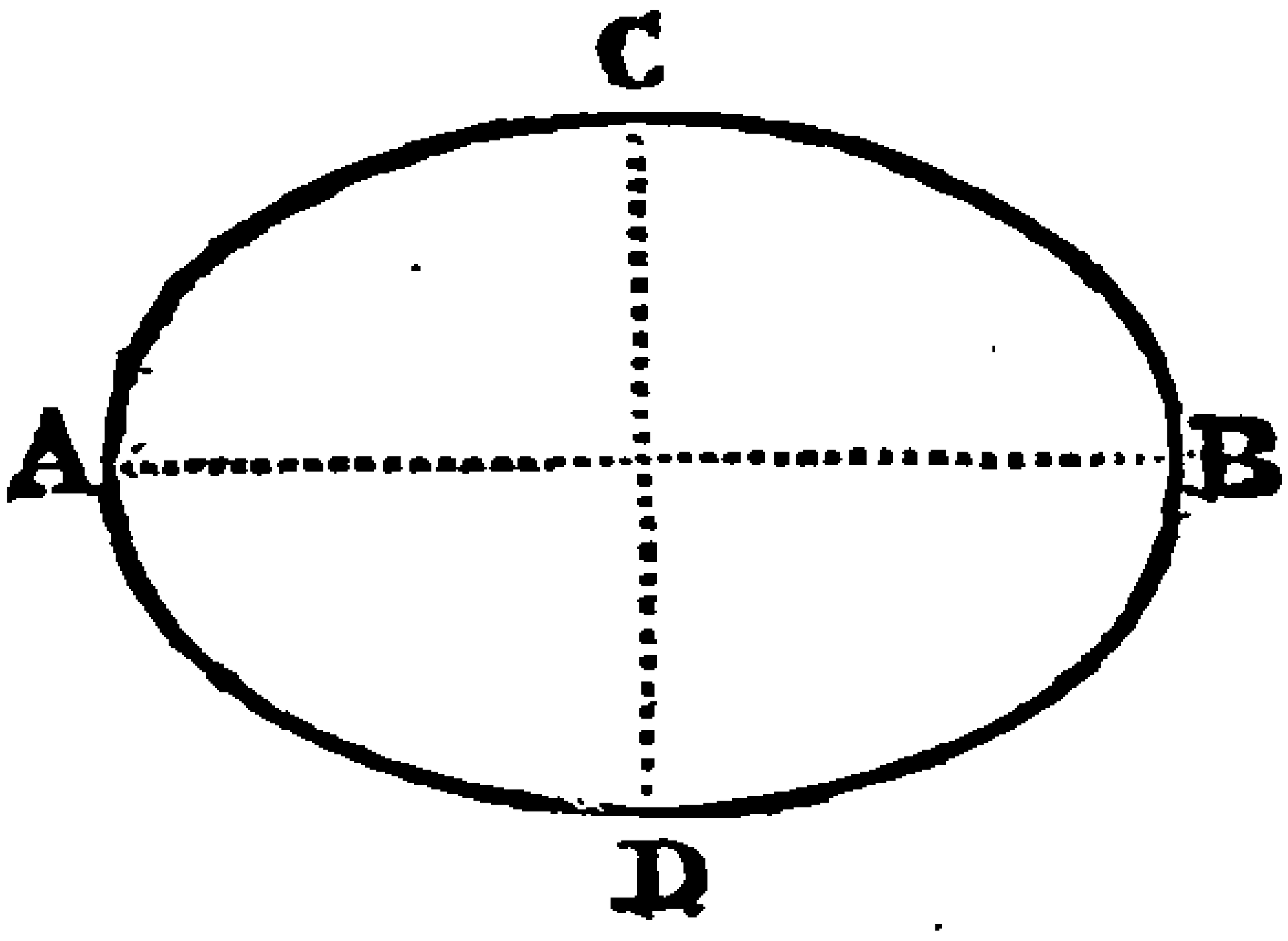
Def. An *Ellipsis* or *Oval* is a Figure bounded by a regular Curve Line returning into itself; having one Diameter longer than the other. The longer Diameter is called the *Transverse*; the shorter, the *Conjugate*.

Rule.

Multiply the Length by the Breadth, and that Product by .7854; this last Product will be the Content.

Example.

Suppose the longer Diameter A B of the following Oval be 61.6 Inches, and the shorter Diameter C D be 44.4 Inches, what is the Content?



Operation. $61.6 \times 44.4 = 2735.04 \times .7854 = 2148.100416$ Inches, the Area in the *Conic Sections*.

Problem 19.

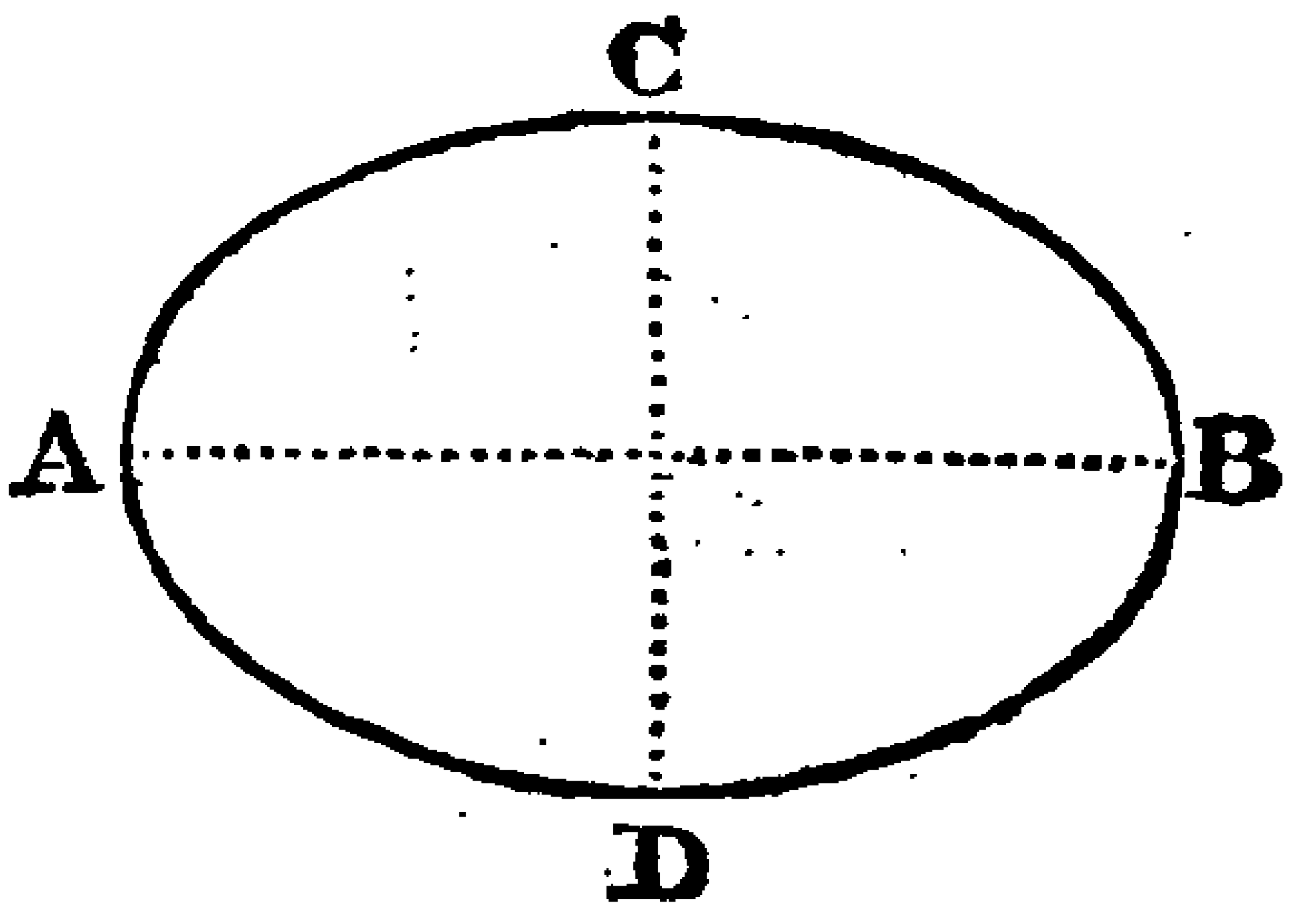
To find the Length of the *Circumference* of an *Oval.*

Rule.

Multiply half the Sum of the two Diameters by 3.1416, and the Product will be the Circumference near enough for Practice.

Example.

Suppose the longer Diameter A B be 50 Inches, and the shorter Diameter C D be 40 Inches, what is the Circumference of the *Oval?*



Operation. Half 2 Diameters$=45\times3.1416=141.372$ Inches, the Circumference required.

See another Method of finding the Circumference of an Oval or Ellipsis in the *Conic Sections.*

Problem 20.

To find the Area of any *irregular Figure.*

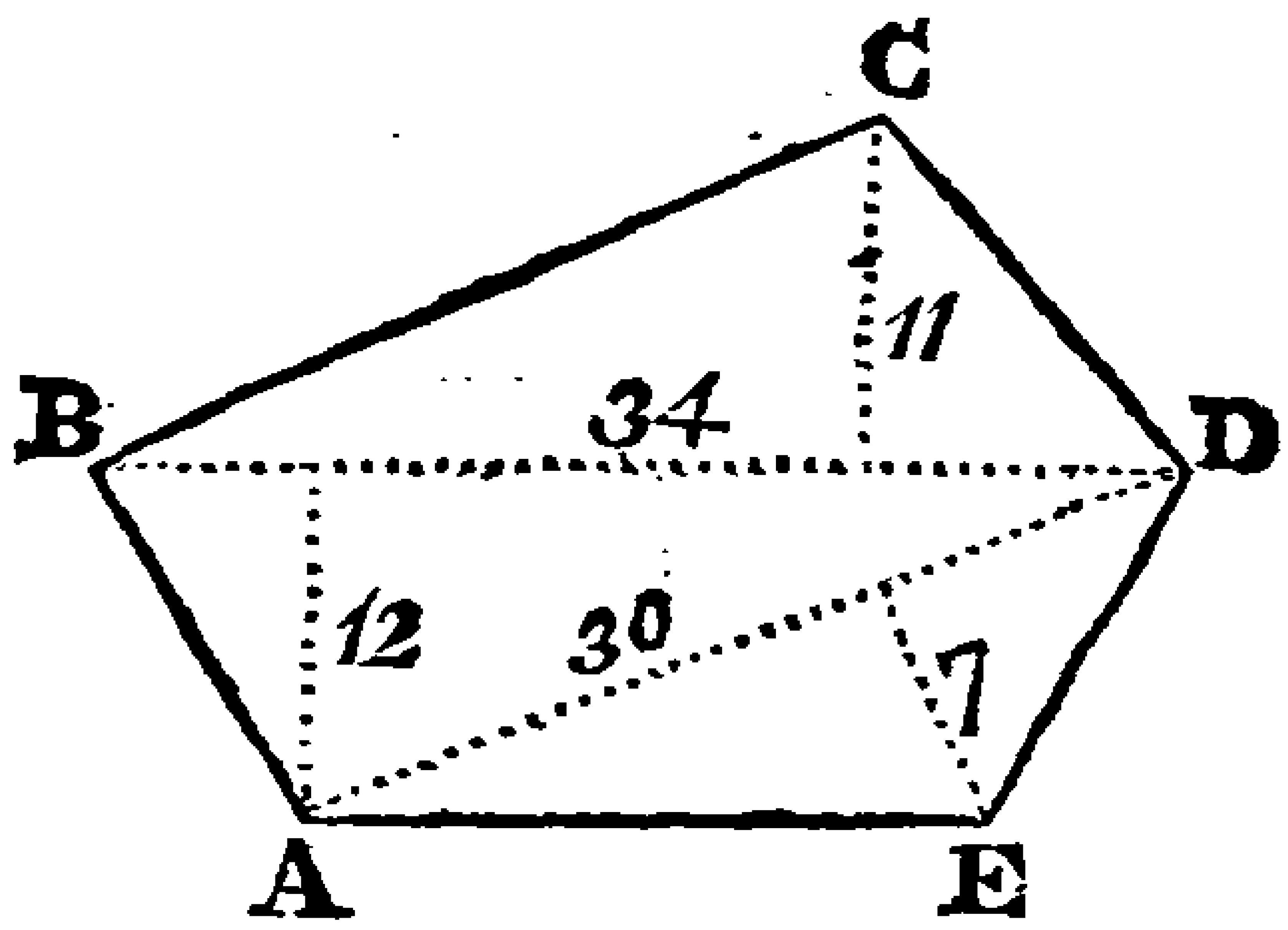
Def. All Figures which have above four Sides, and those all unequal, are called irregular *Polygons*, or *irregular Figures*: Of which Figures there is an infinite Variety; but they may be all reduced to regular Figures in the following Manner.

Rule.

Divide the Figure by *Diagonals* and *Perpendiculars* into *Trapeziums*, *Triangles*, &c. Then find the Areas of the Trapeziums, Triangles, *&c.* severally, and add them all together; so will the Sum be the Area of the whole Figure.

Example.

Suppose A B C D E be an irregular Figure, whose Dimensions are expressed in it; what is its Area?



Having divided the Figure as above into one *Trapezium*, and one *Triangle*, or rather into *three Triangles*, the Areas will be as under.

Area of the Triangle B C D is	187	Inches.
Area of the Triangle A B D is	204	
Area of the Triangle A D E is	105	

Area of the whole Figure 496

Note.

Note. Some irregular Figures will happen to conſiſt of Curve Lines as well as of Right Lines; all which muſt be reduced into ſome of the foregoing Figures, and meaſured by the Rules there delivered.

Note alſo; that if it be required to find the *Side of a Square* equal in Area to any of the foregoing Figures, either *regular* or *irregular*; you need only to extract the Square Root of the Area of that Figure, and the Root will be the Side of the Square ſought.

Pro-

Application of the foregoing Problems.

By the Problems and Rules here laid down, are meaſured *Wainſcot*, *Glaſs*, *Plaiſtering*, *Pavement*, *Boards*, *Land*, &c. by obſerving only, that the Area is always of the ſame Denomination with the Dimenſions, *viz.* If the Dimenſions are taken in *Feet*, the Area will be *Feet*; if in *Yards*, the Area will be *Yards*; if taken in *Chains* or *Miles*, the Area will be *Chains* or *Miles* accordingly.

And though there are other Rules, yet theſe may ſerve alſo for finding the Quantity of Liquor contained in any Superficial Figure at one Inch deep; for when you have found the Superficial Content of any of the foregoing Figures in *Inches*, if you divide that Content by

282, it will give the *Gallons of Ale or Beer*.
231, it will give the *Gallons of Wine or Cyder*.
2150.42, it will give the *Buſhels of Malt or Corn*.

Divided by 144, will give *Square Feet*; which *Feet* divided by 9, will give *Square Yards*.

☞ *Artificers* compute their Work by different Kind of Meaſures; as *Glazing* by the Square Foot; *Plaiſtering*, *Paving*, *Painting*, &c. by the Square Yard; *Roofing*, *Tiling*, *Flooring*, *Partitioning*, &c. by the Square of 100 Square Feet; *Brick-work* by the Rod of 16 ½ Feet, whoſe Square is 272 ¼ Square Feet; and *Land* by a Chain 22 Yards long, of which 10 in Length and 1 in Breadth make 1 Acre.

STEREOMETRY;

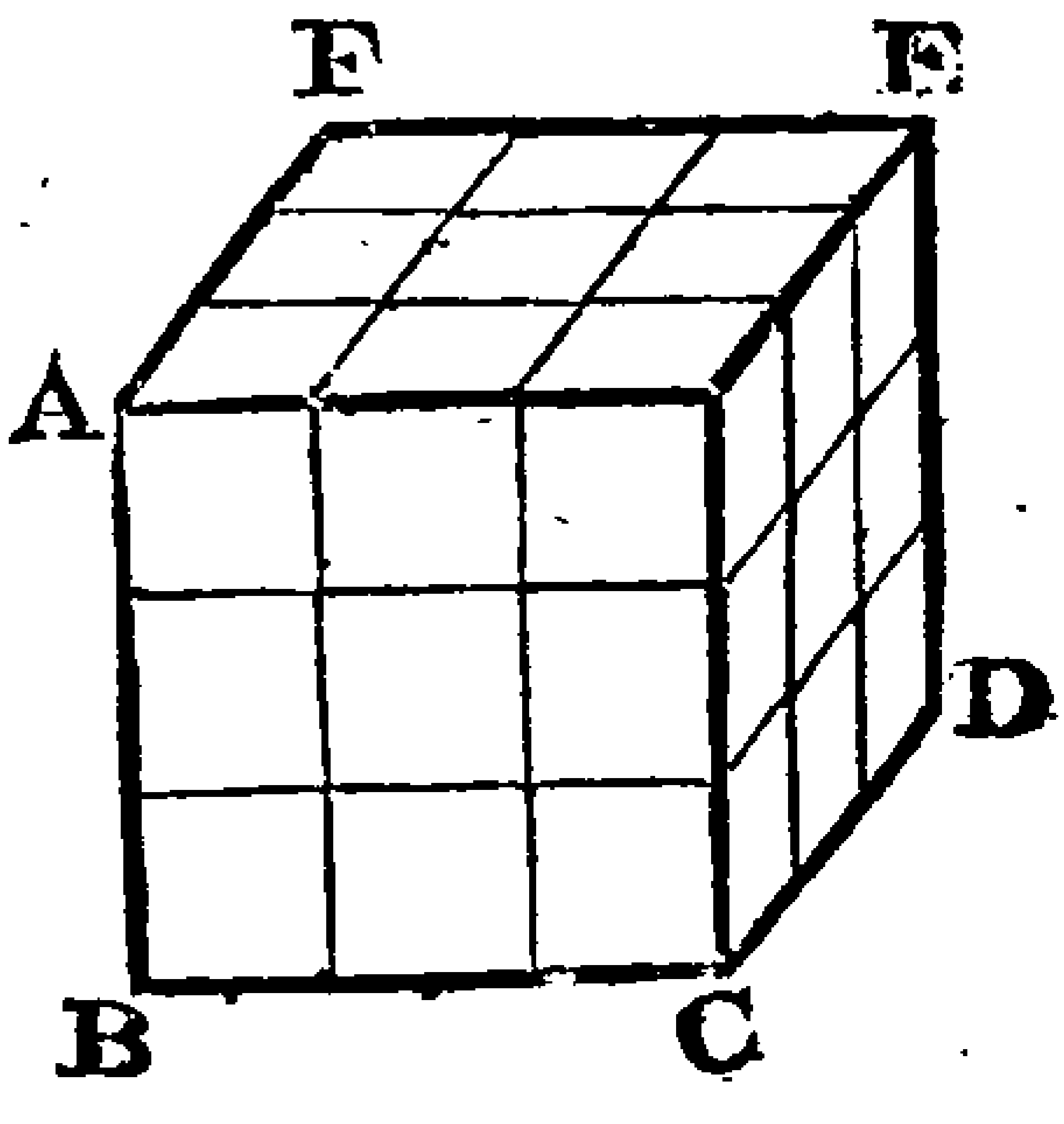
OR, THE

MEASURING OF SOLIDS.

SOLIDS are Bodies or Figures that consist of three Dimensions, *viz. Length, Breadth,* and *Thickness,* as *Timber, Stone,* &c.

The Solid Content of a Body is said to be known, when we know how many less Solids of 1 *Inch,* 1 *Foot,* 1 *Yard,* &c. are contained within it.

As suppose in the following Solid Figure, A B C D E F, each Side is 3 Inches, then is the Number of little Solids contained in it 27 Inches, of 1 Inch each. For dividing each Side into 3 equal Parts, and drawing the Lines through each Side, it becomes divided into 9 Parts, and consequently the Whole into 27; and so many less Solids of 1 Inch are contained in the larger of 3 Inches.



The Solidity or Content is always of the same Name with the Dimensions; if the Dimensions are taken in Inches, the Content will be Inches; and if taken in Feet, the Content will be Feet.

Pro-

Problem I.

To Measure, or find the Solid Content of a *Cube*.

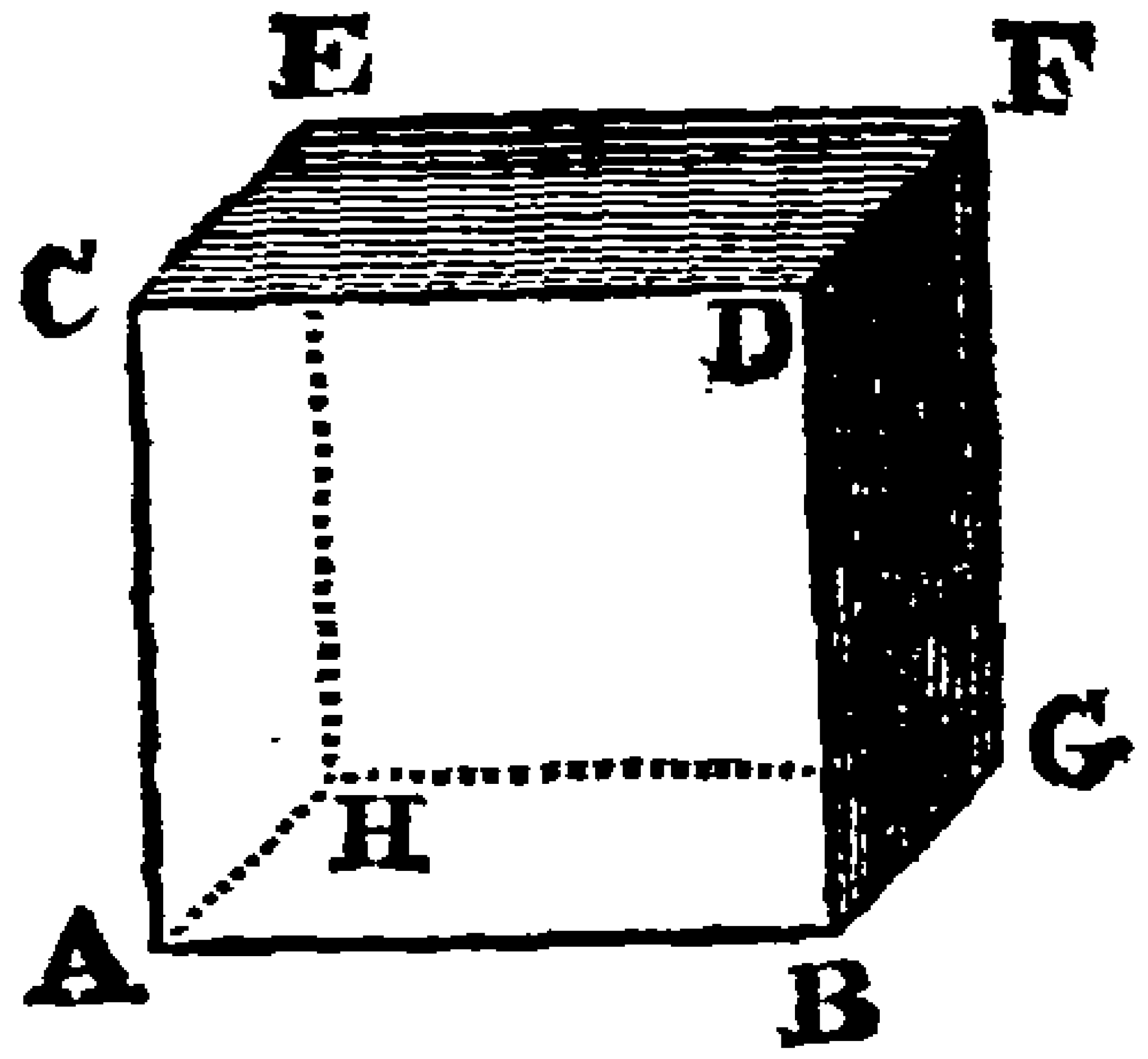
Def. A *Cube* is a Solid bounded with six Square Sides, in the Form of a *Die*.

Rule.

Multiply the Side by itself, and that Product by the Side again; and the last Product will be the Content. *

Example.

Suppose A B C D E F G be a Cubic Piece of Timber or Stone, each Side thereof being 17.5 Inches, what is the Content?



Operation. $17.5 \times 17.5 = 306.25 \times 17.5 = 5359.375$ Inches, the Content required.

☞ The *Superficial Content* is found by finding the Area of one Side, and multiply that by (*6*), the Number of Sides.

* The Reason of this Rule is deduced from the foregoing Figure, where the large Cube of 3 Inches is composed of 27 smaller ones.

To measure a *Parallelopipedon*, or oblong Cube.

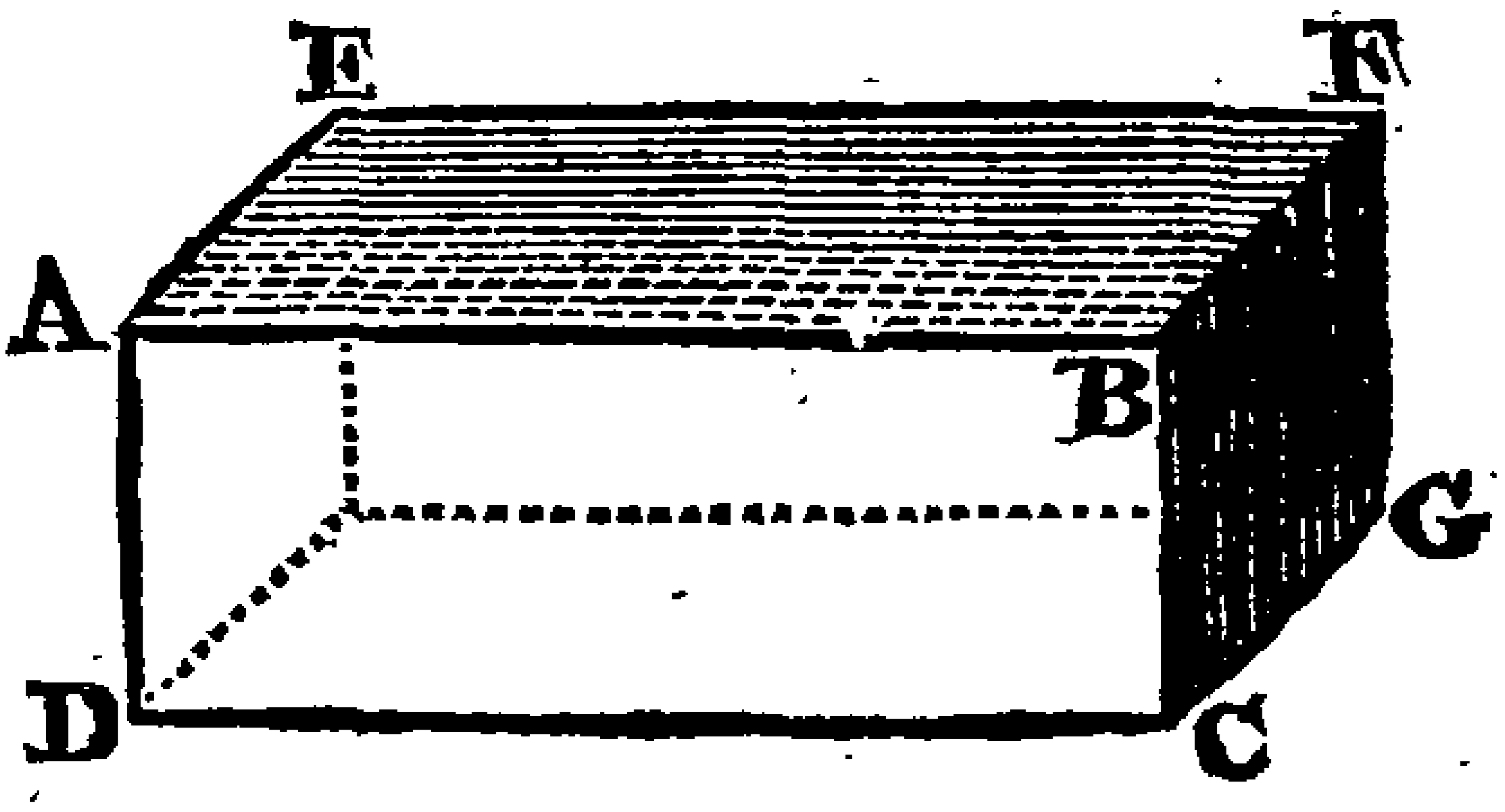
Def. *Parallelopipedon*, or oblong Cube, is a Solid contained under six Parallelograms, the opposite Sides of which are equal and parallel.

Rule.

Multiply the Breadth by the Depth, and that Product by the Length; and this last Product will be the Content. *

Example.

Suppose A B C D E F G be a *Parallelopipedon* representing a Piece of Timber or Stone; the Breadth A E and Depth A D each 21 Inches, and the Length D C ($7\frac{1}{2}$ Feet or) 90 Inches, what is its Solid Content?



Operation. $21 \times 21 = 441 \times 90 = 39690$ Inches, the *Solidity* required.

☞ The *Superficial Content* is the Area of the four Sides and the two Ends added together.

* Every *Parallelopipedon* is composed of a Number of equal Square Surfaces laid upon each other. The Sum of the whole Number of Squares is therefore the Content of the Figure.

Exam-

Example 2.

Suppose a Piece of Timber be 25 Inches broad, 9 Inches deep, and 25 Feet, or 300 Inches long, how many Solid Feet are contained in it?

Operation. $25 \times 9 = 225 \times 300 = 67500$ Inches, which divided by 1728, the Solid Inches in 1 Foot, gives 39 Feet for the Content.

Note. In *Solids* of this Sort, where the Breadth and Depth are different, an Error is often committed by Workmen in their Way of Measuring; which is done by adding the *Breadth* and *Depth* together, and taking Half the Sum for the Side of a Square equal thereto. This Error, though but small, when the Breadth and Depth are nearly equal; yet if the Difference be great, the Error is very considerable. For the Piece of Timber thus measured will be more than the Truth by a Piece, whose Length is equal to the Length of the Piece of Timber to be measured, and the Square equal to Half the Difference of the Breadth and Depth of it.

Operation. $25 + 9 = 34$; Half is $17 \times 17 = 289 \times 300 = 86700$ Inches, which divided by 1728, gives 50 Feet for its Content.

$$\left.\begin{array}{lr} \text{Content the true Way - - -} & 39 \\ \text{by common Way - -} & 50 \end{array}\right\} \text{Feet.}$$

Difference 11 Feet too much.

Pro-

Problem 3.

To Measure a *Prism*.

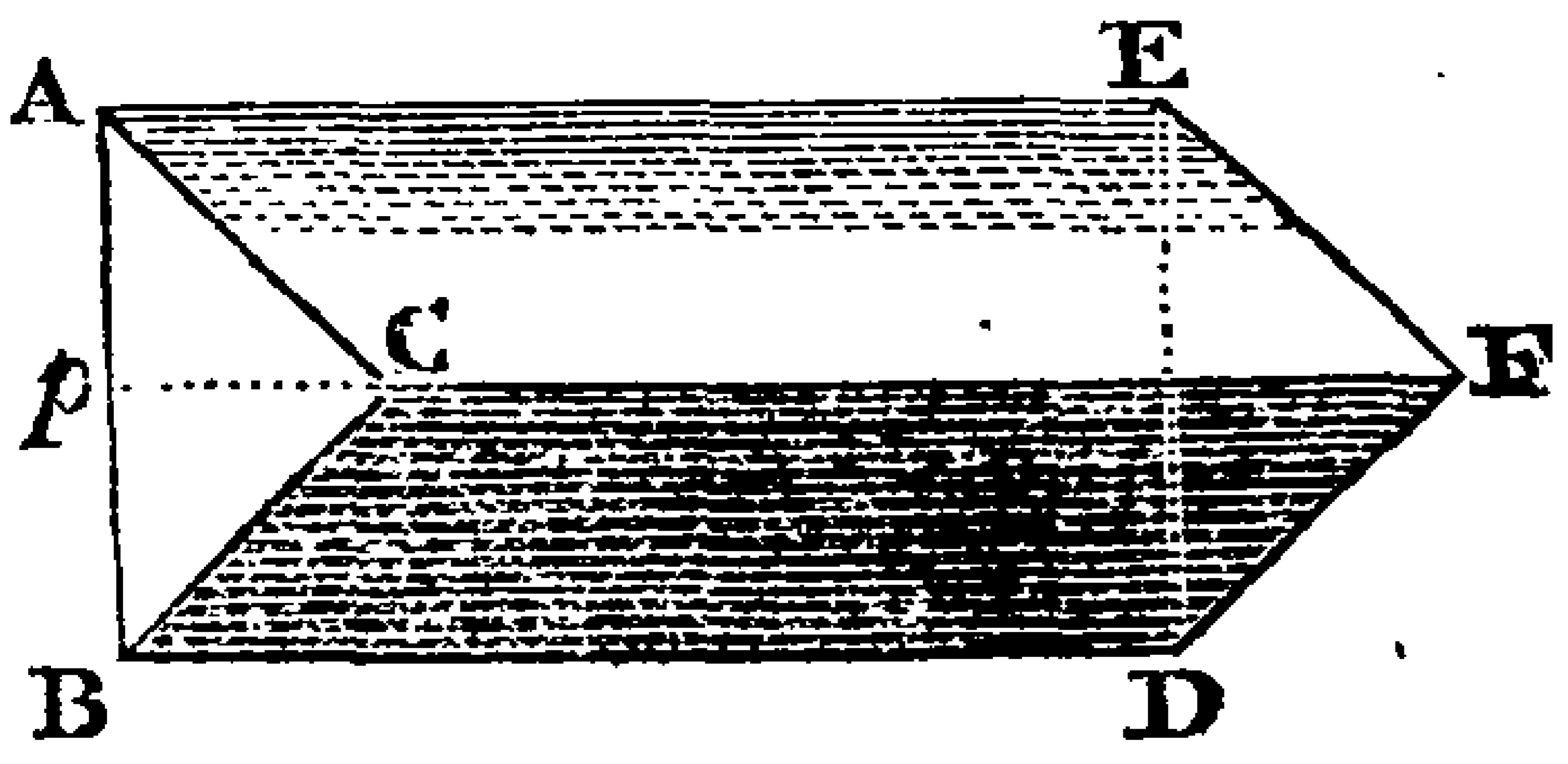
Def. A *Prism* is a Solid Figure contained under several Planes, whose Bases or Ends are equal and alike. It takes its Name from the Figure of its Base, be it either a *Triangle*, *Square*, *Pentagon*, or any other *Polygon*.

Rule.

Find the Area at the End; multiply that by the Length, and the Product is the Content. *

Example.

Let A B C D, &c. be a *Triangular Prism*, each Side of whose End A B C being 15.6 Inches; the Perpendicular *p* C thereof 13.51 Inches; and the Length C F 95 Inches, what is its Solid Content?



Operation. $13.51 \times 7.8 = 105.378 \times 95 = 10010.910$ Inches, the Solidity required.

☞ The *Superficial Content*. Find the Area of one of the Sides, which multiply by the Number of Sides; to this Product add the two Areas of the Ends, and that Sum will be the Content.

* All *Prisms* are composed of several equal and similar Surfaces conceived to be laid upon each other.

Problem 4.

To Meaſure a *Pyramid.*

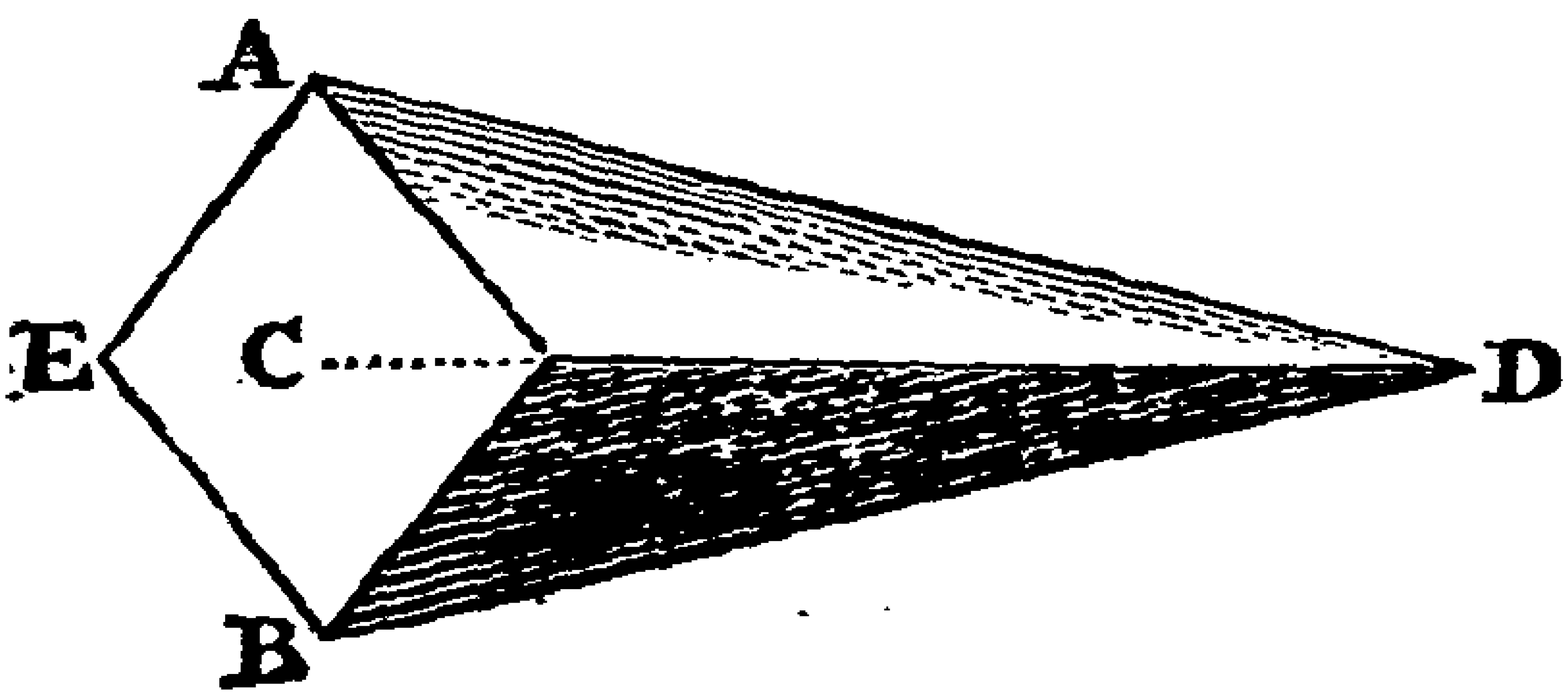
Def. A *Pyramid* is a Solid, whoſe Baſe may be either a *Triangle*, *Square*, or any other Figure, having its ſeveral Sides meeting in a Point at the Top.

Rule.

Multiply the Area of the Baſe, by $\frac{1}{3}$ (one third) of the Perpendicular Height (or Length); and the Product is the Solid Content.*

Example.

Suppoſe A B C D be a *Square Pyramid*, each Side of the Baſe being 18.5 Inches; and the Perpendicular Height C D 180 Inches, what is its Solid Content?



Operation. $18.5 \times 18.5 = 342.25 \times 60 = 20535$ Inches, the Solidity required.

☞ For the *Superficial* Content, multiply Half the Sum of the Sides of the Baſe, by the ſlant Height of the *Pyramid*; to which add the Area of the Baſe, and that Sum will be the *Superficial Content* of the whole Pyramid.

* Every *Pyramid* is equal to $\frac{1}{3}$ of a *Priſm* of the ſame Baſe and Altitude, or Length.

Pro-

Problem 5.

To meaſure a *Cylinder.*

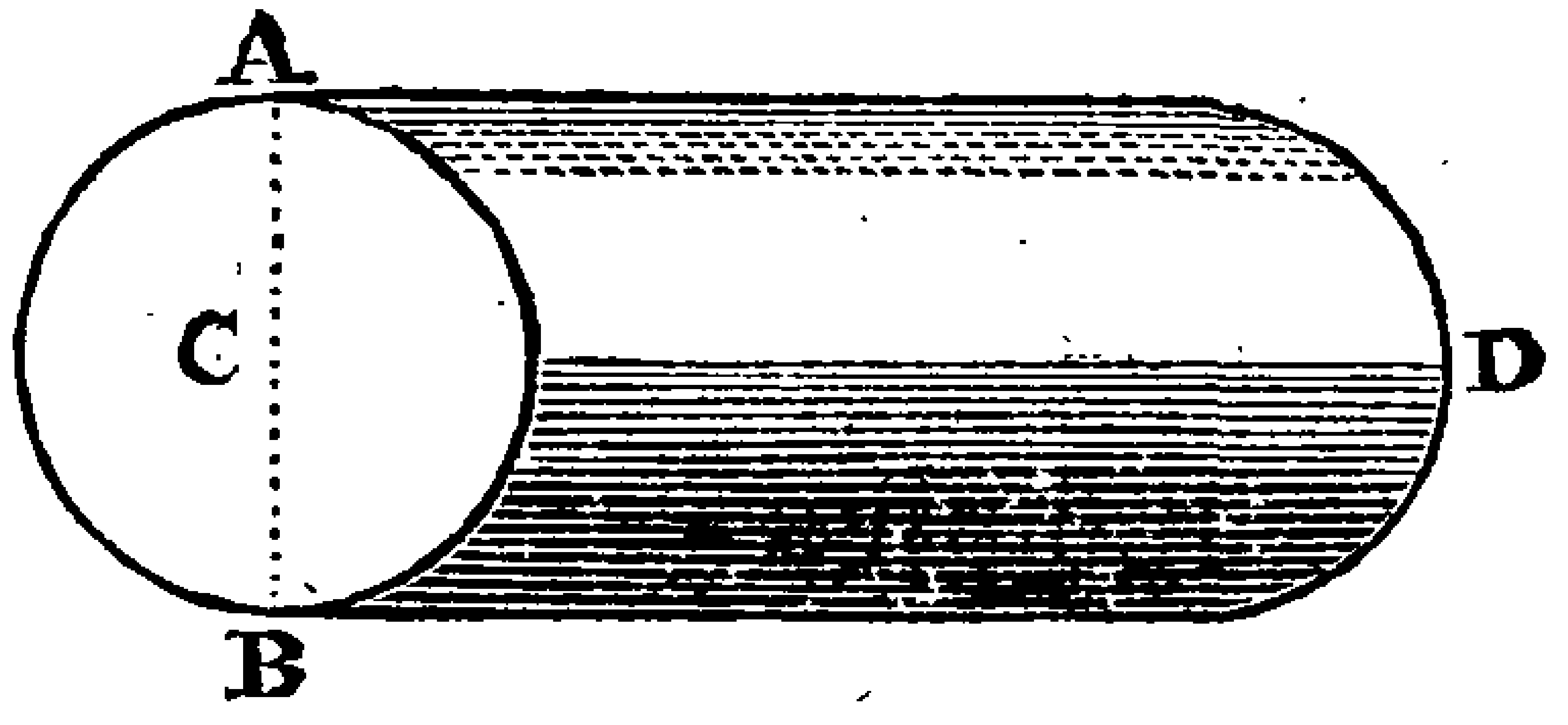
Def. A *Cylinder* is a Solid exactly like a rolling Stone in a Garden: It is in all Parts of an equal Thickneſs or Diameter, and its Ends are Circles, and parallel to each other.

Rule.

Multiply the Area of the Baſe by the Length, and the Product is the Solid Content. *

Example.

Let A B C D be a Cylinder, whoſe Diameter A B is 21.5 Inches, and the Length C D 60 Inches, what is its Solid Content?



Operation. 21.5×21.15=462.25×.7854=363.051150× 60=2178.309 Inches, the Solidity required.

The *Superficial Content* is double the Area of one of the Circles at the End, and of a Parallelogram, one of whoſe Sides is the Circumference of the Cylinder, and the other the Length thereof.

* All *Cylinders* are compoſed of a Number of Circular Surfaces of the ſame Size laid upon each other.

Pro-

Problem 6.

To meaſure a *Cone.*

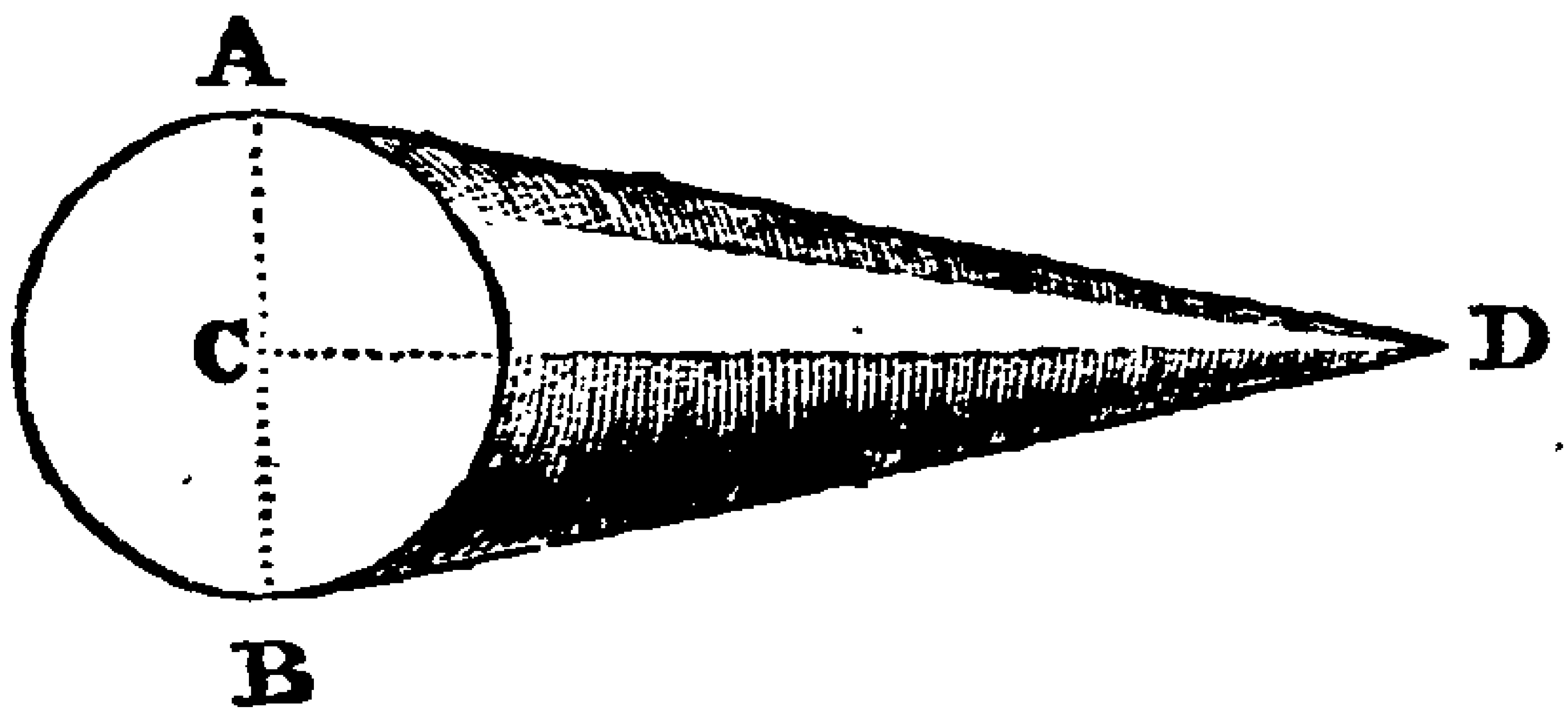
Def. A *Cone* is a Solid, having a Circular Baſe, an growing leſs and leſs till it ends in a Point. It may b juſtly repreſented by the Form of a Sugar Loaf.

Rule.

Multiply the Area of the Baſe, by one third Part of th Perpendicular Height, and the Product is the Soli Content. *

Example.

Let A B C D be a *Cone*, the Diameter of whoſe Baſ A B is 26.5 Inches, and the Perpendicular Height C D 6 Inches, what is its Solid Content?



Operation. $26.5 \times 26.5 = 702.25 \times .7854 = 551.547150 \times 21 = 11582.49015$, the Solid Content in Inches.

For the *Superficial Content*, multiply Half the Circumference at the Baſe, by the ſlant Side; to this Product add the Area of the Baſe, and that Sum is the *Superficial Content*

* Every *Cone* is equal to $\frac{1}{3}$ of a Cylinder of the ſame Diameter and Altitude or Length.

Pro

Problem 7.

To meaſure the *Fruſtum* of a Pyramid.

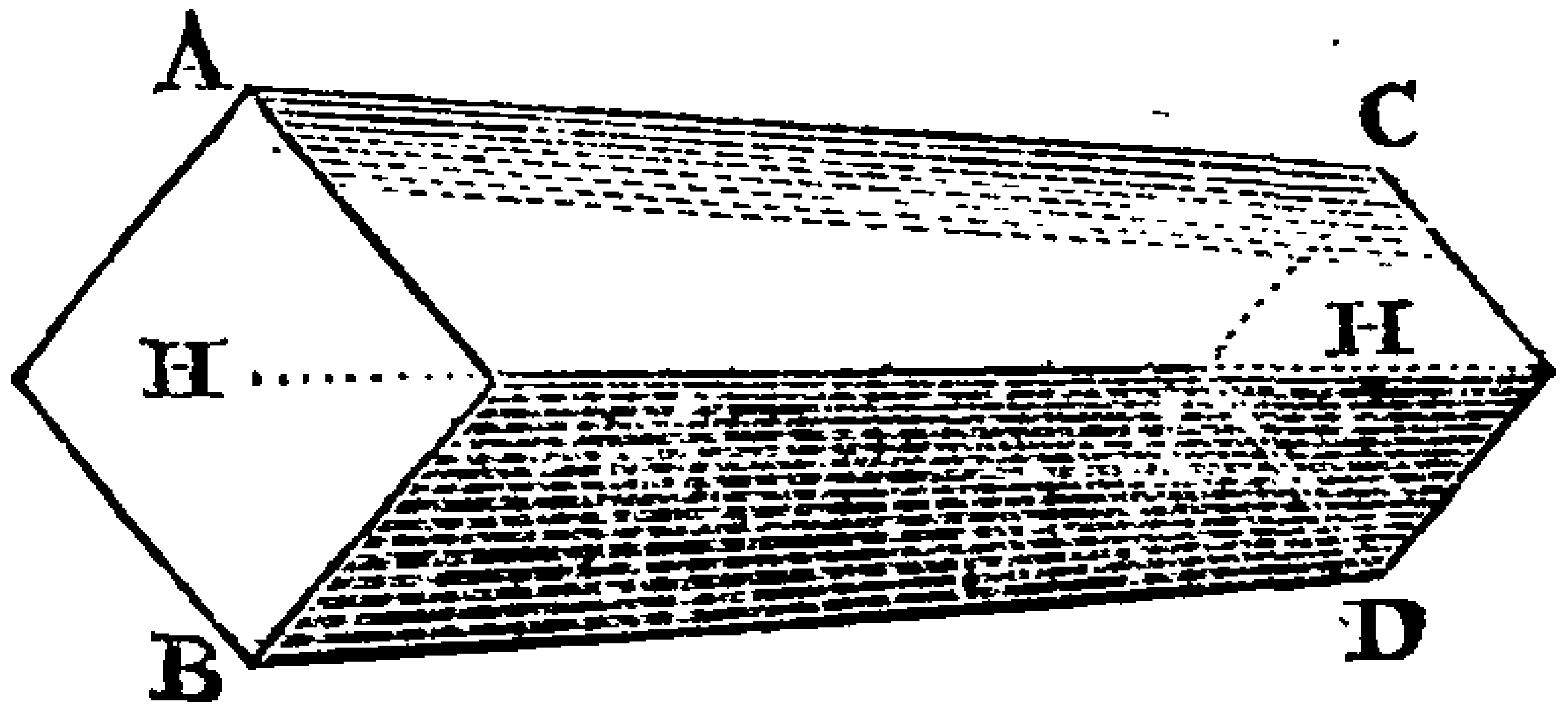
Def. A *Fruſtum* of a Pyramid is the Part remaining, when the Top or End is cut off parallel to the Baſe.

Rule.

Multiply the Areas of the Baſes together, and to the Square Root of this Product, add the two Areas; that Sum multiplied by one third of the Height will be the Content.

Example.

Let A B C D be the *Fruſtum* of a Square Pyramid; the Side of the greater Baſe 18 Inches; the Side of the leſſer 12 Inches; and the Length or Height H H 216 Inches; what is the Solid Content?



Operation. $324 \times 144 = 46656$; whoſe □ Root is $= 216$; then $324 + 144 + 216 = 684 \times 72 = 49248$ Inches, the Solid Content.

☞ For the *Superficial Content*, multiply the Sum of the Sides of the two Ends by Half the ſlant Height; to which Product add the Areas of the two Ends; that Sum will be the Superficial Content of the whole Fruſtum.

Pro-

Problem 8.

To meaſure the *Fruſtum* of a Cone.

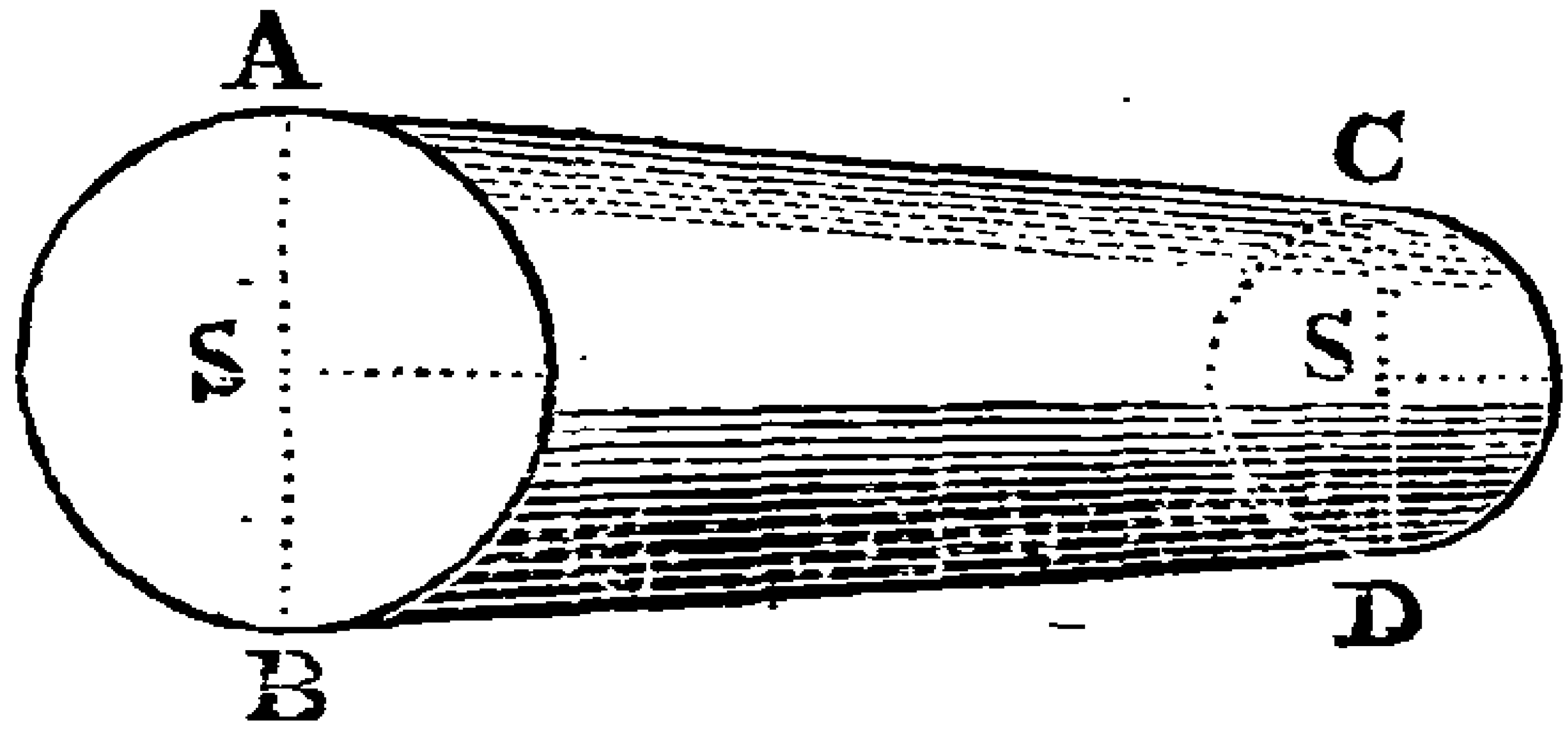
Def. A *Fruſtum* of a Cone is that Part which remains after the Top or End is cut off by a Plain parallel to the Baſe.

Rule.

Multiply the Area of the two Ends together; to the Square Root thereof, add the two Areas; that Sum multiplied by one third of the Height gives the Solid Content.

Example.

Let A B C D be the *Fruſtum of a Cone*; whoſe greater Diameter A B is 18 Inches; and the leſs Diameter C D is 9 Inches; and the Height or Length S S 171 Inches; what is its Solid Content?



Operation. $63.6174 \times 254.4696 = 16188.69433104$, whoſe Root is 127.2348. Then $127.2348 + 63.6174 + 254.4696 = 445.3218 \times 57 = 25383.3426$ Inches; the Solidity required.

For the *Superficial Content*, multiply the Sum of the Circumferences of the two Ends, by half the ſlant Height of the Fruſtum, to which Product add the Areas of the two Ends, and that Sum will be the Superficial Content of the whole Fruſtum.

Pro-

Problem 9.

Another Way to find the Content of a *Frustum* of a *Pyramid* or *Cone*, having the top Part cut off parallel to the Base.

Rule.

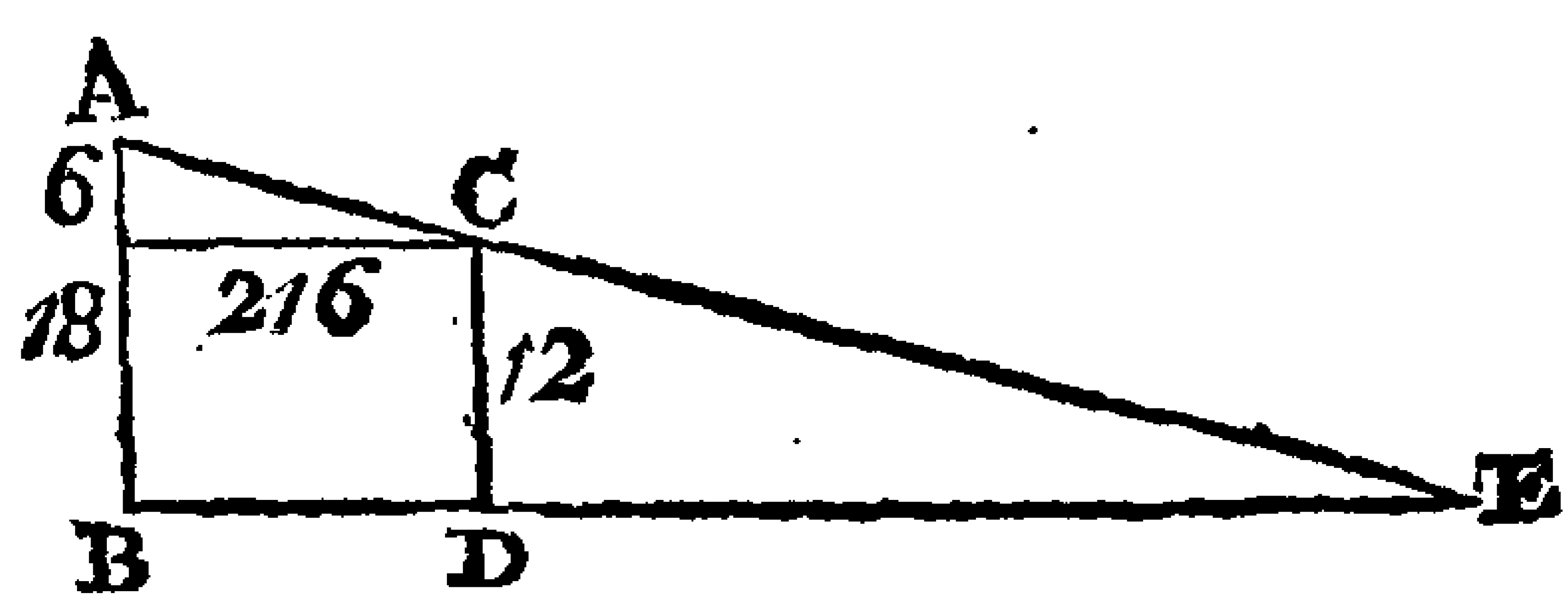
Find the whole Length of the *Pyramid* or *Cone* thus:

As the Difference of the Breadths of the two Ends,
Is to the Length between them:
So is the Breadth of the less End,
To the Length of the Part cut off.

Then find the Content of the whole Pyramid or Cone; and also the Content of the top Part; and subtract the Top from the Whole, there will remain the true Content of the *Frustum* sought.

Example 1.

Suppose, as in Problem the 7th, the Side A B of the greater End of a Square *Pyramid* be 18 Inches; the Sides C D of the less End 12 Inches; and the Length B D 216 Inches, what is its Content by this Rule?



Operation. As 6 : 216 : : 12 : 432 = Length of the top Part D E; then 432 + 216, the Length of the bottom Part, is = 648, the Length of the whole *Pyramid* B E. Then □ 18 = 324 × 216 = $\frac{1}{3}$ of whole Length is = 69984, the Content of the whole Pyramid. And □ 12 = 144 × 144 = $\frac{1}{3}$ of top Part is = 20736, the Content of the top Part, which subtracted from the Whole leaves 49248 Inches, the Content of the *Frustum* as before.

Example 2.

Suppose, as in Problem the 8th, the greater Diameter of a *Frustum* of a Cone be 18 Inches; the less Diameter 9 Inches, and the Length 171 Inches, what is its Solid Content?

Operation. As 9 : 171 : : 9 : 171, the Length of the top Part. Then $171+171=342$, the whole Length of the *Cone*.

Then $18\square=324\times.7854=254.4696\times114=29009.5344$, the Content of the whole Cone.——Next, $9\square=81\times.7854=63.6174\times57=3626.1918$, the Content of the top Part; which taken from the Whole, leaves 25383.3426, the Content of the *Frustum*, and is the same as before.

*** By this, and the two last Problems, tapering Timber, both round and square, is accurately measured.

Pro-

Problem 10.

To measure a Sphere or *Globe*.

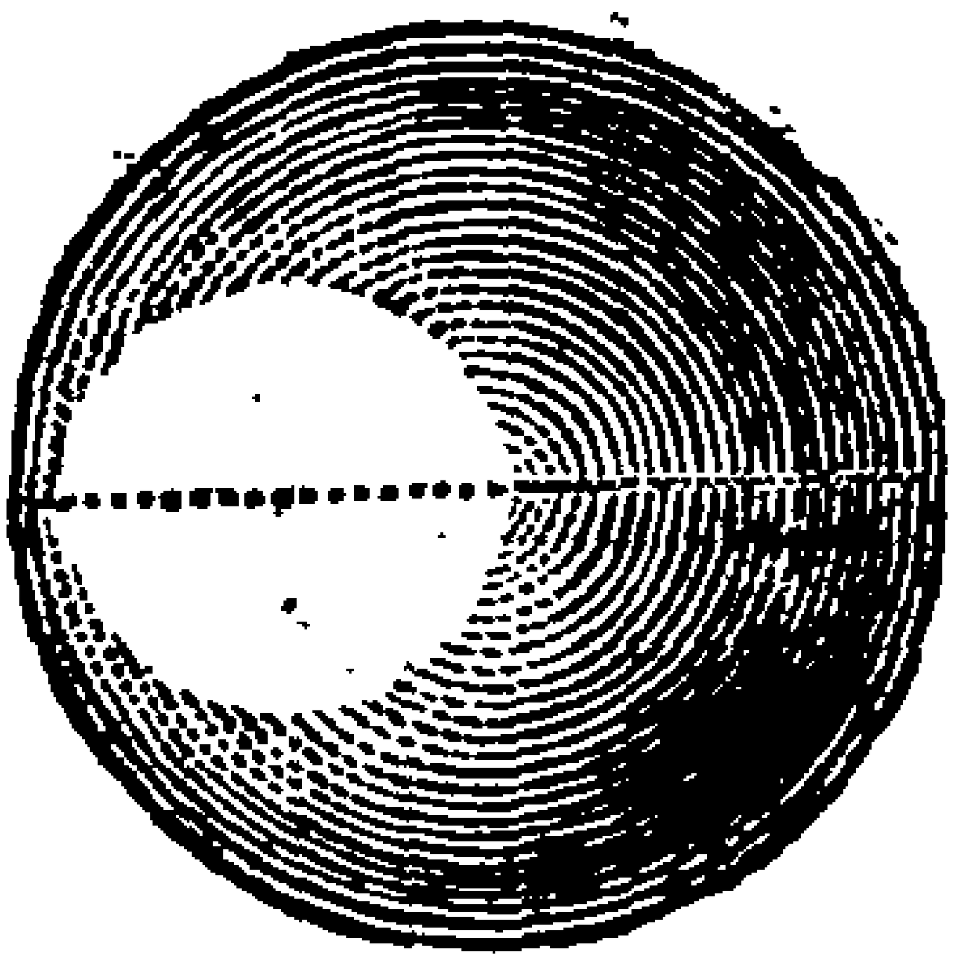
Def. A *Sphere* or *Globe* is a round solid Body, every Part of whose Surface is equally distant from a Point within, called its Center.

Rule.

Find the Area of a Circle in the middle of the *Globe*, and multiply it by $\frac{2}{3}$ (two thirds) of the Height, that Product will be the Solid Content. *

Example.

Let A B C D be a *Globe* whose Diameter is 20 Inches, what is its Solid Content?



Operation. $20 \times 20 = 400 \times .7854 = 314.16 \times 1.33333 =$ 4188.798 Inches, &c. the Solidity required.

For the *Superficial Content*, multiply the Diameter by the Circumference, and that Product will be the Superficial Content required.

* Every *Globe* is equal to $\frac{2}{3}$ of a Cylinder of the same Diameter and Altitude.

Problem II.

Another Way to meaſure a *Globe*.

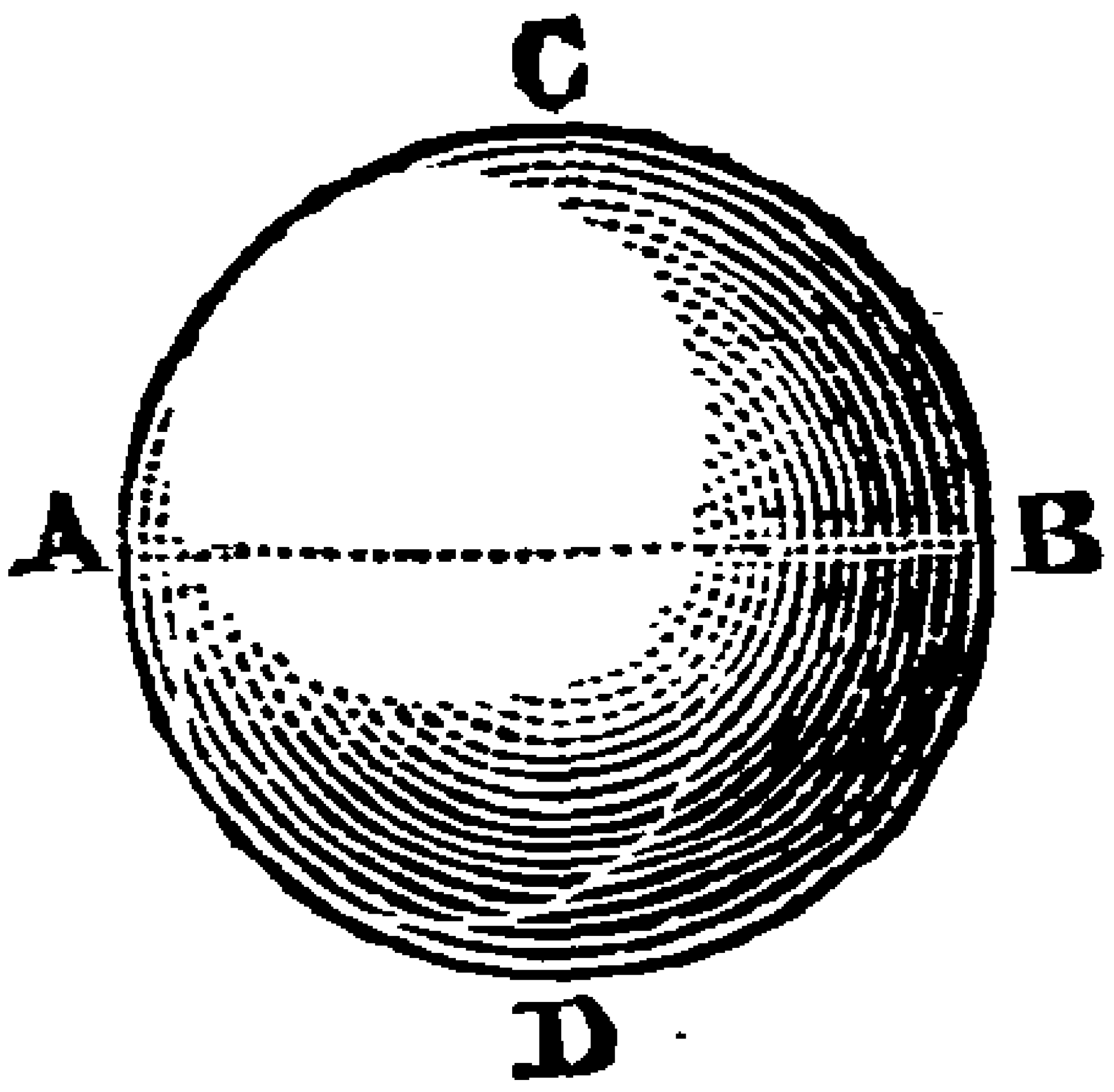
*** All *Globes* are to each other as the Cube of their Diameters; and it is found, that if a Globe be 1 in Diameter, its Solid Content will be .5236; therefore this

Rule.

Cube the Diameter, and multiply that by .5236, it will give the Solidity required.

Example.

Let A B C D be a *Globe*, whoſe Diameter A B is 20 Inches, (as before) what is its Content?



Operation. $20 \times 20 \times 20 = 8000 \times .5236 = 4188.8$, nearly the ſame as in the laſt Problem.

Another Way to meaſure a Globe. A 21 : 11 : : Cube of the Diameter to the Solid Content. This Rule will ſerve for thoſe who do not underſtand Decimals.

Pro-

Problem 12.

To meaſure the *Fruſtum* of a Globe.

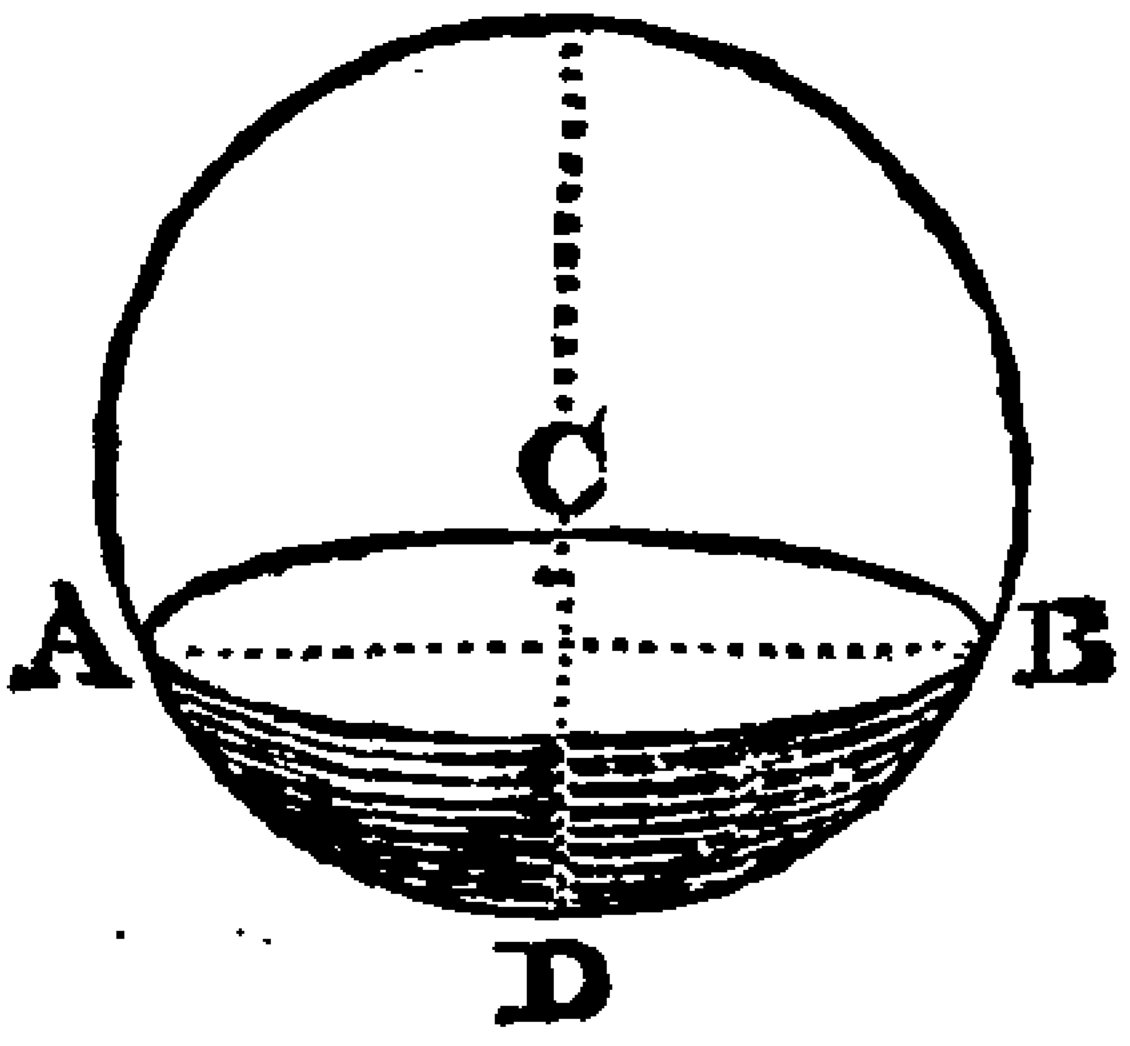
Def. The *Fruſtum* of a Globe is a Part cut off leſs than Half the Globe.

Rule.

To three Times the Square of the Semidiameter of the *Fruſtum*'s Top, add the Square of the *Fruſtum*'s Height; this Sum multiplied by the *Fruſtum*'s Height, and that Product again multiplied by .5236, will give the Solid Content.

Example.

Let A B C D be the Fruſtum of a Globe; let A B, the Diameter at Top, be 16 Inches, and C D, the Height, be 4 Inches, what is the Solidity?



Operation. $8 \times 8 = 64 \times 3 = 192 + 16 = 208 \times 4 = 832 \times .5236 = 435.6352$ Inches, the Solidity required.

For the Curved or Superficial Content, multiply the whole Circumference of the Sphere by the Height of the Segment, and the Product will be the Content required. The Diameter and Circumference of the Sphere may be found by Problems 15 and 10 of *Planometry*.

Problem 13.

To meaſure an *Oblong*, or an Oblate Sphere.

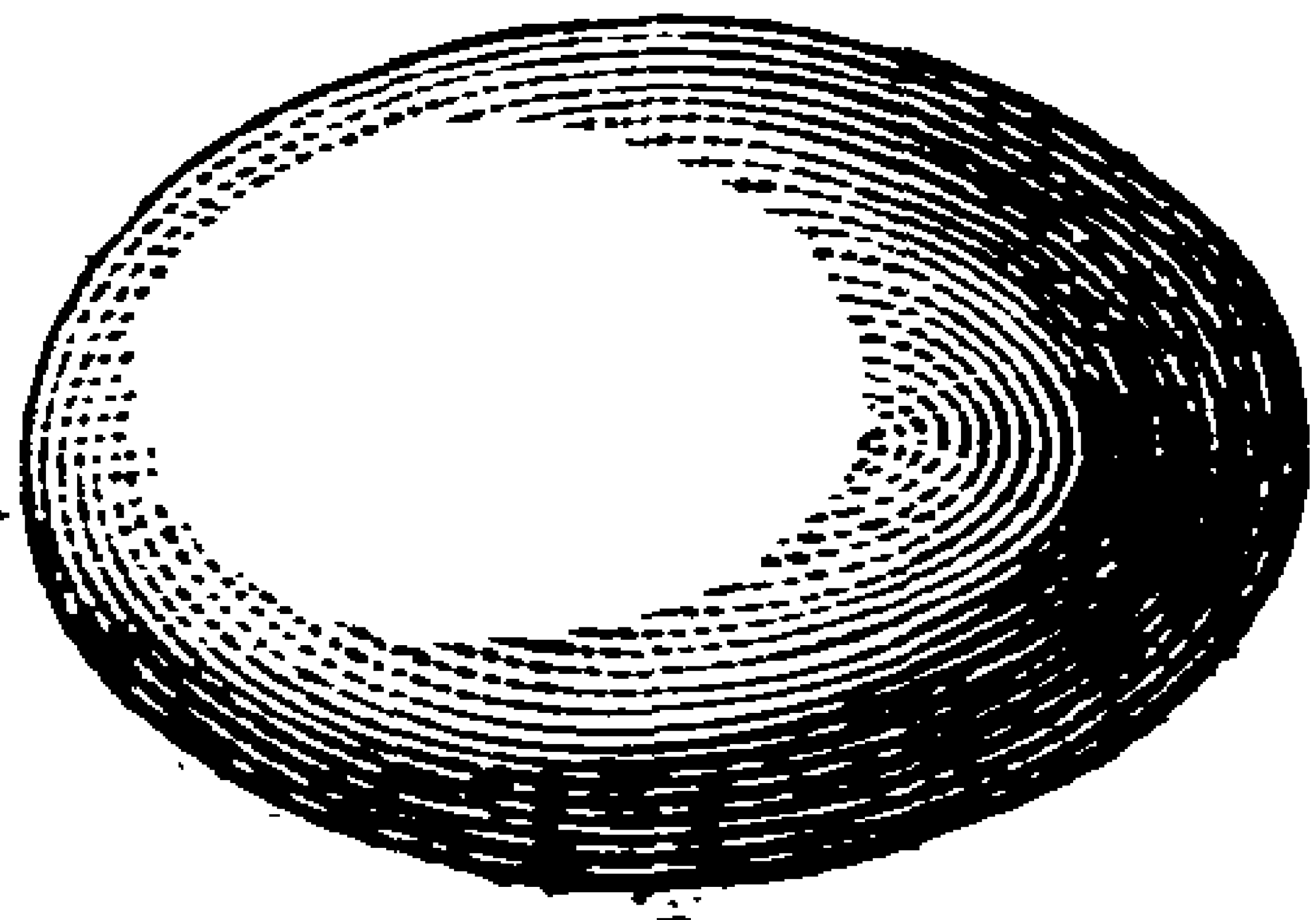
Def. If a Globe be ſuppoſed to be puſhed out of its real Figure, ſo as to become longer than before, like an Egg, it is called an *Oblong Sphere*; if it be compreſſed ſo as to make it flatter, like an Orange, it is called an *Oblate Sphere*; and the Rule for meaſuring each is thus.

Rule.

Square the Diameter in the Middle, and multiply that Product by the Length, which Product multiply again by .5236, and it will give its true Solidity or Content. *

Example.

Let A B C D be an *Oblong* Sphere; and let the Diameter D C be 30 Inches, and the Length A B 50 Inches, what is its Solid Content?



Operation. 30×30=900×50=45000×.5236=23562 Inches, the Content ſought.

* See the Reaſon of this Rule in the Cubature of the *Conic Sections*.

Pro-

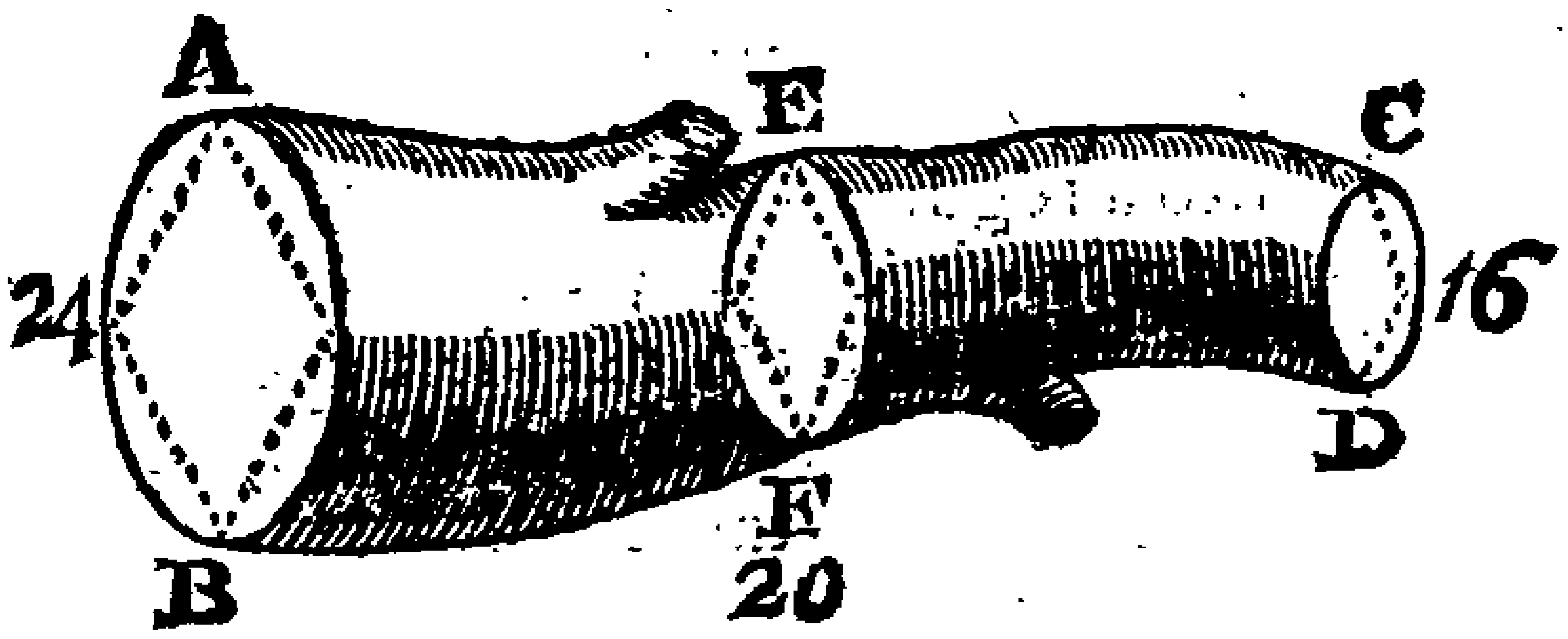
Problem 14.

To measure an *Irregular Solid.*

All *Irregular Bodies* must be reduced as near as possible to regular ones; no particular Rule therefore can be laid down; but from a due Consideration of the foregoing Rules, a Method may be found for taking the Dimensions of any solid Body, how irregular soever, that shall give the Content near enough the Truth.

Example.

Let A B C D be a Piece of Timber 28 Feet long; and the Circumference at A B 24 Inches, at E F 20 Inches, and at C D 16 Inches, what is its Content?



Workmen usually add the three Circumferences together, taking $\frac{1}{3}$ of the Sum for a mean Circumference; then $\frac{1}{4}$ of which Circumference they make the mean Side of a Square, and measure it as square Timber, by multiplying that Side by itself, and that Product by the Length, and dividing by 1728, if the Length was Inches; or by 144, if the Length was given in Feet; this Quotient gives the Content in square Feet.

This Method of finding the mean Dimensions of a Square, though much in Use, is in many Cases very erroneous, and gives the Content less than the Truth; which is reckoned by Workmen as an Allowance for the *Slabs*. But to have the true Content without any Deduction of *Bark* or *Slabs*, it must be measured as the Frustum of a Cone; or it may be reduced to a *Cylinder*, making the mean Circumference, when truly found, the Circumference of a *Cylinder*, and finding its Content accordingly.

But to get the Content *very near* the Truth, by having the Circumference given; multiply the Square of $\frac{1}{5}$ of the Girth by twice the Length, and the Product will be the Solidity near enough.

Problem 15.

To measure an *Irregular Body* another Way, and more exactly.

Rule.

Put the Body into a regular Vessel (either square or round) and fill it to the Brim with Water; then take out the Body, and measure the Vacuity of the Vessel left between the Surface of the Water and the Top of the Vessel, and that will give the Solidity of the Body taken out.

Pro-

Problem 16.

To find the *Side* of a *Cube* equal to any given Solid, whether *Parallelopipedon*, *Cone*, *Cylinder*, *Sphere*, &c. Only extract the Cube Root of the Content of the given Solid in Inches, and that Root will be the Side of a Cube equal to it.

Note. By these Problems are measured *Timber*, *Stone*, &c. as well as the *Vacuity of any Vessel* in Shape of, or reducible to any of those Figures.

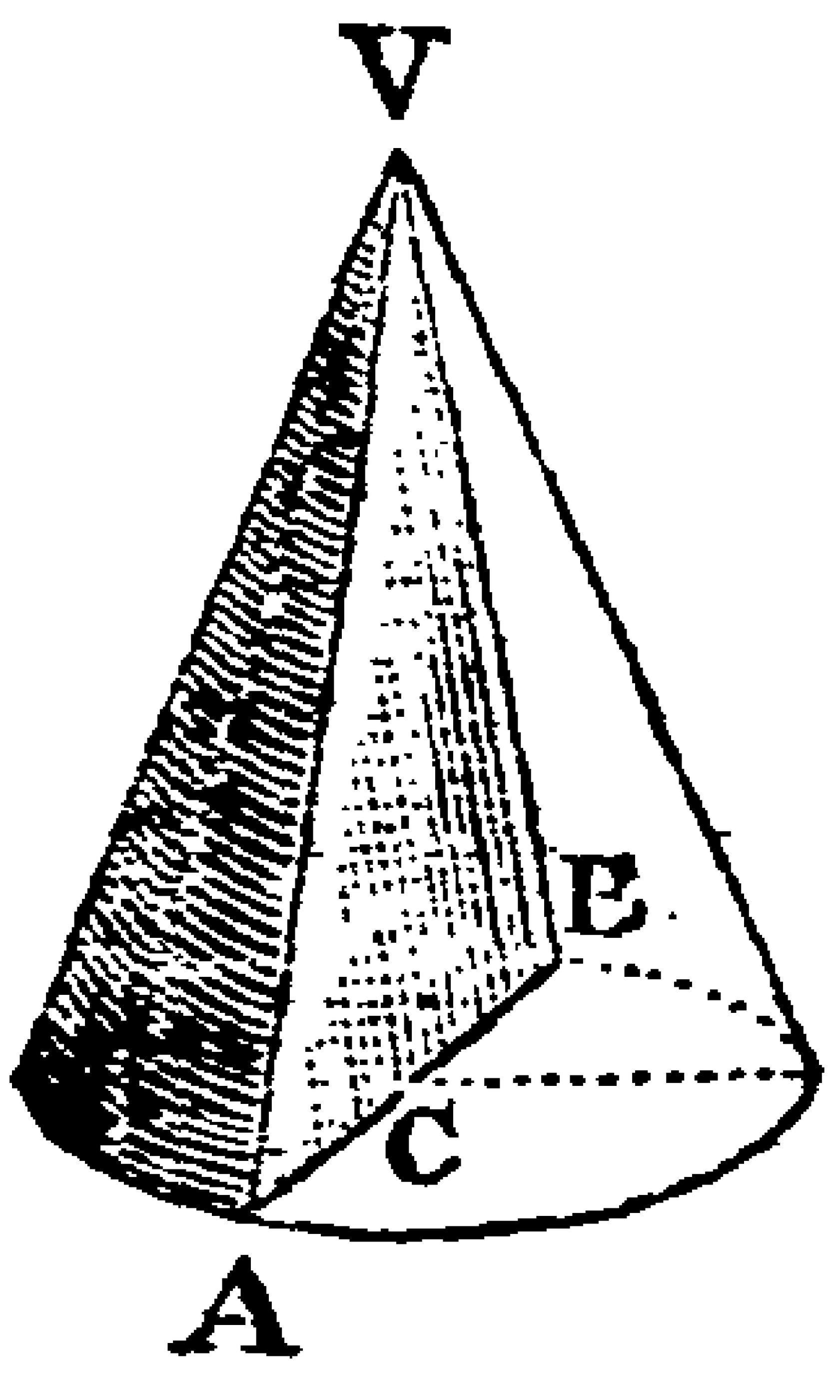
Also note. If any *Chest*, *Bing*, *Vessel*, &c. in Form of any of the foregoing Figures, have its Content found in Inches, the Quantity of *Water*, *Ale*, *Wine*, *Malt*, *Corn*, &c. it will contain may be found by dividing the Inches by

282 for Gallons of Water, Ale, Beer.
231 for Gallons of Wine, Cyder.
2152.42 for Bushels of Malt, Corn.
1728 for Feet Solid.

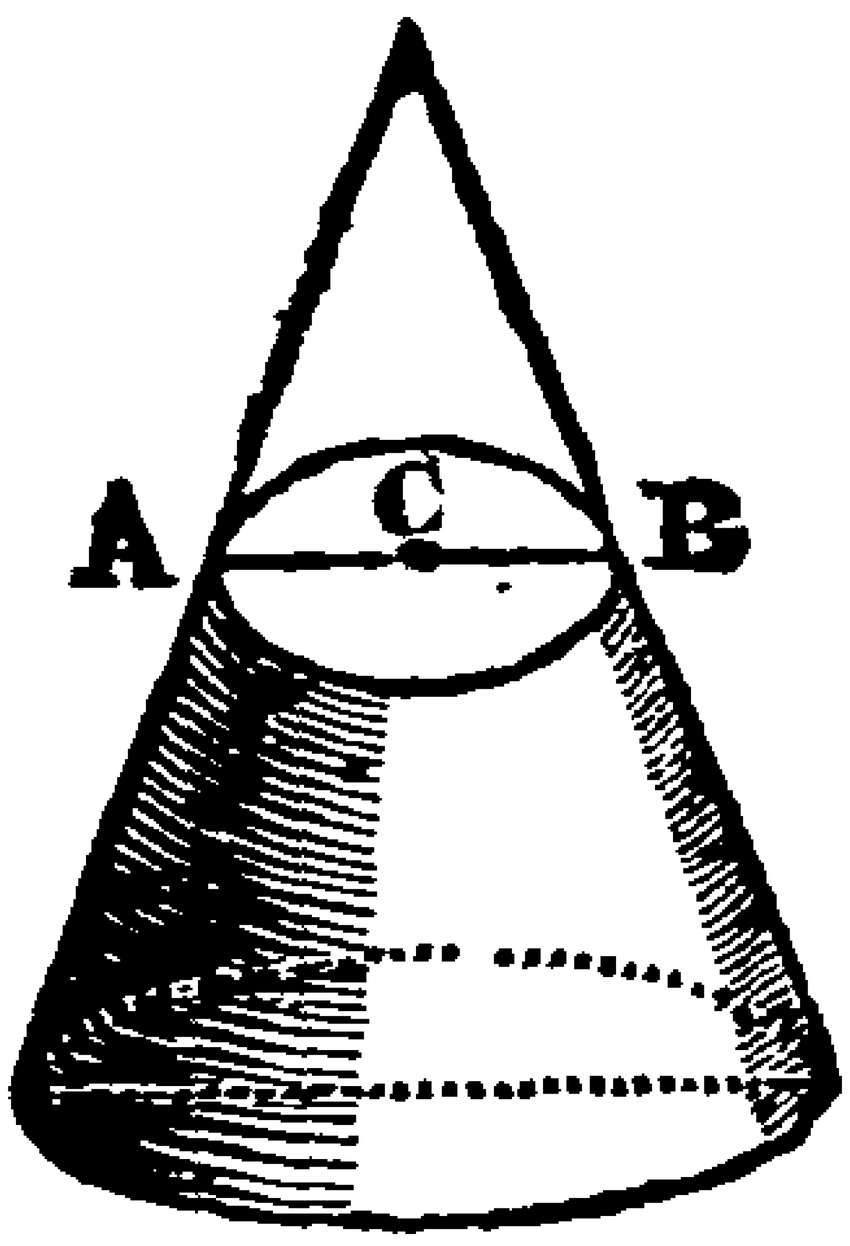
THE CONIC SECTIONS.

A *Cone* is a solid Figure having a circular Base, as A B, and growing regularly smaller till it ends in a Point at the Top, called its Vertex, at V. Every such Solid may be cut by Planes into five Sections following:

(1st.) If a *Cone* be cut directly down the Middle or Axis V C, the Plane or Superficies of that Section will be a *Triangle*, as A V B.

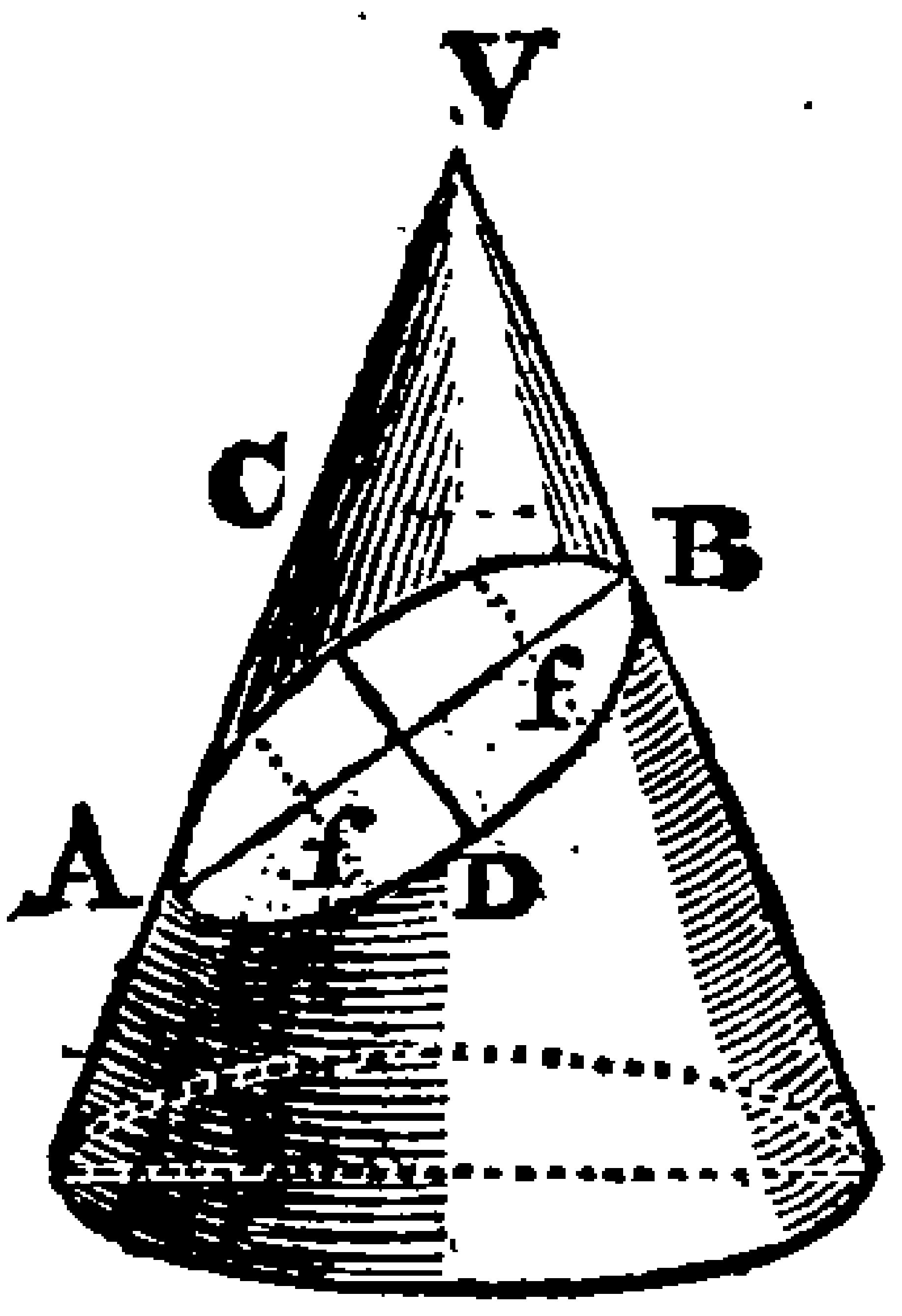


(2d.) If the *Cone* be cut through any where parallel to its Base, the Plane of that Section will be a *Circle*. The Point C in the Middle is called its Center or *Focus*; and the Line going through it, its Diameter, or *Latus Rectum*, as A B.



(3d.)

(3d.) If the *Cone* be cut through any where in an oblique Position, as at A B, the Plane of that Section will be an *Ellipsis*, or oblong Circle, as A B C D. The Line A B is called the *Transverse Diameter*, and the Line C D the *Conjugate Diameter*.

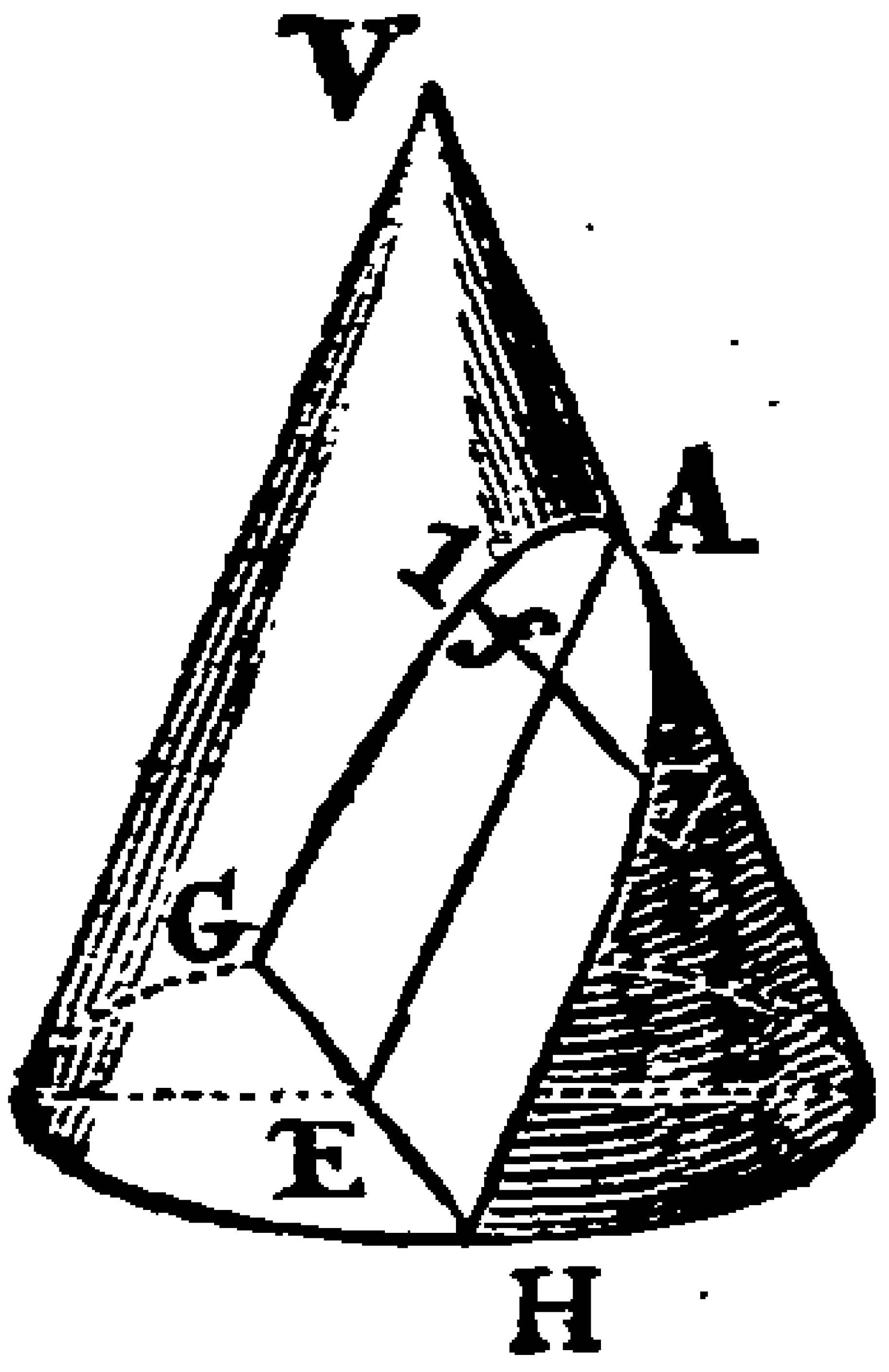


As every Circle hath one Center, called its *Focus*, or Burning Point; so every *Ellipsis* hath two Centers, called the *Foci*, or Burning Points of that *Ellipsis*, as *f f*.

All Lines drawn across the *Transverse* Diameter at right Angles are called *Ordinates*; and those two Lines which pass through the two *Foci* are more observable than the rest, and are called each—the *Latus Rectum*.

Note. The *Comets* all revolve round the Sun in Orbits of this Elliptic Figure; the Sun being situated in one of the *Foci*.

(4th.) If a *Cone* be cut in two Parts by a Line A E *parallel* to one of its Sides, this Section will be a *Parabola*, as G A H. The Line A E drawn through the Middle is called its *Axis*; any Line crossing this at Right Angles is called an *Ordinate*; and that Part of the Axis which is contained between the Vertex and the Ordinate is called the *Abscissa*. The *Focus* or Burning Point (for every *Parabola* hath but one) is always in the *Axis* towards the Top, as at *f*; and the *Ordinate*, or Line that goes through it, is called the *Latus Rectum*, as *l l*.



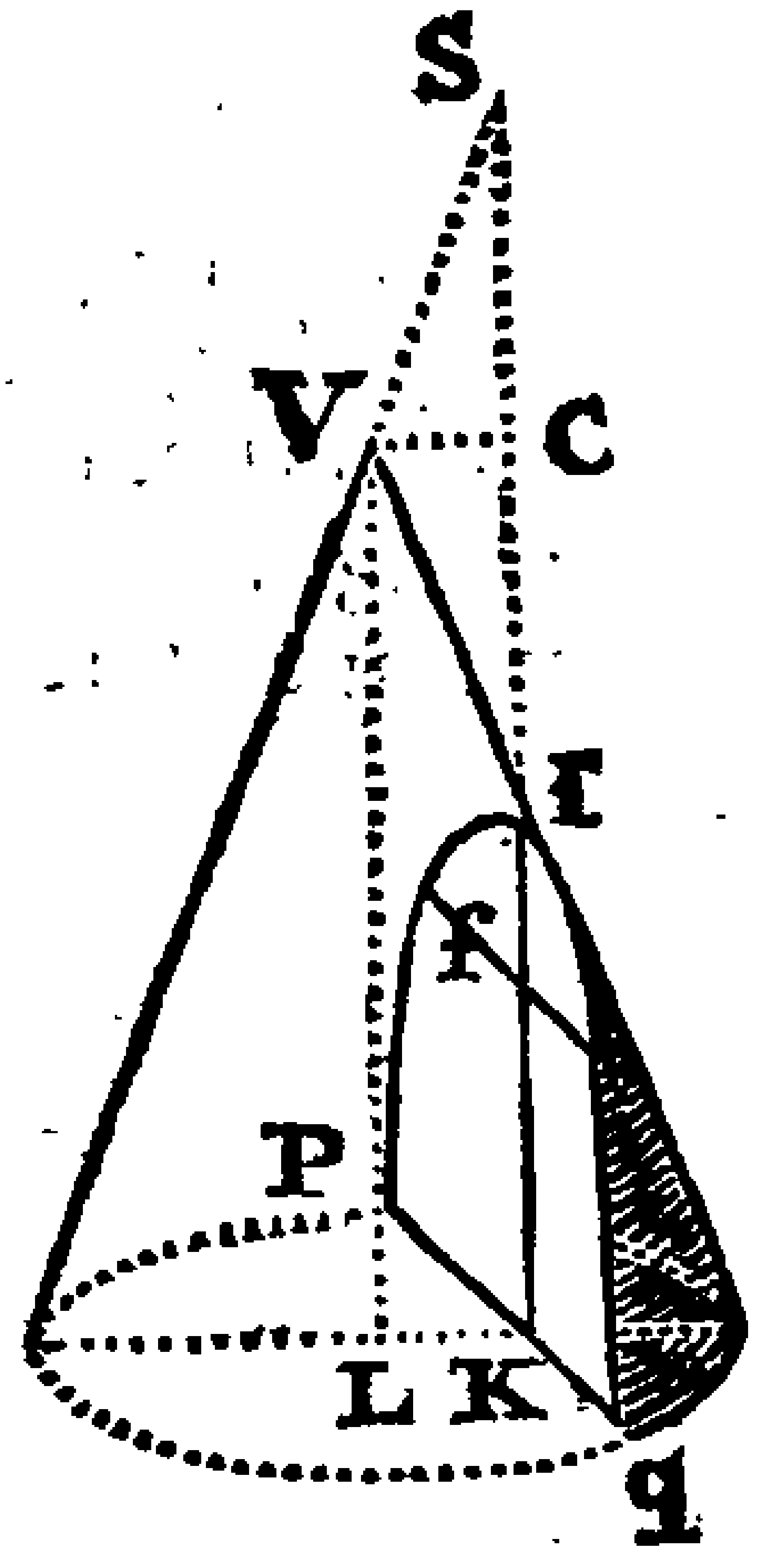
All Bodies projected, as *Arrows*, *Stones*, *Cannon Balls*, and *spouting Fluids*, move (whether upward or downward) in a Curve Line of this Kind, which never returns into itself, but opens wider and wider, as extended and carried on.

N. B. On this Principle depends the whole Art of *Gunnery*.

(5th.)

(5th.) If a *Cone* be cut by a Plane, as I K, any where parallel to its *Axis* V L, the Section ſo made will be an *Hyperbola*, as P I q K. The Line drawn down through the Middle of the *Hyperbola*, as I K, is called its *Axis*, or intercepted Diameter. The Part produced above the Section till it meet the other Side of the Cone continued, as S I, is called the *Tranſverſe Diameter*; and the Line drawn from the Vertex of the Cone V, perpendicular to the Tranſverſe Diameter S I, to C, is called the *Semi-conjugate Diameter* of the *Hyperbola*.

All Lines drawn at Right Angles acroſs the Axis are called *Ordinates*; and that which paſſes through its *Focus* (for every *Hyperbola*, as well as *Parabola*, hath one *Focus* or Burning Point near the Top of the Plane, as at *f*) is called the *Latus Rectum*, as in the *Parabola*, *Ellipſis*, and *Circle*.



That Part of the Axis which is contained between the Vertex I, and the Ordinate, wherever drawn, is called the *Abſciſſa*, as in the Parabola.

The

The Middle of the *Tranſverſe Axis* or Diameter S I is called the *Center*, as at C, of the *Hyberbola*. From which Point may be drawn two Right Lines out of the Section, called *Aſymtotes*, becauſe they will always approach nearer to the Sides of the *Hyberbola*, yet would never touch them, though both they and the Sides of the Hyperbola were infinitely extended.

Theſe are all the Sections that can be cut from a Cone; and from a due Obſervation of them, it is evidently ſeen how one Section degenerates into another. As the *Circle* into an *Ellipſis*; the *Ellipſis* into a *Parabola*; the *Parabola* into a *Hyperbola*; and the *Hyperbola* into a plain *Triangle*. And the Center of the Circle, which is its Focus, divides itſelf into two Foci ſo ſoon as the Circle begins to degenerate into an Ellipſis; but when the Ellipſis changes into a Parabola, one End of it flies open, one of its Foci vaniſhes, and the remaining Focus goes along with the Parabola when it degenerates into a Hyperbola. And when the Hyperbola degenerates into a plain Triangle, this *Focus* becomes the vertical Point of the Triangle, that is, the Vertex of the Cone. So that the Center of the Cone's Baſe may be truly ſaid to paſs gradually through all the Sections until it arrive at the Vertex of the Cone, ſtill carrying its *Latus Rectum* along with it, where both, the *Focus* and *Latus Rectum*, become coincident, and terminate in the Vertex of the Cone.

THE

THE QUADRATURE; OR, MENSURATION of SURFACES arising from the SECTIONS of a CONE.

Problem I.

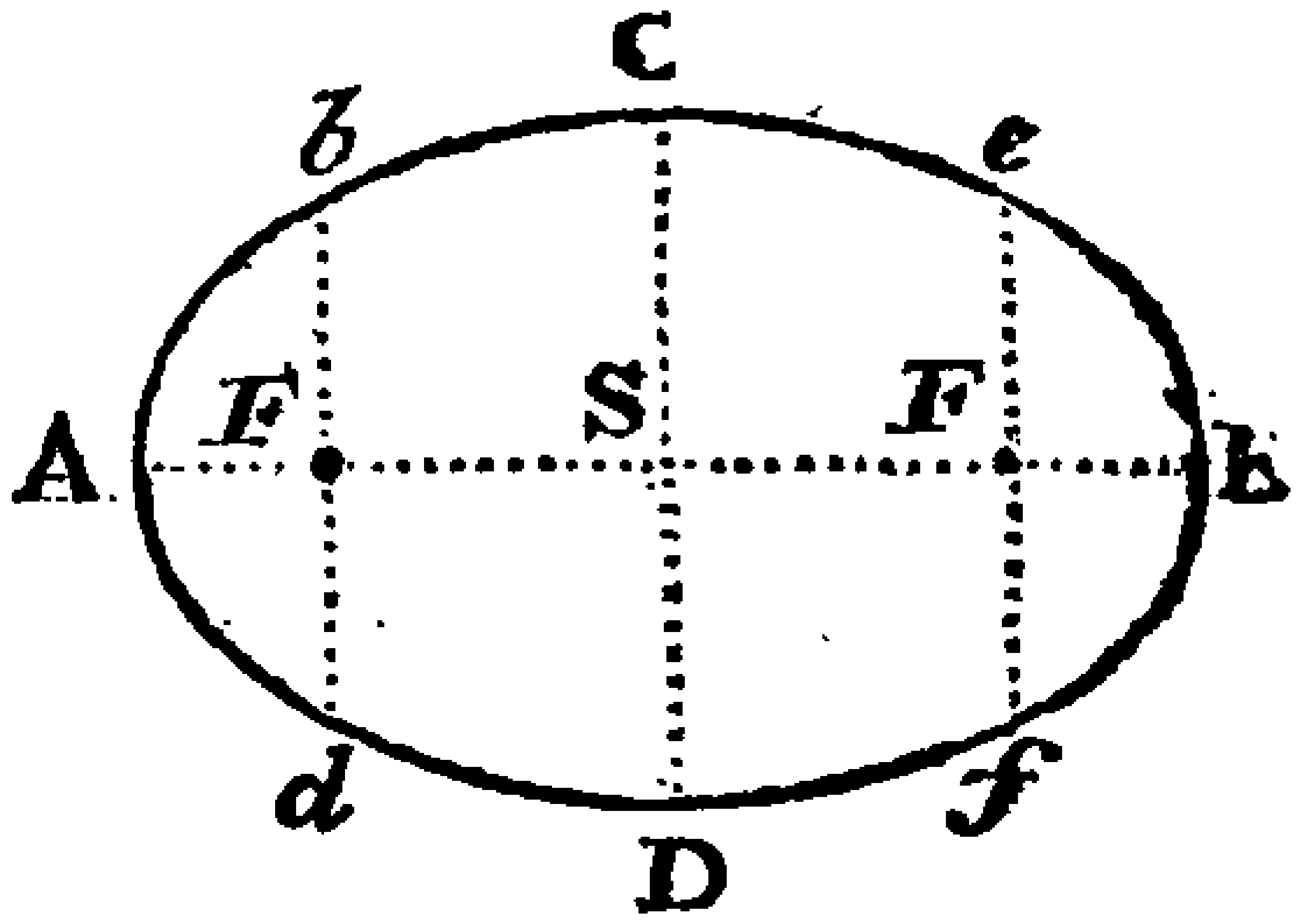
TO find the *Foci* of any Ellipsis.

Rule.

From the Square of $\frac{1}{2}$ the Transverse Diameter, subtract the Square of $\frac{1}{2}$ the Conjugate; the Square Root of the Remainder will be the Distance of each *Focus* from the Center, or Middle of the Ellipsis.

Example.

Suppose the Transverse Diameter A B of an Ellipsis be 24 Inches, and the Conjugate C D be 18 Inches, what is the Distance of each *Focus* F from the Center at S?



Operation. $144-81=63$, whose Root is 7.938 Inches, the Distance of each Focus from the Center S.

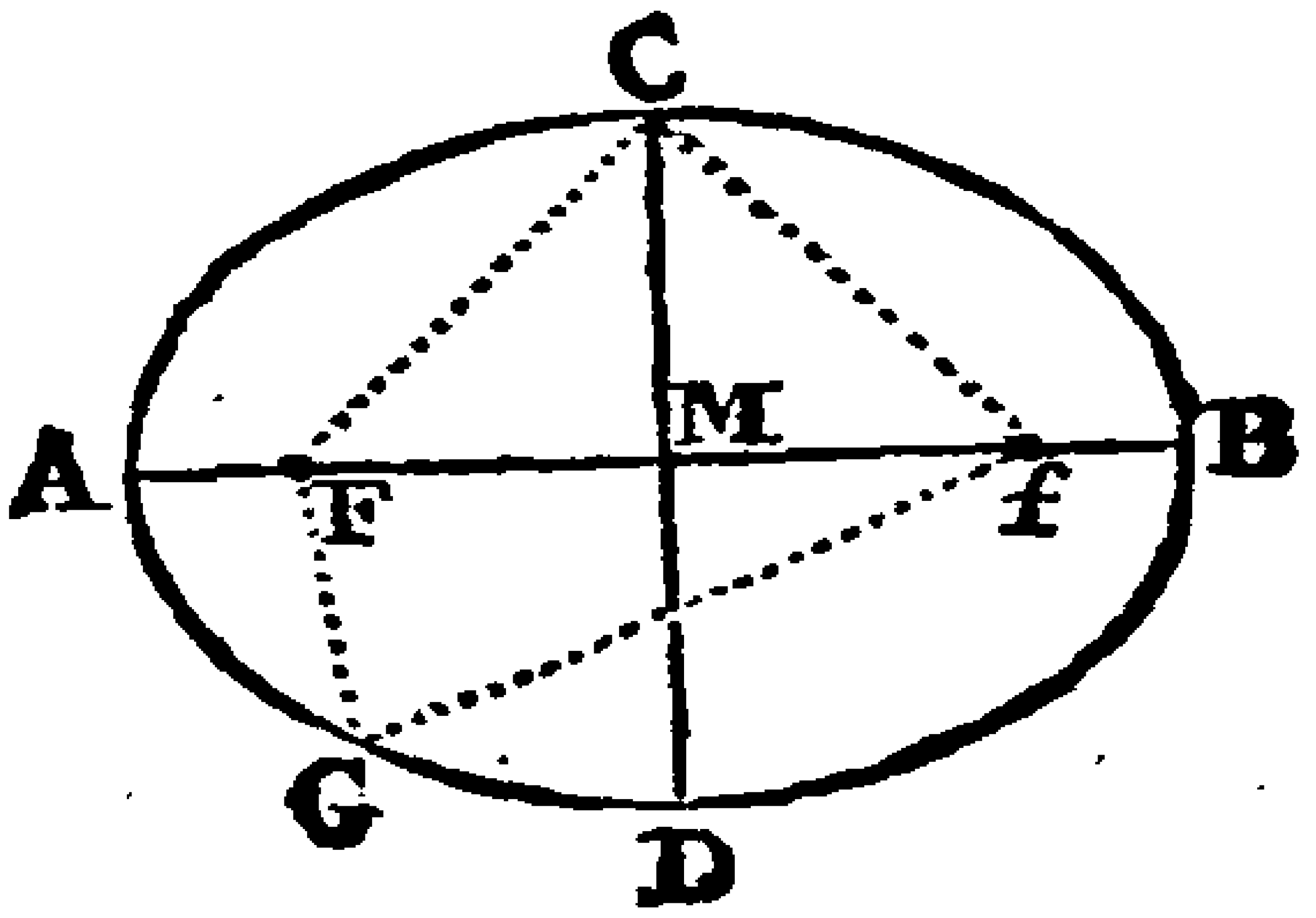
Each of the dotted Lines drawn through the two *Foci* parallel to the Conjugate is called the *Latus Rectum*; and the Length may be found by this Proportion:—As the Transverse Diameter A B, is to the Conjugate C D; so is the Conjugate C D, to the *Latus Rectum* *b d*, or *e f*.

Pro-

Problem 2.

To delineate an Ellipsis, having the Tranverse and Conjugate Diameter given.

Let A B be the longer Diameter, and C D the shorter, as in the following Figure.



Two Right Lines drawn from any Point in the Curve of the Ellipsis to the two *Foci*, are together always equal to the Transverse Diameter, *viz.* $f c + C F = A B$, or $f G + G F = A B$; and this holds good for every Point in the Circumference of the Ellipsis. Whence is derived the following Method of delineating that Figure.

Construction. First set the two Diameters across each other at Right Angles, and exactly in the Middle at M. Next, take Half the longer Diameter A M or M B in the Compasses, and setting one Foot in C or D, make a Dash each Way across the longer Diameter A B, at the Points F and *f*, which will be the two Foci or Centers on which the Ellipsis must be described. Then in the Points F and *f* stick up two Pins, and round them put a Thread so long, that it being doubled may reach from F to B, or from *f* to A, and tie it fast. Lastly, inserting a Black Lead Pencil between the Threads, draw them strait, and move the Pencil round, keeping the Threads moderately stretched, so will the Point of the Pencil describe the Periphery of the Ellipsis required.

Note. The Orbits of all the Planets are Elliptical, and the Sun is placed in or near to one of the Foci of each of them; and *that* in which he is placed, is called the *lower Focus*.

Problem 3.

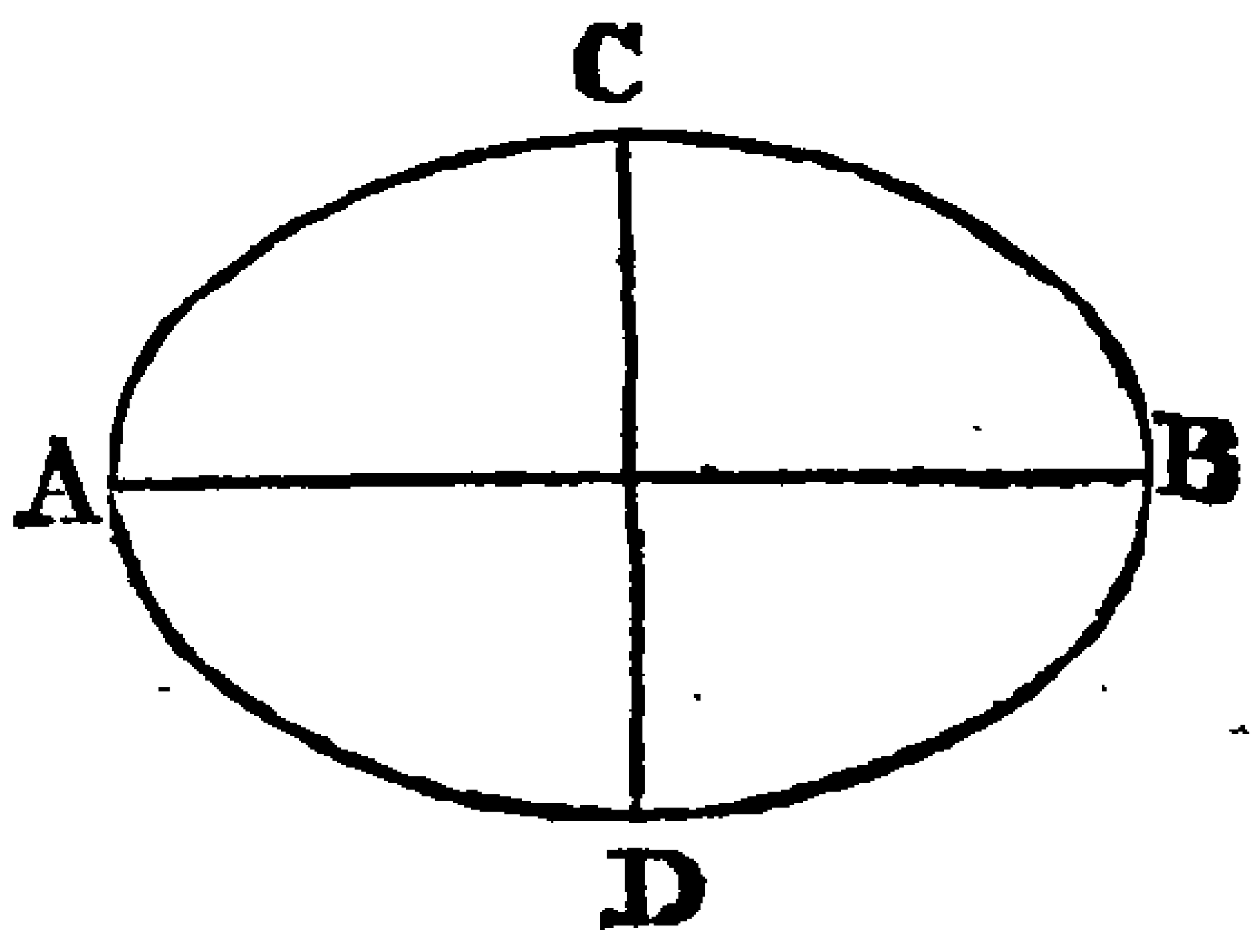
The two Diameters of an *Ellipsis* being given, to find its Circumference.

Rule.

Add the Squares of the two Diameters together, and extract the Square Root of their Sum; then to the double of that Root add $\frac{1}{3}$ of the shorter Diameter; this last Sum will be the Circumference very near.

Example.

Suppose the Tranverse Diameter A B be 24 Inches, and the Conjugate C D 18 Inches, what is the Circumference of the *Ellipsis*?



Operation. $\square$ A B $576 + \square$ C D $324 = 900$, whose $\sqrt{30}$; then $30 \times 2 = 60$; and $60 + 6 = 66$ Inches, the Circumference required.

Pro-

Problem 4.

To find the Area of an *Ellipsis*.

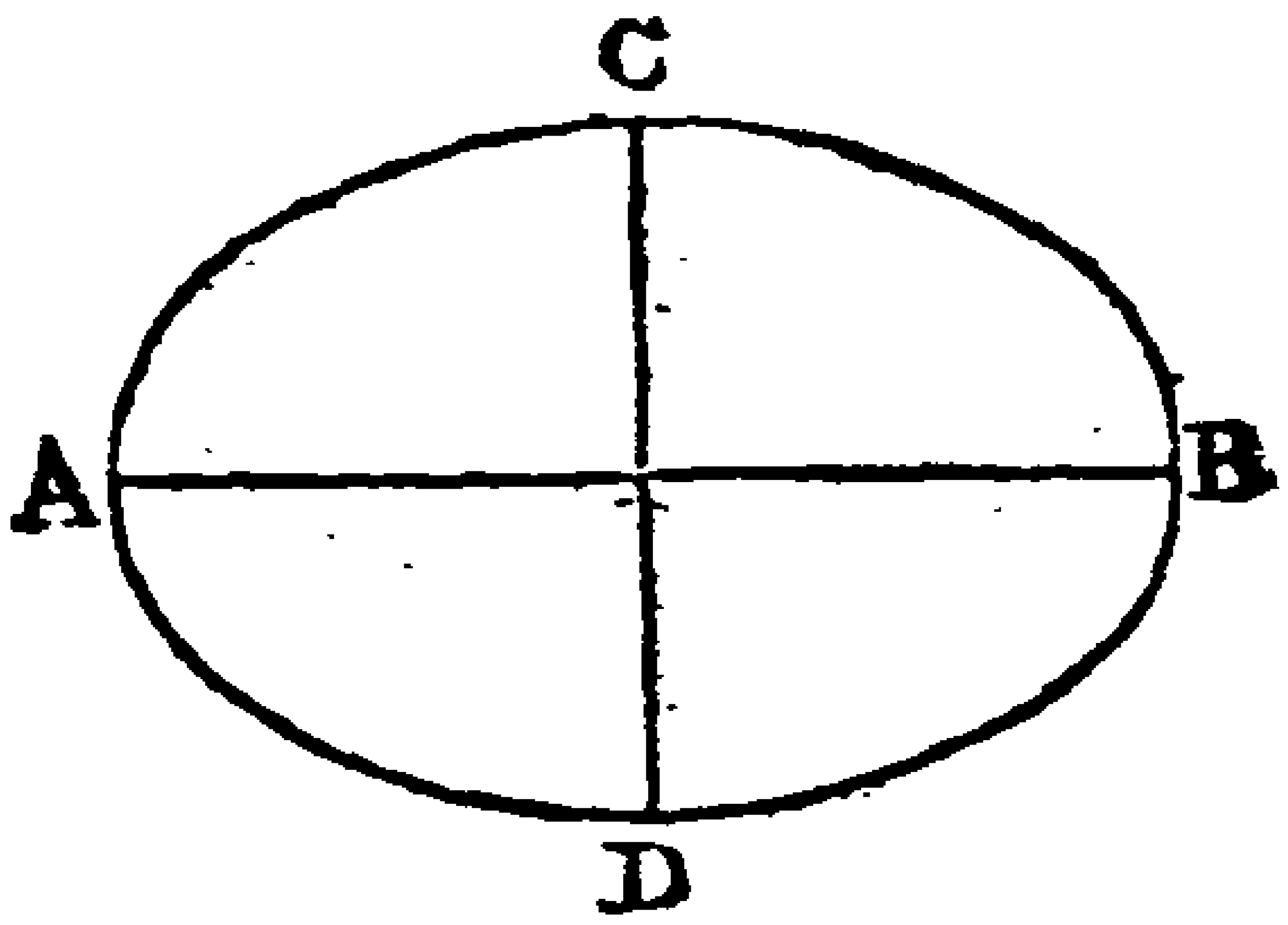
Note. Every Ellipsis is equal in Area to a Circle, whose Diameter is a mean Proportional between the two Diameters of the Ellipsis; whence this

Rule.

Multiply the Transverse Diameter by the Conjugate; and multiply that Product by .7854, the last Product will be the Area required.

Example.

If the Transverse Diameter of an Ellipsis, as A B, be 24 Inches; and the Conjugate Diameter C D be 18 Inches, what is its Area?



Operation. $24 \times 18 = 432 \times .7854 = 339.2928$ Inches, the Area required.

Pro-

Problem 5.

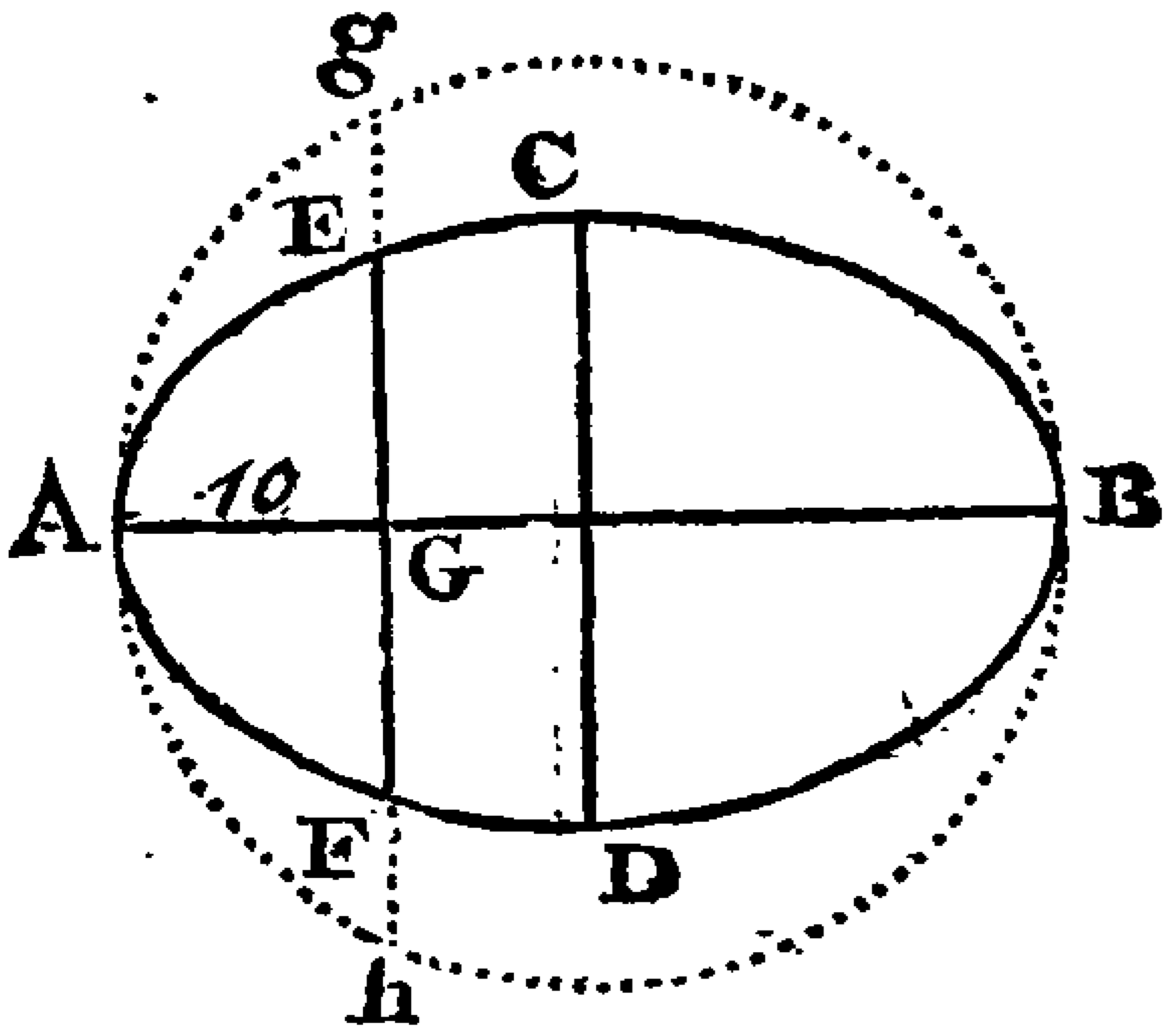
To find the Area of a *Segment* of an Ellipsis.

Rule.

As the Tranſverſe Diameter is to the Conjugate Diameter; ſo is the Area of the Circular Segment, to the Area of the Elliptical Segment correſponding thereto.

Example.

Suppoſe in the Ellipſis A C B D, a Segment, as E A F, be cut off parallel to the Conjugate Diameter C D; and ſuppoſe the Height A G 10 Inches, the Tranſverſe Diameter A B 35 Inches, and the Conjugate C D 25 Inches, what is the Area of the Segment?



Operation.

Firſt, Half the Chord *g h* = *g* G is a mean Proportional between A G 10, and G B 25, and by Problem 22 at Page 94, is found to be = 15.811 Inches.

Next, find the Area of the Circular Segment *g* A *h* by Problem 13 of Planometry, which will be found = 227 Inches.—Then ſay,

Tranſ. Dia.	Conj. Dia.	Circ. Area.	Ellipt. Area.
As A B 35 :	C D 25 : :	227 *g* A *h* :	162 E A F required.

☞ After the ſame Manner may the Area be found, was the Segment cut off parallel to the Tranſverſe Diameter; only then you muſt operate with the Circle deſcribed on the Conjugate Diameter.

Pro-

Problem 6.

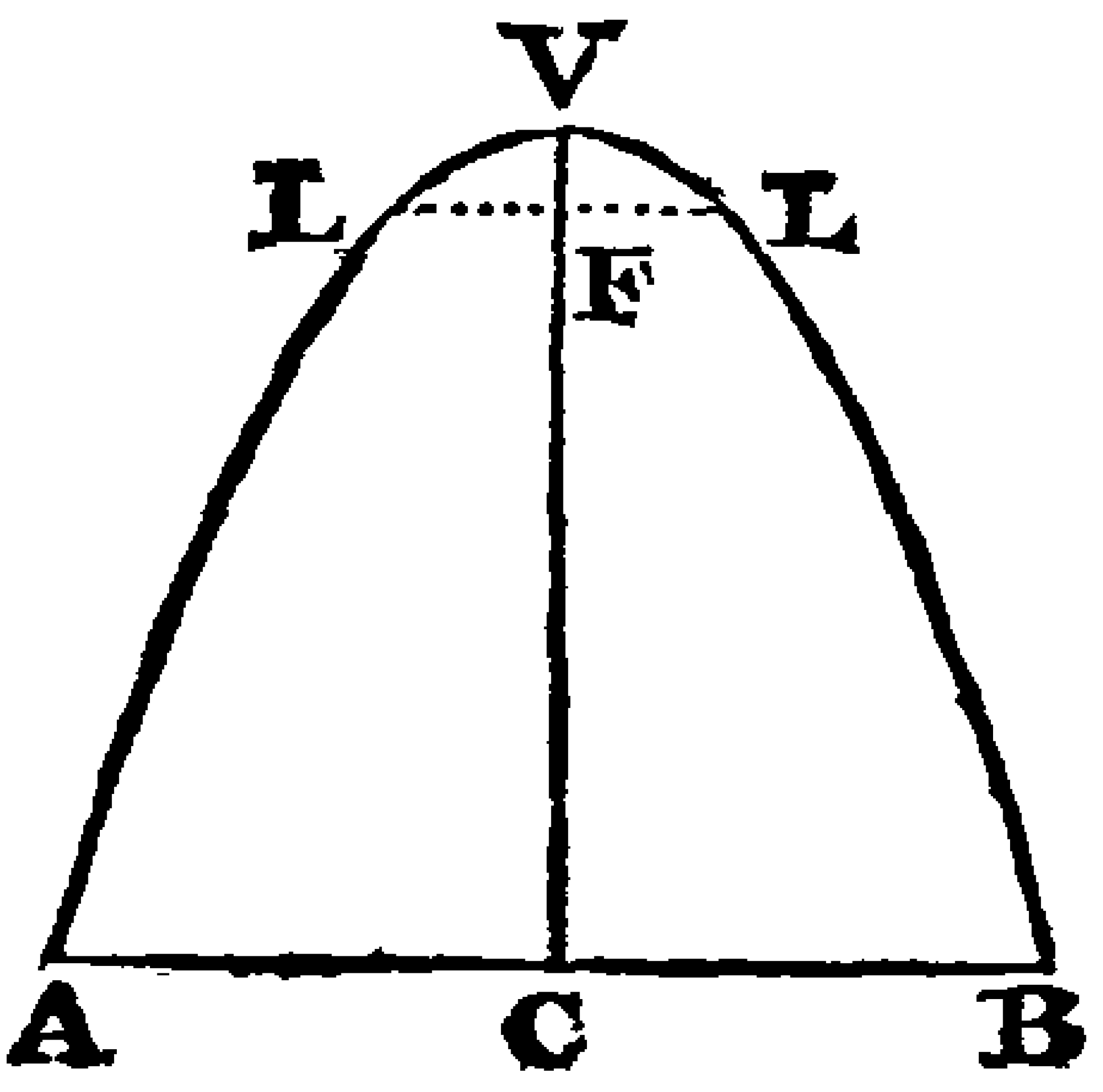
To find the Focus of a *Parabola*.

Rule.

Divide the Square of $\frac{1}{2}$ the Baſe by 4 Times the Height; the Quotient will be the Diſtance of the *Focus* from the Vertex or Top of the Parabola.

Example.

Suppoſe the Baſe A B of the following *Parabola* be 30 Inches, and its Height V C be 24 Inches, what is the Diſtance of the *Focus* from the Vertex of the *Parabola*?



Operation. $\square\ \frac{1}{2}$ A B $= 225 \div 4$ V C $= 96$ gives 2.343 Inches, the Diſtance of the Focus from the Vertex V.

Note. The Baſe A B is often called by Mathematicians the *Ordinate*; and the Height, the *Abſciſſa* (or Axis) of the *Parabola*.

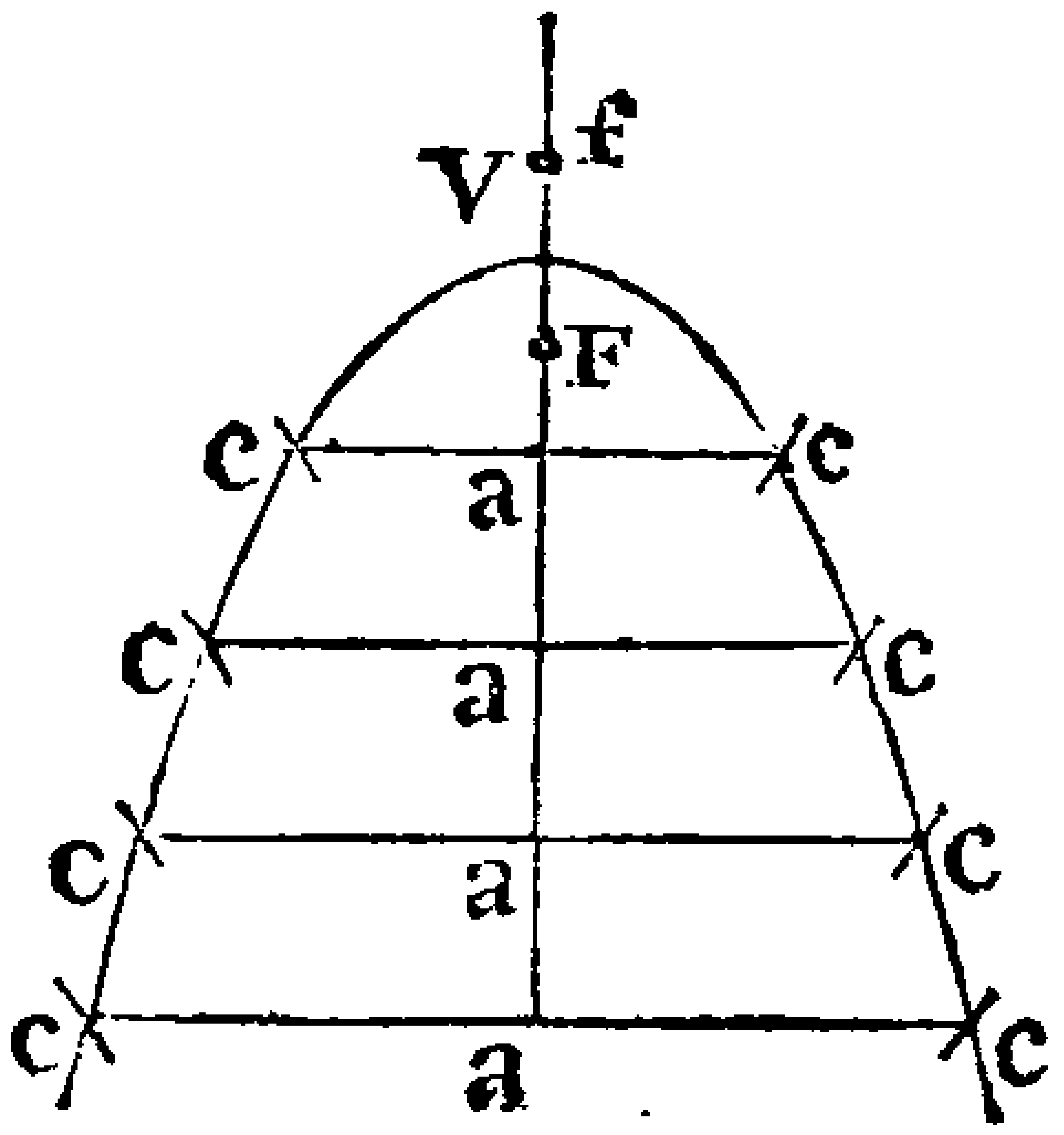
The dotted Line drawn through the Focus is called the *Latus Rectum*; and its Length may be thus determined. As the *Abſciſſa* or Height V C, is to the *Semi-ordinate*, or $\frac{1}{2}$ Baſe C B : : ſo is $\frac{1}{2}$ Baſe C B to the *Latus Rectum* L L. The *Focus* is diſtant from the Vertex always $\frac{1}{4}$ of the *Latus Rectum*.

Pro-

Problem 7.

To delineate a *Parabola*, the Baſe and Height being given.

Let the Baſe of the Parabola to be delineated be 30, and its Height 24.



Conſtruction. Firſt, find the Diſtance of the *Focus* from the Vertex V, by the laſt Problem. Then ſet off that Diſtance from the Vertex, upwards on the Axis continued at *f*; and alſo on the Axis itſelf at F. Next, make a ſufficient Number of Points in the Axis, as at *a a a a*, and through thoſe Points draw parallel Lines cutting the Axis at Right Angles, as *c c c c*. Then, with the Compaſſes, take off ſeverally the Diſtances *a f*, and ſetting one Foot in the Focus F, with the other deſcribe Arches croſſing the parallel Lines in *c c c c*, &c. Laſtly, with an even Hand draw a Curve through thoſe Interſections, and it will be the Parabola required.

Problem 8.

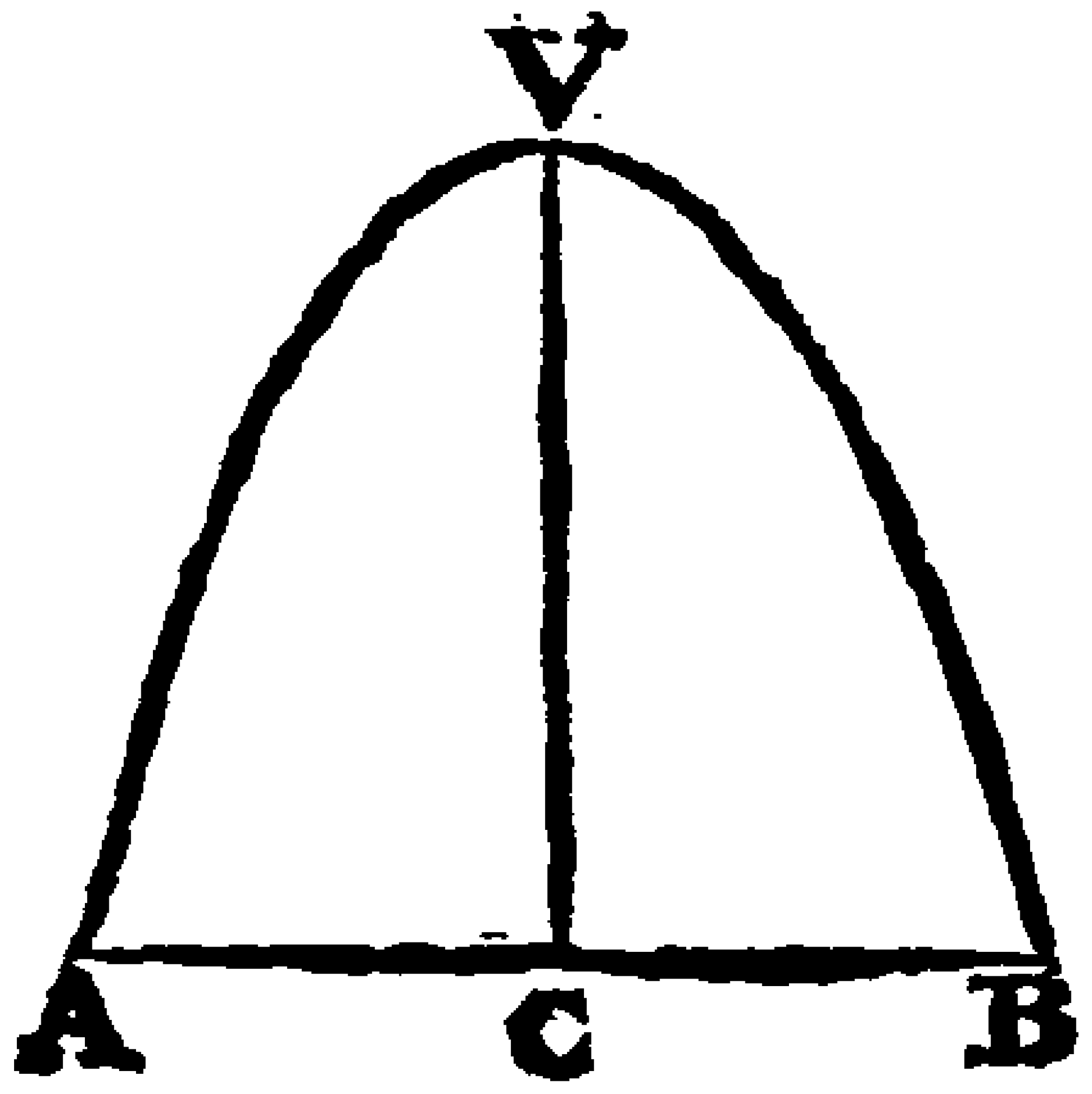
To find the Length of an *Arch* of a *Parabola.*

Rule.

To the Square of $\frac{1}{2}$ the Base add $\frac{4}{3}$ of the Square of the Height; then twice the Square Root of that Sum will be the Length of the Parabolic Arch required, very near.

Example.

Let the Base A B of the Parabola be 30 Inches, and its Height V C 24 Inches, what is the Length of the Arch A V B?



Operation. $15 \times 15 = 225 + 768 = 993$, whose Square Root is $= 31.51 \times 2 = 63.02$ Inches, the Length of the Arch A V B sought.

Pro-

Problem 9.

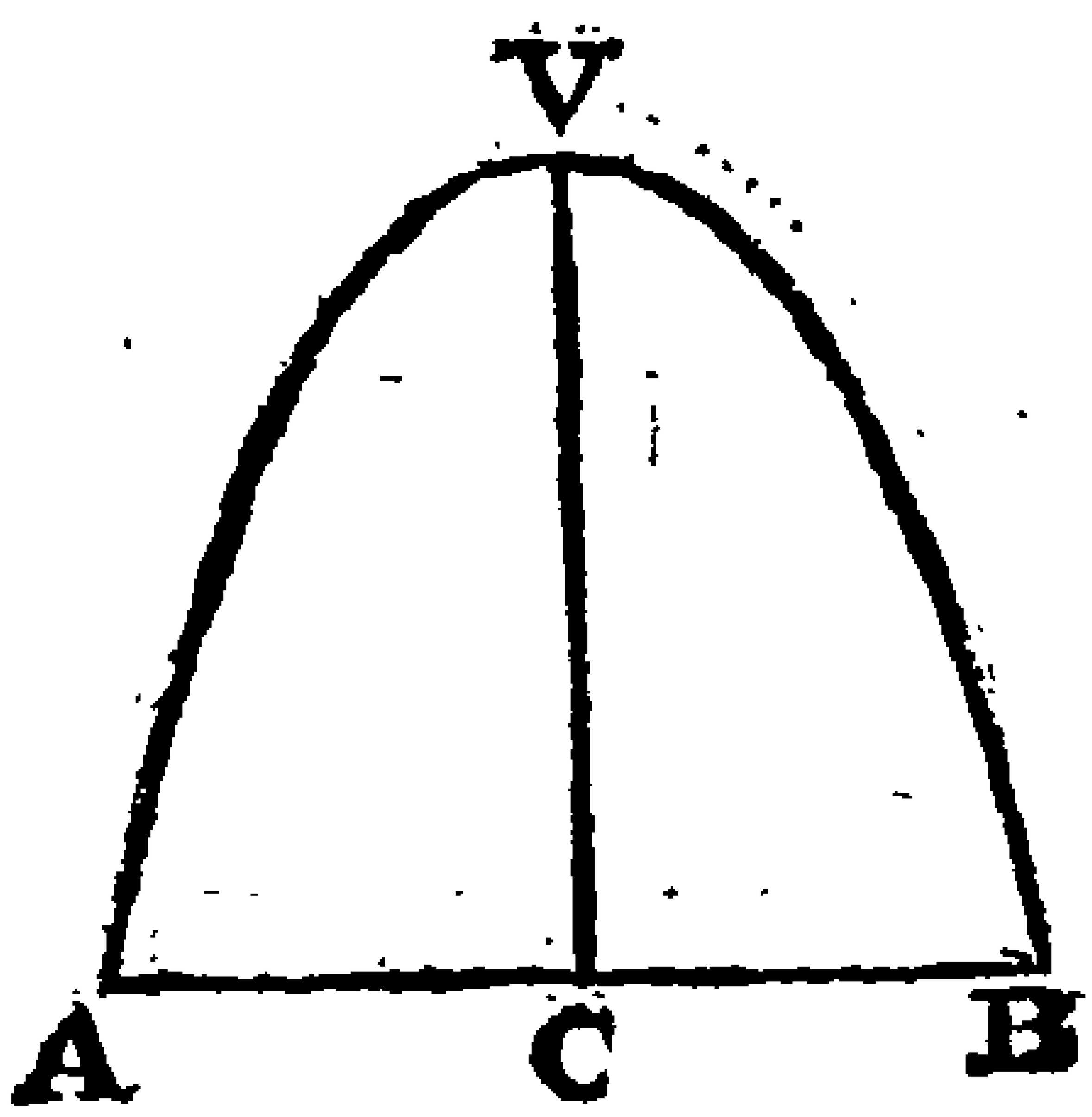
To find the *Area* of a *Parabola*.

Rule.

Multiply the Base by the Height; then multiply that Product by 2, and divide that last Product by 3, the Quotient will be the Area required.

Example.

Suppose the Base A B of the Parabola A V B be 30 Inches, and the Height V C 24 Inches, what is its Area?



Operation. $30 \times 24 = 720 \times 2 = 1440 \div 3 = 480$ Inches, the Area sought.

Note. Every *Parabola* is $= \frac{2}{3}$ of its Circumscribing Parallelogram. Consequently the Base multiplied by $\frac{2}{3}$ of the Height will be the Area as above.

Pro-

Problem 10.

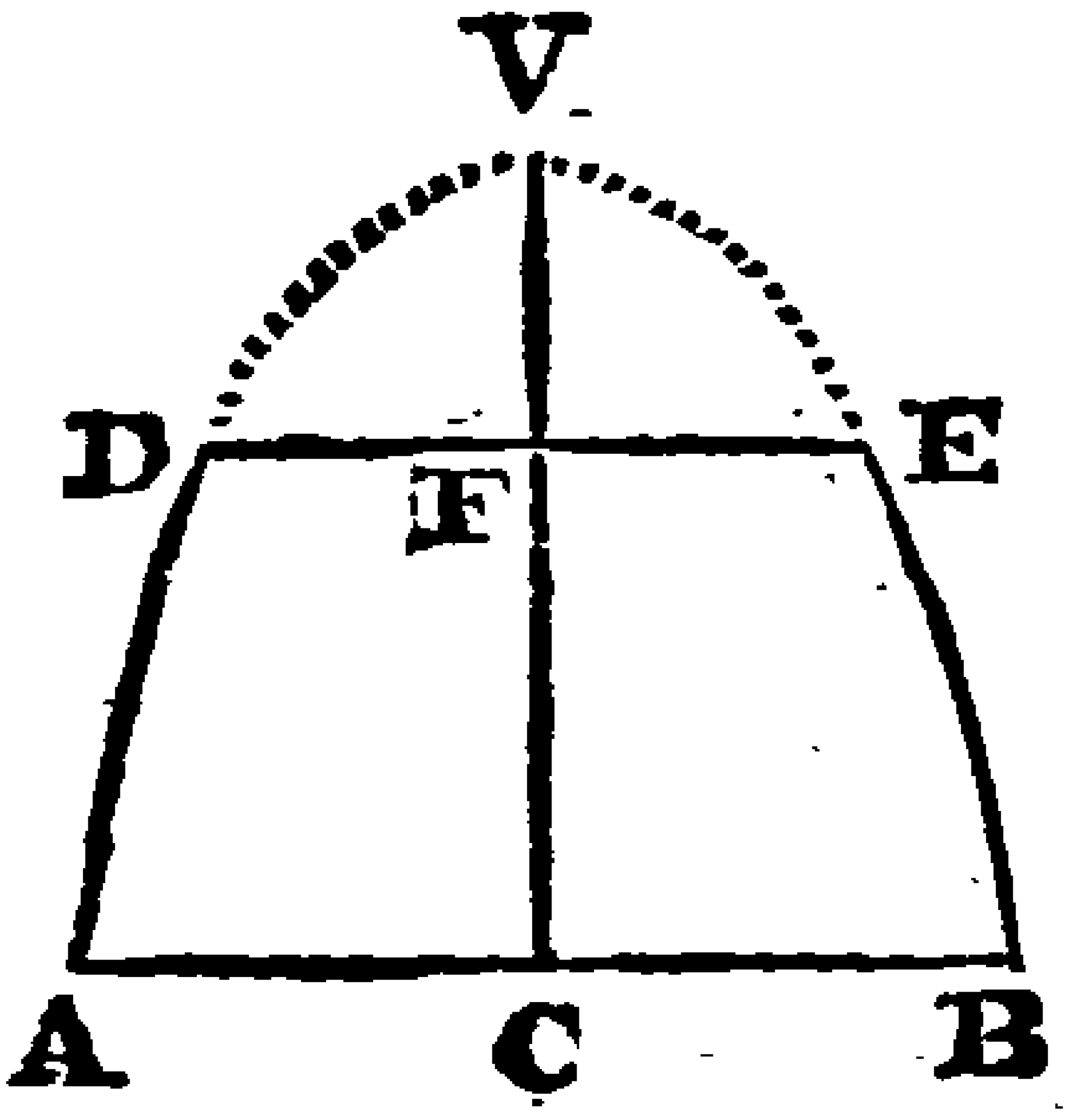
To find the Area of a *Frustum* of a *Parabola*.

Rule.

Find the Area of the top Part, and alſo the Area of the whole Parabola, by the laſt Problem; then ſubtract the Area of the Top from the Area of the Whole, and the Remainder will be the Area of the Fruſtum required.

Example.

Suppoſe in the *Parabolic Fruſtum* A D E B, the Side or Baſe A B be 10 Inches, and the Side D E be 6 Inches, the Height F C 3 Inches, and the remaining Part F V 1.687 Inches, what is the Area of the Fruſtum A D E B?



Operation.

The Area of the upper Parabola D V E = 6.744 } per laſt
The Area of the whole Parabola A V B = 31.240 } Problem

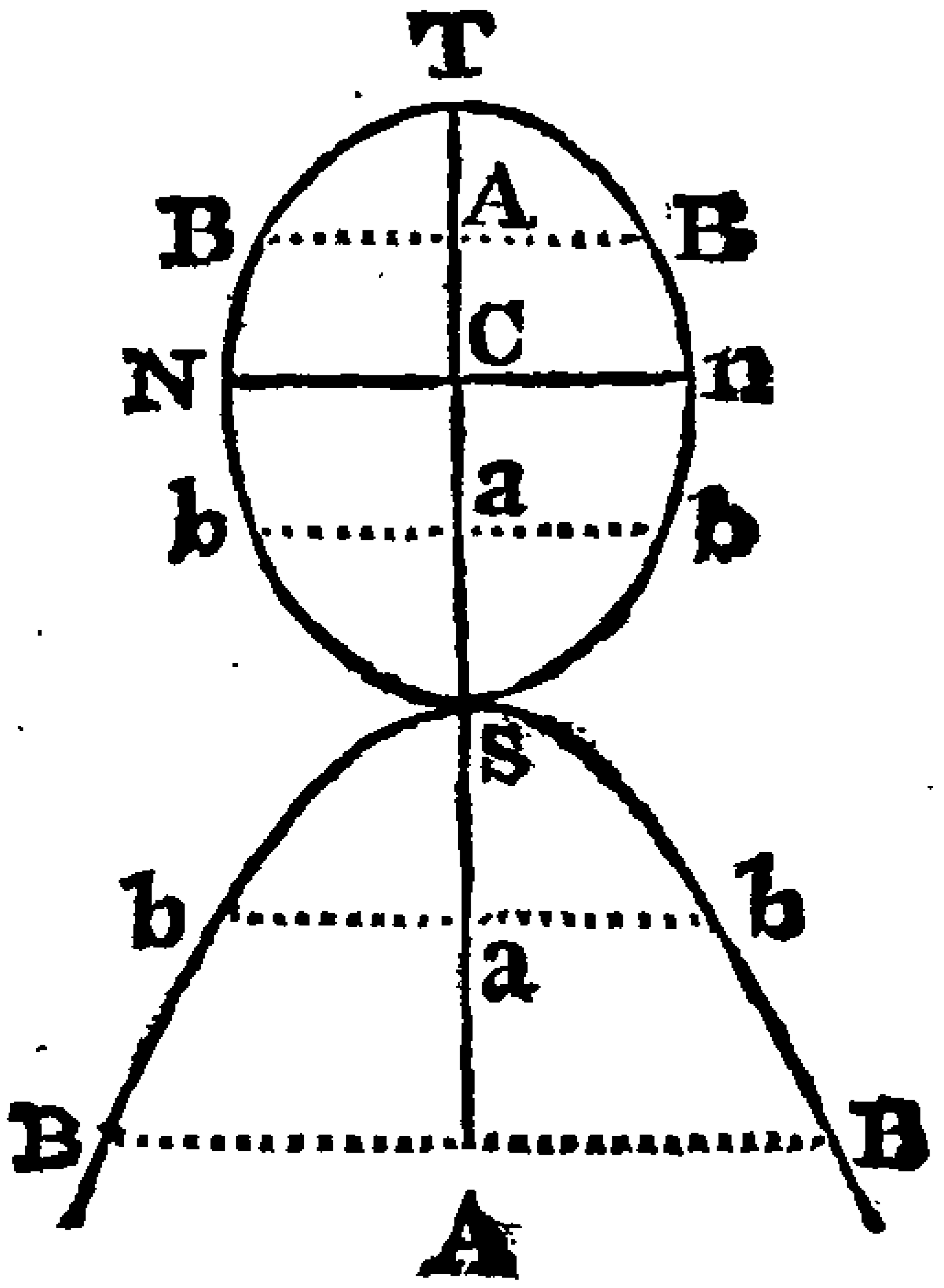
the Difference 24.496 Inches = the Area of the Fruſtum A D E B ſought.

Pro-

Problem 11.

Of the *Hyperbola.*

The Properties of an *Hyperbola* and an *Ellipsis* differ only in the Signs + (more) and — (less), as is evident in the following Figure, where T S and N *n* are the Transverse and Conjugate Diameters of both Sections.



For in the *Ellipsis* it will be,

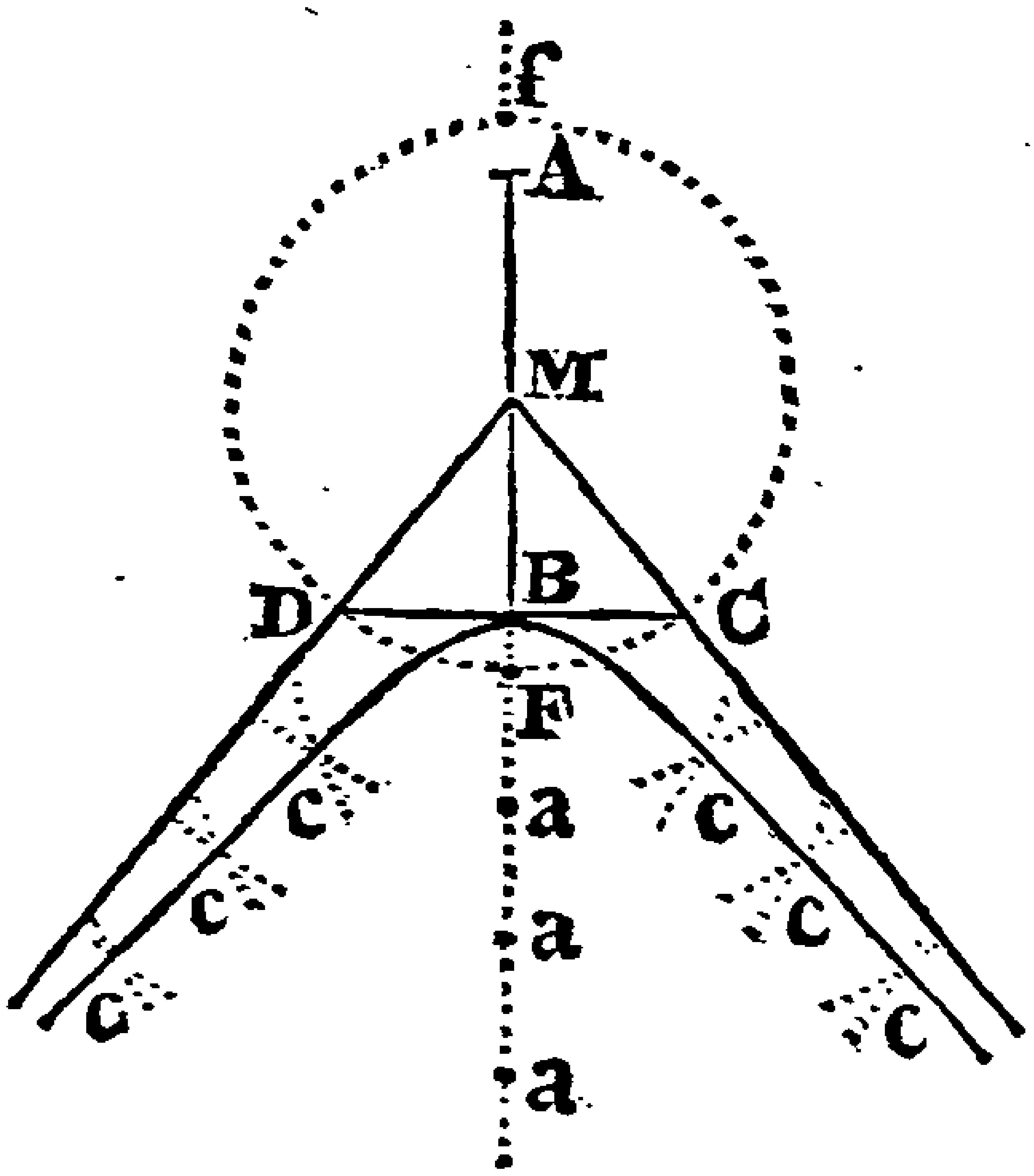
As $\overline{TS-Sa}\times Sa : ab\times ab :: \overline{TS-SA}\times SA : AB\times AB.$

But in the *Hyperbola* it will be,

As $\overline{TS+Sa}\times Sa : ab\times ab :: \overline{TS+SA}\times SA : AB\times AB.$

Problem 12.

To delineate an *Hyperbola*, having the Tranſverſe and Conjugate Diameters given.



Conſtruction. Firſt, draw the Horizontal Line D C for the Conjugate Diameter, and upon the Middle of it erect perpendicularly the Tranſverſe Diameter A B at B. Next, ſet one Foot of the Dividers in the Middle of the Line A B, as at M; and opening the other to the End C or D of the Conjugate Diameter, deſcribe the Circle D F C *f*, cutting the Line A B continued upwards and downwards in F and *f*, which Points will be the two *Foci* of the *Hyperbola*.

On the Line A B (continued downward) take any Number of Points, as *a*, *a*, *a*, and from *f* and F as Centers,

ters, with the Diſtances A *a*, and B *a*, deſcribe Arches, cutting each other in *c*, *c*, *c*, on each Side the Figure; then through the Points *c*, *c*, *c*, draw with a ſteady Hand the Curve *c* B *c*, and it will be the Hyperbola required.

If Right Lines be drawn from the Point M, in the Middle of the Tranſverſe Diameter, by the Ends of the Conjugate C D, they will be the *Aſymtotes* of the *Hyperbola*, whoſe Property it is to approach continually nearer the Curve, and yet never to meet it.

Note. If the *Tranſverſe* and *Conjugate* Diameters (or Axes) of the *Hyperbola* are not given, their Lengths may be determined from the Dimenſions of the Hyperbola itſelf. A Problem to this Effect is inſerted among the additional Problems at the End of the Book.

Problem 13.

To find the Length of an Arch of an *Hyperbola*, the *Base* and *Height* being given.

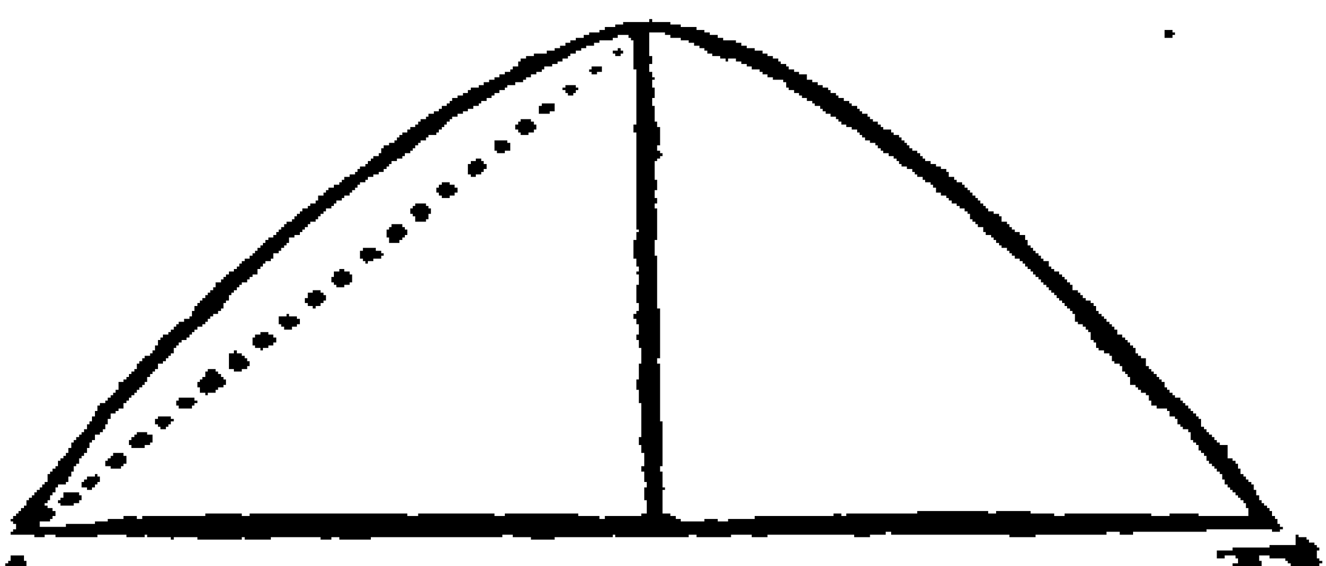
Many different *Hyperbolas* may be cut from the same Cone; but all of them will be less (in Area) than a *Parabola*, and greater than a *Triangle*, whence we may deduce this

Rule.

First, find the Length of the Curve as if it were a *Parabola*; and afterwards find the Length of the *Hypothenuse* as a Triangle of the same Base and Altitude; then add the two Sums together, and half this last Sum will give the Length of the *Hyperbolic* Arch, near enough in Practice.

Example.

Suppose the Base A B of the following *Hyperbola* be 20 Inches, and the Height V C 2.163; what is the Length of the Arch V A, and also the whole Arch A V B?



Operation.

The Length of the Arch V A as a *Parabola* - - - - - -	10.307	by Problem 8th
The Length of the *Hypothenuse* V A as a Triangle - -	10.230	
the Sum	= 20.537	

the Half = 10.268 = the Hyperbolic Arch V A, which doubled, gives the Length of the whole Arch A V B required.

Pro-

Problem 14.

To find the Area of an *Hyperbola*.

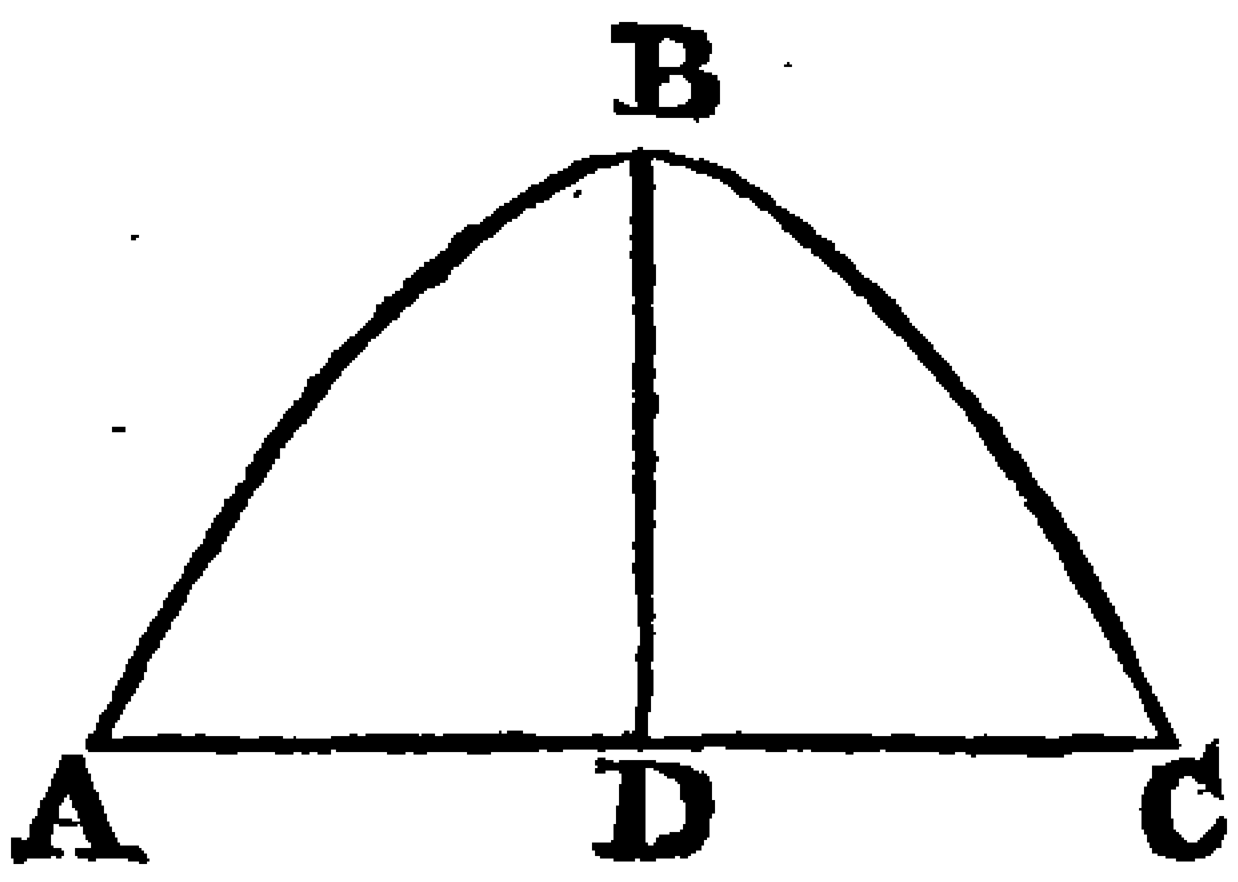
In the foregoing Problem we observed, that every Hyperbola is greater than *Half*, and less than *two-Thirds* of its circumscribing Parallelogram: It will therefore be accurate enough to take all Hyperbolas at a Mean; that is, supposing every *Hyperbola* is $\frac{5}{8}$ of its circumscribing Parallelogram, whence this

Rule.

Multiply the Base by its Height, and that Product multiply again by 5, and divide this last Product by 8, the Quotient will be (nearly) the Area required. *

Example.

Let A B C be an *Hyperbola*, whose Base A C is 22 Inches, and the Height B D 14 Inches, what is its Area?



Operation. A C 22 × B D 14 = 308 × 5 = 1540 ÷ 8 = 192.5 Inches, the Area.

* To determine the *Length of an Arch*, as well as *Area* of an *Hyperbola*, to Mathematical Precision, other Lines, besides the above, must be taken into the Calculation, which will be difficult for the Learner to understand, till he has made some Progress in *Algebra* and the *Conic Sections*.

THE CUBATURE; OR MENSURATION OF SOLIDS, ARISING FROM THE SECTIONS of a CONE.

Problem I.

TO find the Solidity of a *Spheroid.*

Def. A *Spheroid* is a Solid, approaching to the Figure of a Sphere, having one of its Diameters longer than the other, and is formed by the Revolution of a Semi-Ellipsis about its Axis. If the Spheroid be formed by the Rotation of a Semi-Ellipsis round its *tranverse* Axis, it is called an *oblong Spheroid*; if generated by the Rotation of a Semi-Ellipsis round its *conjugate* Axis, it is called an *oblate Spheroid.*

Rule.

Find the Area of the Circle in the Middle, and multiply it by the Length; then multiply that Product by 2, and divide by 3, the Quotient will give the Solidity. *

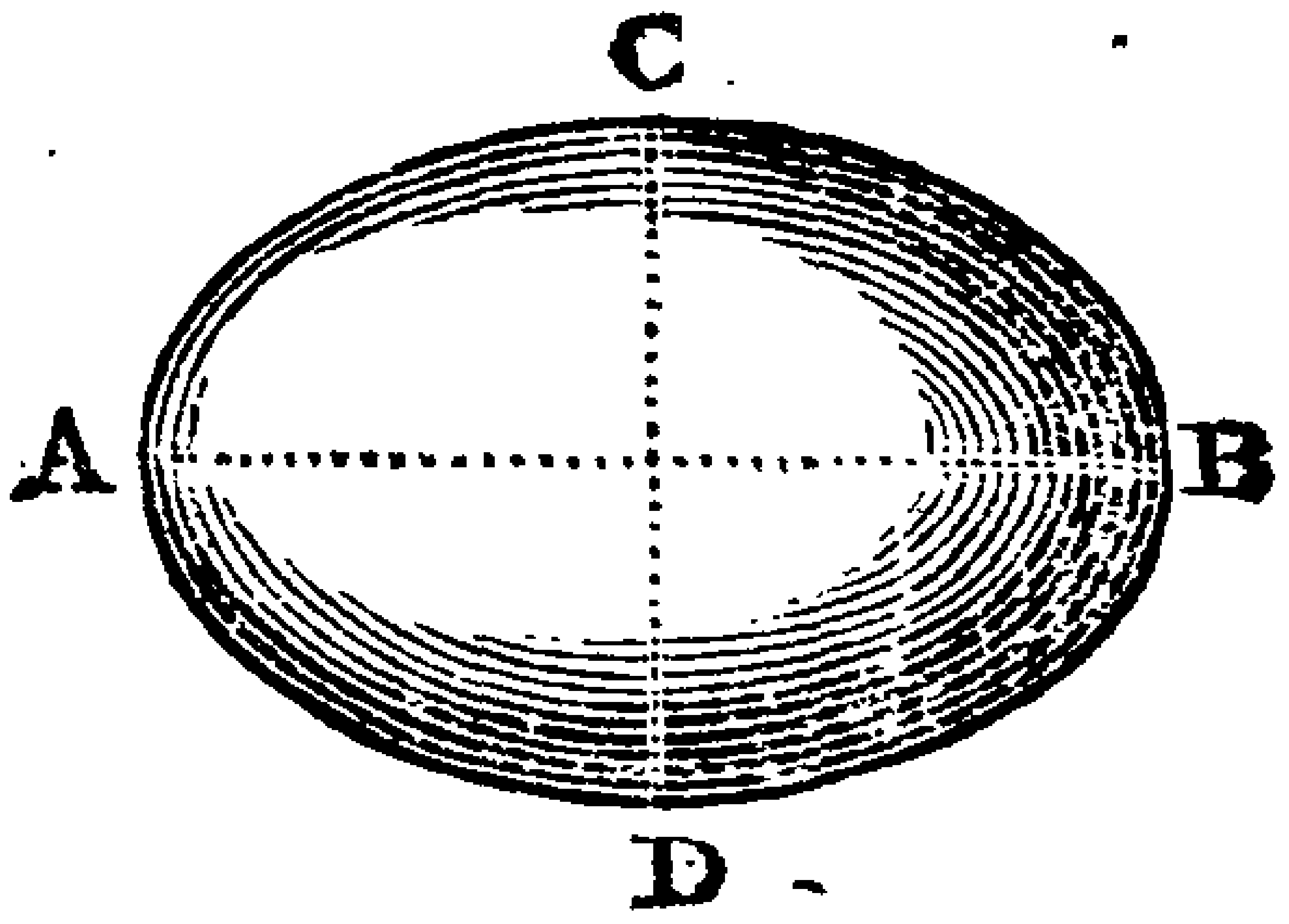
* Every *Sphere* and every *Spheroid* is equal to $\frac{2}{3}$ of a Cylinder of the same Diameter and Length.

See another Method of solving this Problem at Page 150 in *Stereometry.*

Exam-

Example.

Suppose ABCD be an oblong Spheroid, whose Diameter in the Middle CD is 33 Inches, and whose Length AB is 55 Inches, what is its Solid Content?



Operation.

□ CD $33 \times 33 = 1089 \times .7854 = 855.3006$, the Area in the Middle, which $\times 55 = 47041.5330$, which multiplied by 2, and divided by 3, gives 31361.0220 Inches, the *Solidity* required.

Another Way of finding the Solidity of a *Spheroid.*

Multiply the Square of the revolving Axe, by the fixed Axe, and this Product again by .5236,* and it will give the Solidity required.

* .5236 is $\frac{2}{3}$ of .7854, the Area of a Circle whose Diameter is Unity, or 1.

Problem 2.

To find the Solidity of a *Segment* of a *Spheroid.*

Def. A *Segment* of a Spheroid is a Part cut off by a Plane perpendicular to one of its Axes or Diameters.

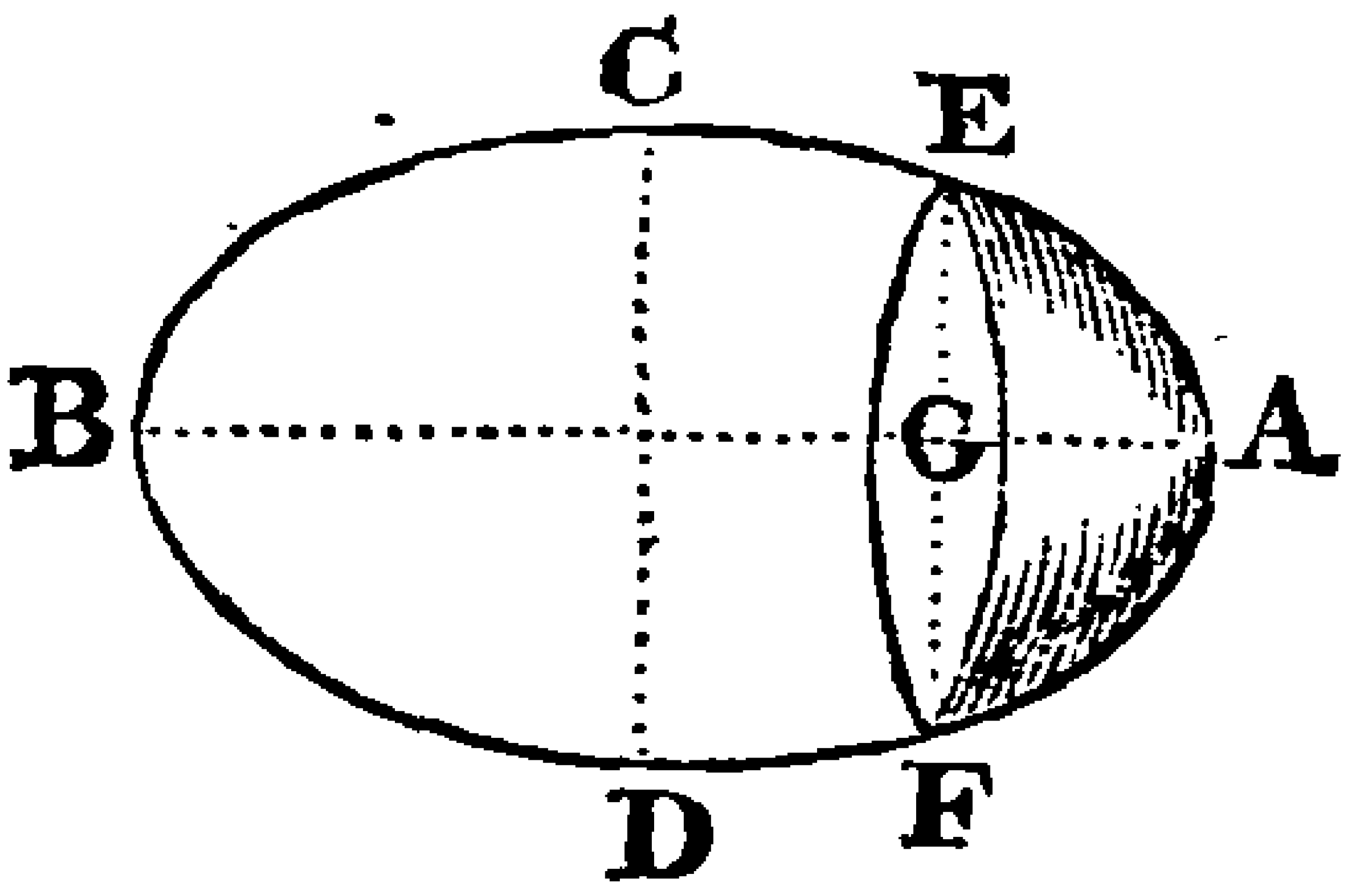
If the Segment be cut off *perpendicular to the Transverse Axe*, observe this

Rule.

Divide the Square of the Conjugate Axe by the Square of the Transverse Axe; multiply the Quotient by the Difference between *three Times* the Transverse Axe, and *twice* the Height of the Segment; multiply the Product by the *Square* of the said Height, and this Product multiplied again by .5236, will give the Solidity required.

Example.

Suppose in the following Spheroid, the Transverse Axe A B be 50 Inches, and the Conjugate C D 30, and the Height of the Segment A G 5 Inches; what is the Solidity of the Segment E A F *cut off perpendicular to the Transverse Axe?*



Operation. □ of Conjugate 900 ÷ 2500 □ of Transverse is = .36; this × by 140 (the Difference between *three Times* the Transverse and *twice* the Height of the Segment) is = 50.40, which × by 25 □ of the Height is = 1260; this × by .5236 gives 659.7360 for the Solidity required.

Note. If the Segment *be cut off perpendicular to the Conjugate Axis*, then divide the Transverse Axe by the Conjugate Axe; multiply the Quotient by the Difference between *three Times* the Conjugate Axe, and twice the Height of the Segment; multiply the Product by the *Square* of the said Height, and this last Product multiplied again by .5236, will give the Solidity of the Segment sought.

Pro-

Problem 3.

To find the Solidity of the *Middle Zone* of a *Spheroid*.

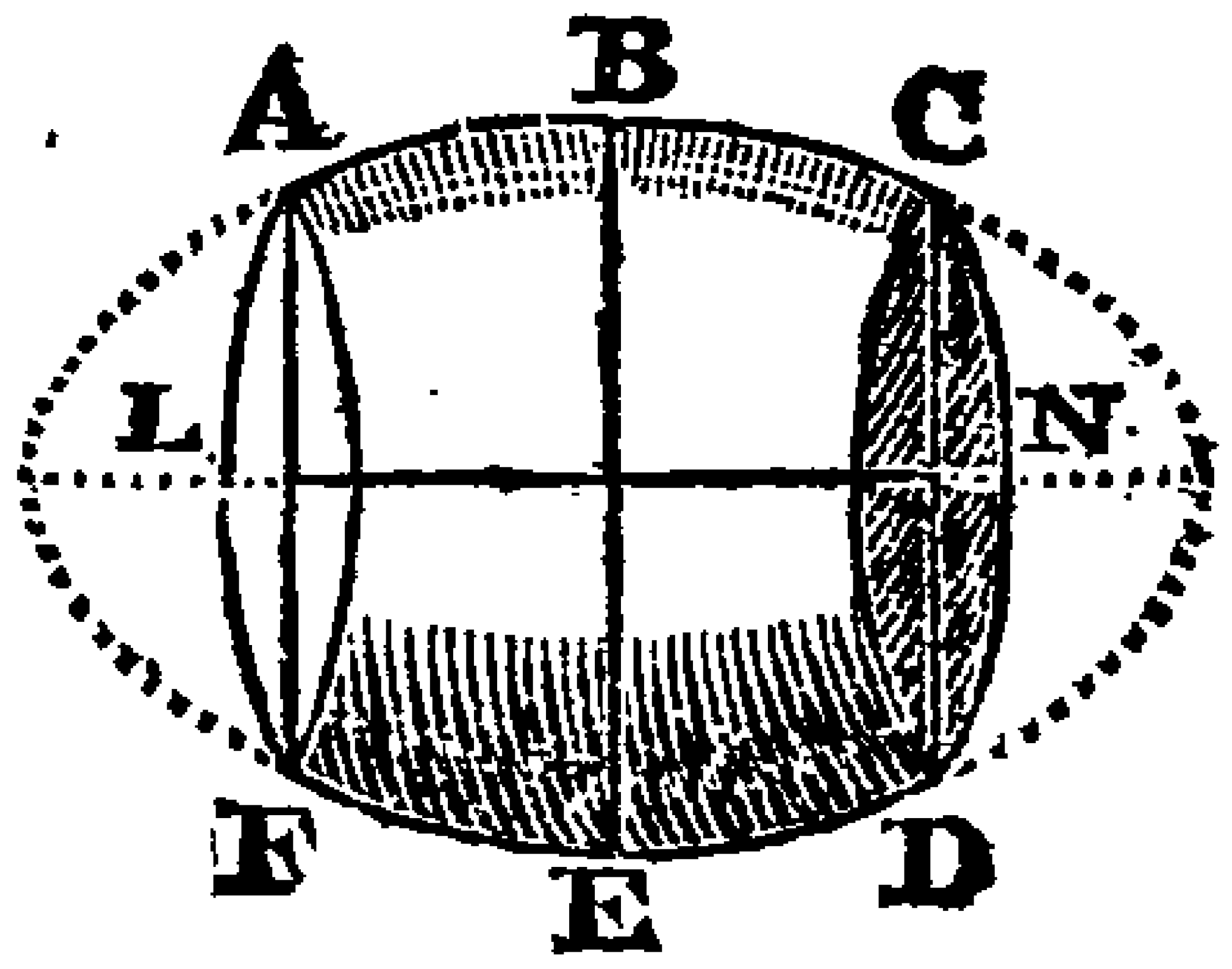
Def. The *Zone* of a *Spheroid* is the Part remaining when the two Ends are cut off by Planes parallel to each other; if the Planes are equally distant from the Middle, it is called the *Middle Zone* of the Spheroid.

Rule.

To *twice* the Square of the Diameter in the Middle, add the Square of the Diameter of either End; multiply this Sum by the Length of the Zone, and that Product multiply again by .2618, a Decimal, and the last Product will give the Solid Content.

Example.

Suppose A B C D E F be the *Frustum*, or *Middle Zone* of a *Spheroid*; the Middle Diameter B E is 50 Inches; the Diameter of either End A F or C D 40 Inches; and the Length L N 18 Inches; what is its Solid Content?



Operation. 2 □ B E 5000 + □ C D or A F = 1600 = 6600 × L N 18 = 118800, which × .2618 gives 31101.8400 Inches, the Solidity sought.

Problem 4.

To find the Solidity of a *Parabolic Conoid*.

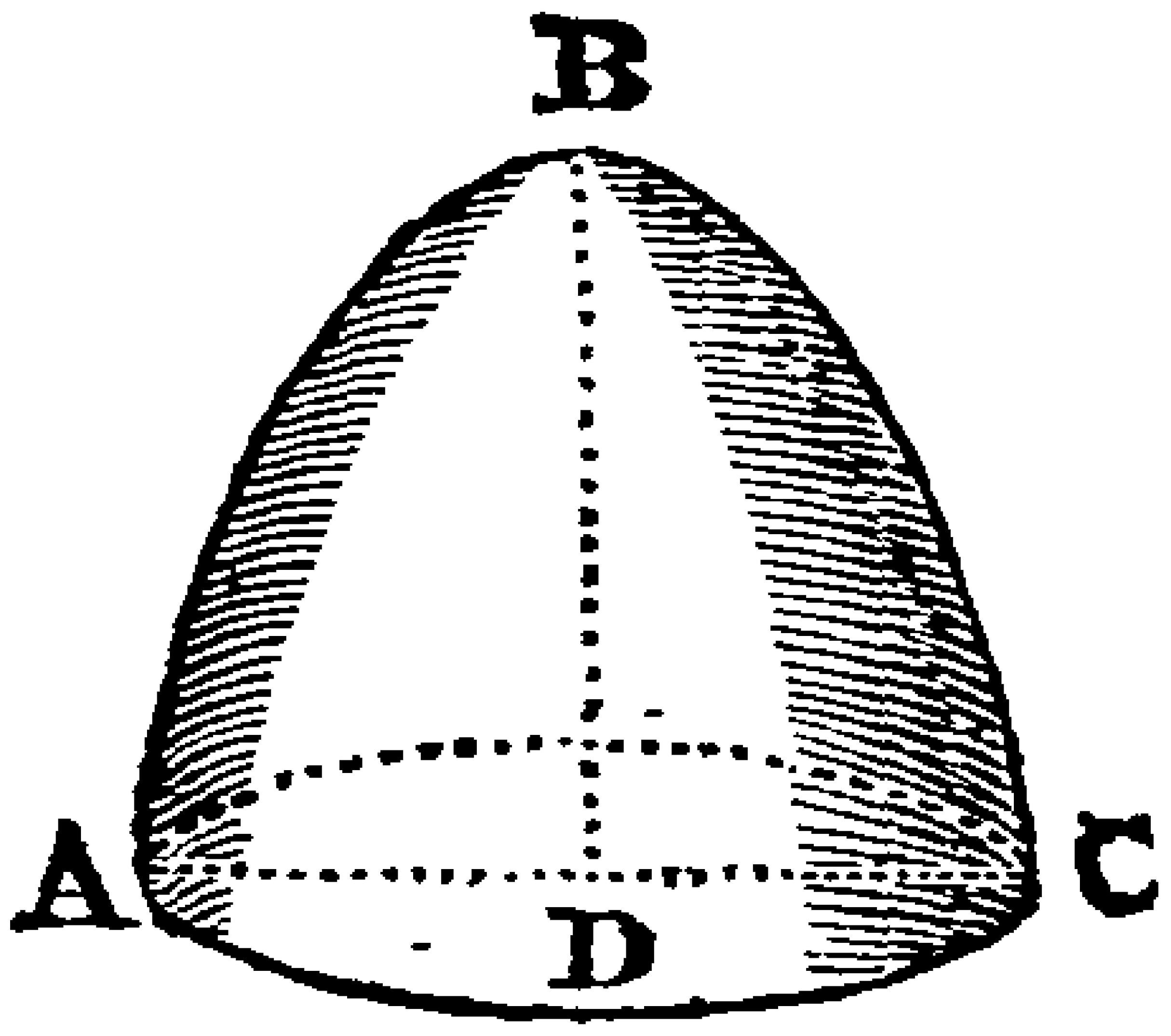
Def. A *Parabolic Conoid* is a Solid, formed by the Revolution of a *Semi-Parabola* about its Axis.

Rule.

Multiply the Area of the Base, by Half the Height, and the Product will be the Solidity required. *

Example.

Suppose A B C D be a *Parabolic Conoid*, whose Diameter of its Base A C is 40 Inches, and Height B D is 30 Inches, what is its Solid Content?



Operation. A C 40 × 40 = 1600 × .7854 = 1256.6400, the Area of the Base, which × 15 half B D = 18849.6000 Inches, the Solidity required.

* Every *Parabolic Conoid* is equal to ½ its circumscribing Cylinder.

Problem 5.

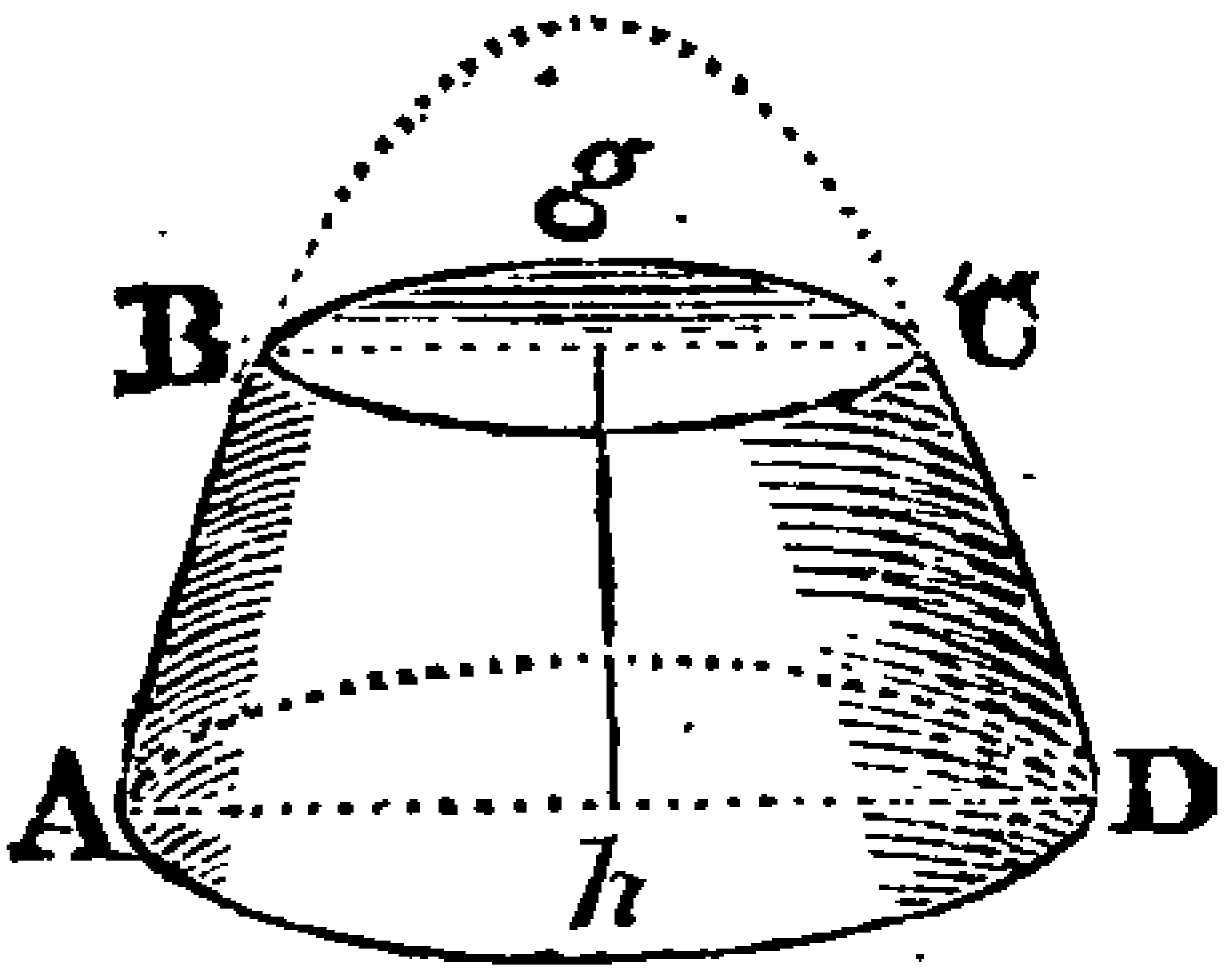
To find the Solidity of a *Frustum* of a *Parabolic Conoid*.

Rule.

Multiply the *Sum of the Squares* of the Bottom and Top Diameters by the Frustum's Height, and that Product multiply again by .3927, * a Decimal, and it will give the Solidity required.

Example.

Suppose A B C D be the *Frustum* of a *Parabolic Conoid*, the greater Diameter A D being 58 Inches, the lesser Diameter B C 30 Inches, and the Height *g h* 18 Inches, what is the Solid Content?



Operation. □ A D 3364 + □ B C 900 = 4264 × *g h* 18 = 76752 × 3927 = 30140.5104 Inches, the Solidity required.

* .3927 is the Half of .7854, the Area of a Circle whose Diameter is 1.

Problem 6.

To find the Solidity of a *Parabolic Spindle*.

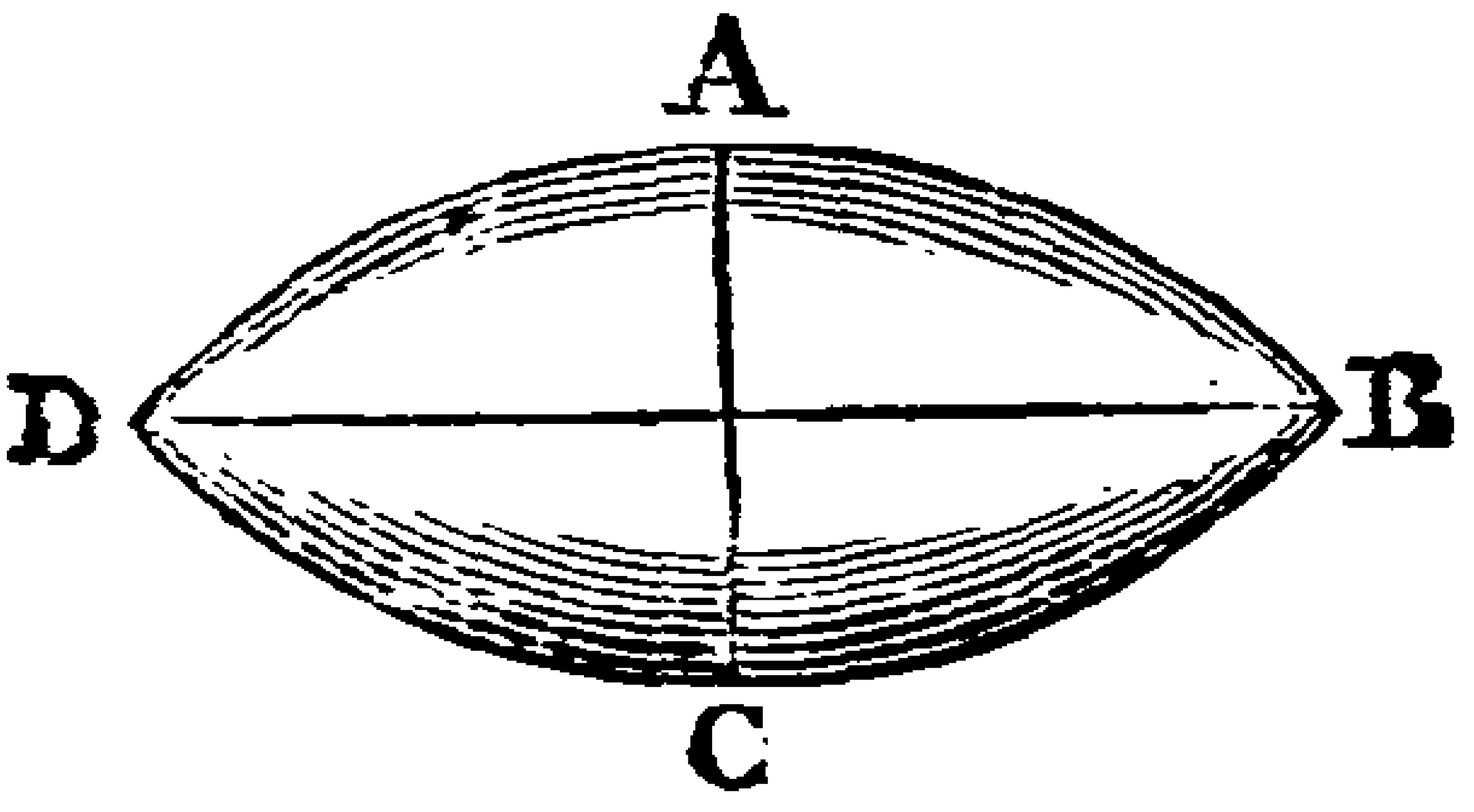
Def. A *Parabolic Spindle* is a Solid generated by the Revolution of a *Parabola* about its *greatest Ordinate* or *Base*.

Rule.

Multiply the *Area* of the Circle in the Middle by the Length, which Product multiplied again by 8, and that Product divided by 15, gives in the Quotient the Solidity required. *

Example.

Suppose A B C D be a *Parabolic Spindle*, whose Diameter in the Middle A C is 34 Inches, and the Length B D is 60 Inches, what is its Solid Content?



Operation. A C $= 34 \times 34 = 1156 \times .7854 =$ 907.9224, the Area of the Circle in the Middle, which $\times$ B D $= 60 = 54475.3440$, which $\times\ 8 = 435802.7520$ $\div\ 15 = 29053.5168$ Inches, the Solidity sought.

* Every *Parabolic Spindle* is equal to $\frac{8}{15}$ of its circumscribing Cylinder; or, multiply the Square of the Diameter in the Middle by the Length, and that Product again by $.418879 = \frac{8}{15}$ of $.7854$.

Pro-

Problem 7.

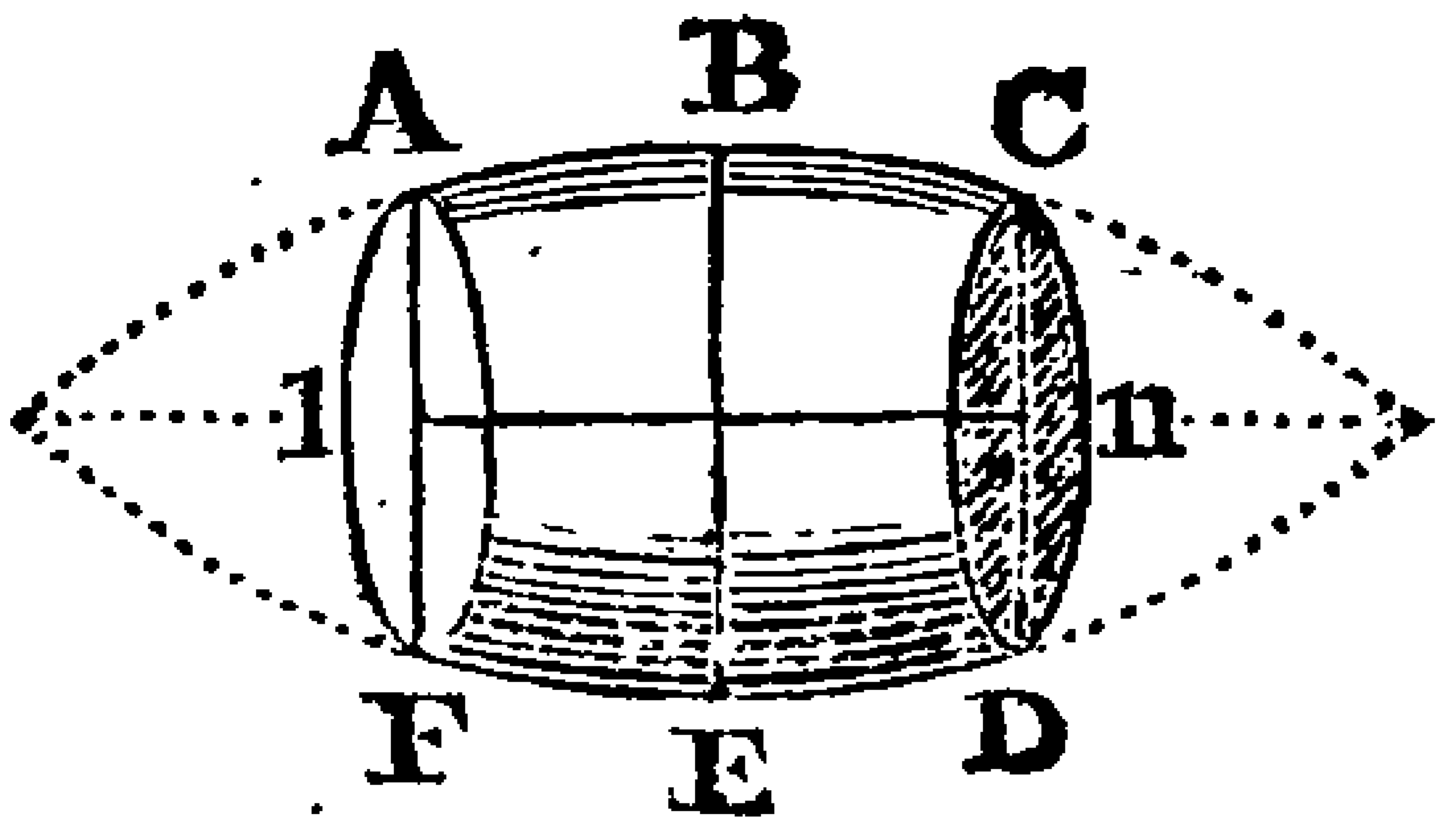
To find the Solidity of the *Middle Zone* of a *Parabolic Spindle.*

Rule.

To *twice* the Square of the Middle Diameter add the Square of the Diameter of the End; from this Sum subtract .4 (Tenths) of the Square of the Difference of the two Diameters; multiply the Remainder by the Length, and that Product divided by 3.8196 will give the Solidity of the *Zone* required.

Example.

Suppose A B C D E F be the *Middle Zone* of a *Parabolic Spindle*; the Diameter of which in the Middle B E is 36 Inches; the Diameter at the End C D or A F is 20 Inches; and the Length *l n* 36 Inches, what is its Solid Content?



Operation. B E 36 × 36 × 2 + C D 20 × 20 = 2992; and B E 36 — C D 20 = 16, the Difference of the two Diameters. Again, 16 × 16 = 256, 4-tenths of which is = 102.4. Then, 2992 — 102.4 × *l n* 36 = 104025.6, which ÷ 3.8196 gives 27234.68, *&c.* Inches, the Solidity sought.

Pro-

Problem 8.

To find the Solidity of an *Hyperbolic Conoid.*

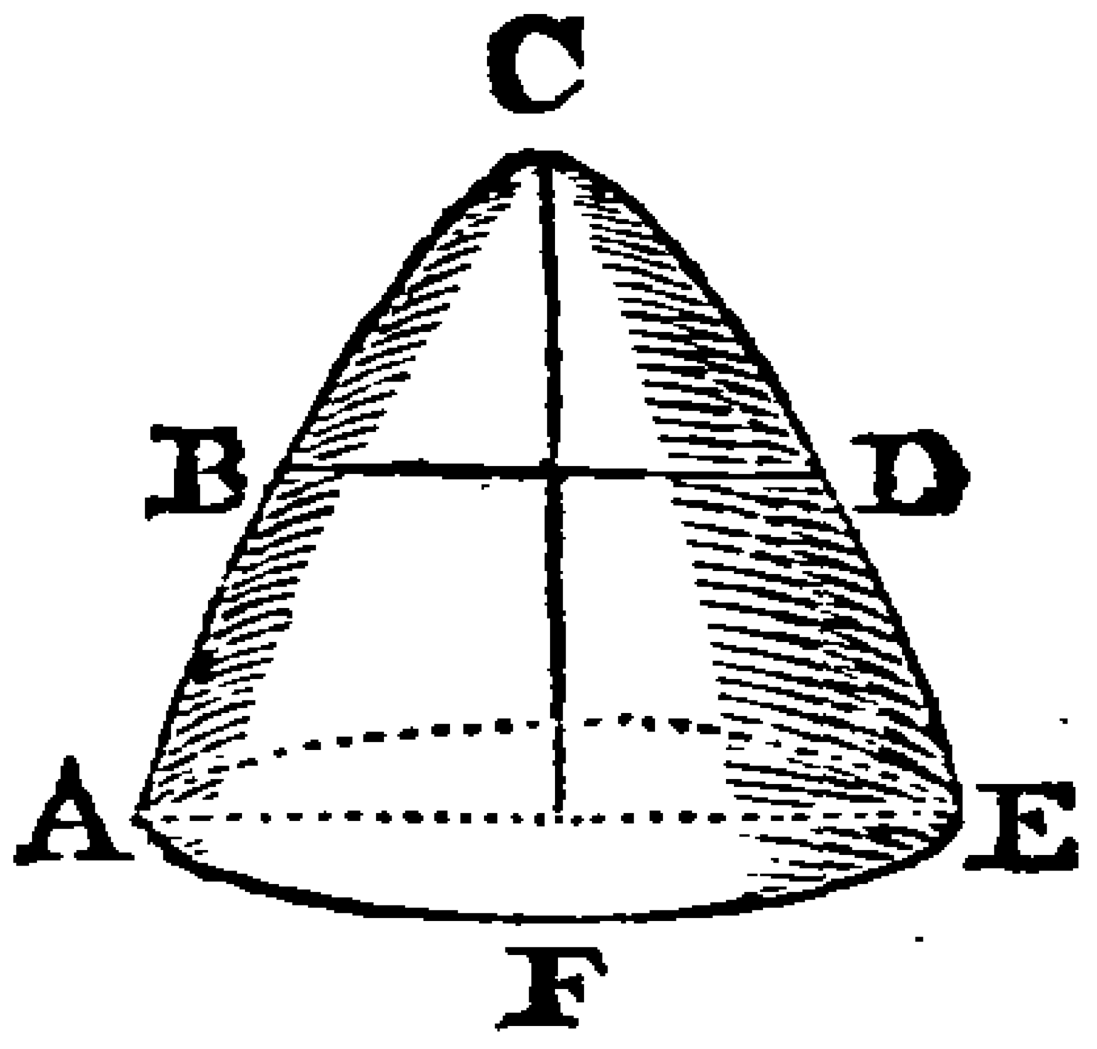
Def. An *Hyperbolic Conoid* is a Solid made by the Revolution of a Semi-hyperbola about its Axis. *

Rule.

To the Square of the Radius of the Base, add the Square of the Diameter in the Middle between the Top and Bottom; this Sum multiplied by the Height, and the Product multiplied again by .5236, will give the Solid Content.

Example.

Suppose A B C D E F be a Hyperbolic Conoid; the *Semi-diameter* A F of whose Base A E is 52 Inches; the *Diameter* in the Middle B D 68 Inches, and the Height C F 50 Inches, what is its Solid Content?



Operation. □ A F 52 = 2704 + □ B D 4624 = 7328 × C F 50 = 366400 × .5236 = 191847.04 Inches, the Solid Content required.

* An *Hyperbolic Conoid* is a Solid whose Sides are straiter than a *Parabolic Conoid,* yet more curved than a *Cone.*

Pro-

Problem 9.

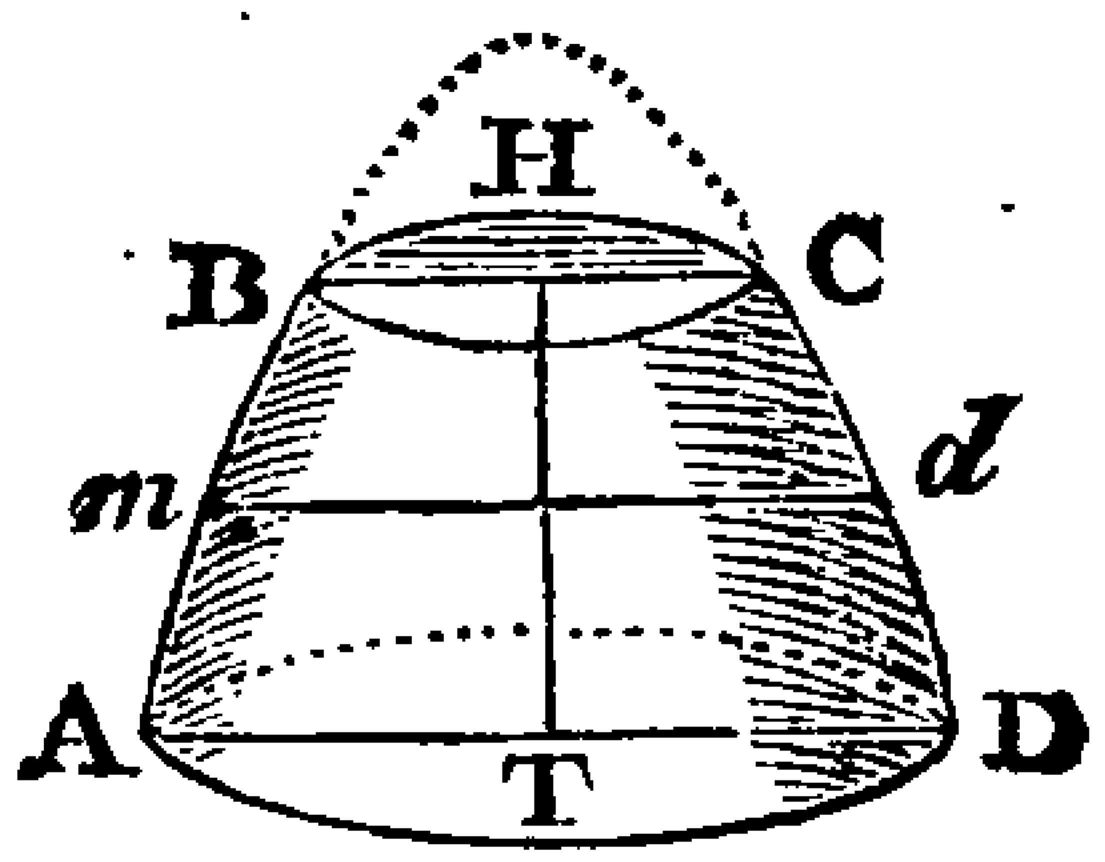
To find the Solidity of the *Frustum* of an *Hyperbolic Conoid*.

Rule.

To the Sum of the Squares of the *Semi-diameters* of the Bottom and Top of the *Frustum*, add the Square of the whole *Diameter* in the Middle; this Sum being multiplied by the Height, and that Product again by .5236, will give the Solid Content.

Example.

Suppose A B C D be the Frustum of an *Hyperbolic Conoid*; the Semi-diameter A T of the Bottom measures 16 Inches; the Semi-diameter B H of the Top 12 Inches; the Middle Diameter *m d* 28.17 Inches, and the Height H T 20 Inches, what is its Solid Content?



Operation. $\square$ A T 256 + $\square$ B H 144 + $\square$ *m d* 793.5489 = 1193.5489 × H T 20 = 23870.9780 × .5236 = 12498.84408080 Inches, the Solidity required.

Note.

Note. As many Houſhold Utenſils are in the Shape of ſome of the foregoing Figures, as, for Example, *Tuns* and *Tubs* in Form of Fruſtums of *Cones* or *Conoids*; *Furnaces* and *Coppers* in Form of *Parabolic* or *Hyperbolic Conoids*; * *Caſks* in Form of the *Middle Zones* of *Spheroids*, *Parabolic Spindles*, Double Fruſtums of *Parabolic Conoids*, and *Double Fruſtums of a Cone*; the Quantity of Liquor contained in each may be eaſily aſcertained by dividing (as before in Planometry and Stereometry) the Solid Content in Inches by

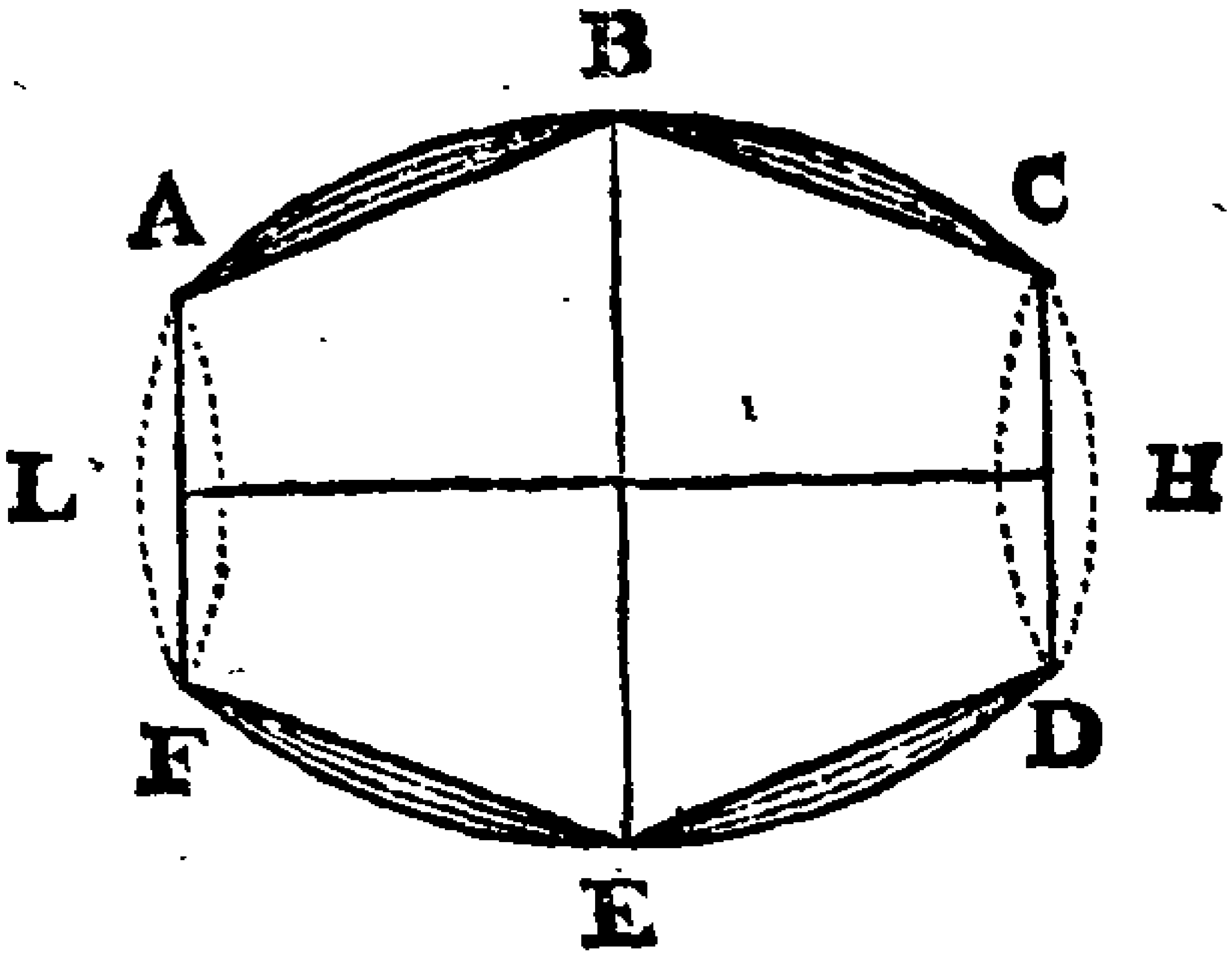
282 for Ale Gallons.
231 for Wine Gallons.
2150.42 for Corn Buſhels.

With reſpect to *Caſks*, it may be difficult, on Account of the different Bending of the Staves, to aſcertain exactly the Form to which they belong; for though the Dimenſions of ſeveral Caſks may be exactly the ſame, yet their Contents will be very different, as is clear from a Sight of the following Figure.

* The riſing *Crowns* of *Stills* are *Segments of Spheres*; the remaining Part generally the Fruſtum of a *Parabolic Conoid*; *Bowls* and *Baſons* are generally the Segments of Spheres, and meaſured accordingly.

Suppoſe

Suppose A B C D E F to represent a Cask; then, it is evident, that if the outer curved Lines A B C and D E F are the Boundaries or Staves of the Cask, it must of Course hold more than if the inner and straiter Lines were the Bounds and Staves of it, yet the Dimensions of the Bung Diameter B E, and Head Diameters A F and C D, and the Length L H, are the same in all the Casks.



If the Staves of the Cask are very much curved or arching, as the outer Line in the foregoing Figure, it is supposed to be in the Form of the *Middle Zone of a Spheroid*, and its Content may be found by Problem 3d.

If the Staves are not quite so much curved or arching, as represented in the second Line in the Figure, it is taken for the *Middle Zone of a Parabolic Spindle*, and is measured by Problem 7th.

When the Staves are but little curved or arching, as the third Line in the Figure, it is supposed to be in the Form of the lower *Frustums* of two equal *Parabolic Conoids* joined together upon one common Base in the Middle, and its Content may be found by Problem 9th.

If

If the Staves are quite ſtrait from Bung to Head, as the inner Lines in the Figure, it is then conſidered as the lower *Fruſtums* of two *equal Cones* joined together upon one common Baſe, and its Content may be found by Problem 8th. in Stereometry.

Note. Caſks made in the firſt Form hold the moſt; and thoſe of the laſt Form hold the leaſt of any other Kinds.

But ſince we can only at laſt gueſs, as it were, at the Variety or Form which the Caſk belongs to, the eaſieſt and beſt Way of finding its Content is to be preferred in Practice, which is to find ſuch a *mean Diameter* between the Bung and Head Diameters as will reduce the Caſk to a *Cylinder* equal to it, which may be done by the following

Rule.

Multiply the Difference between the Bung and Head Diameters by .7, or by .65, or by .6, or by .55, according as the Staves are more or leſs arching; add the Product to the Head Diameter, and that Sum will be a *mean Diameter*, *i. e.* it will be the Diameter of a Cylinder, whoſe Length and Content are equal, as near as can be, to that of the Caſk.

Example.

Suppoſe a Caſk whoſe Bung Diameter is 31.5 Inches; the Head Diameter 24.5 Inches, and its Length 42 Inches, what is its Content in Ale Gallons?

Operation.

Operation. The Bung Diameter 31.5 less the Head Diameter 24.5 is equal to 7; which multiplied by the Numbers in the above Rule, according as the Cask is more or less arching, and added to the Head Diameter, the several *mean Diameters* will be for the

1 Variety	29.4	and Content of each Cask will be found to be	101.	Ale Gallons.
2 Variety	29.05		98.71	
3 Variety	28.7		96.35	
4 Variety	28.35		94.03	

But if the Young *Geometrician* wishes to see the whole Method made use of in Practice by the Officers of the Excise Duty, for finding the Contents of all Kinds of Vessels, he may find an ample Account in the *Young Gauger's best Instructor*, written by the Author of this Work. In which Work is inserted an easy Method of finding the Content of any Cask, without considering the bending of the Staves, or regarding what Variety it belongs to.

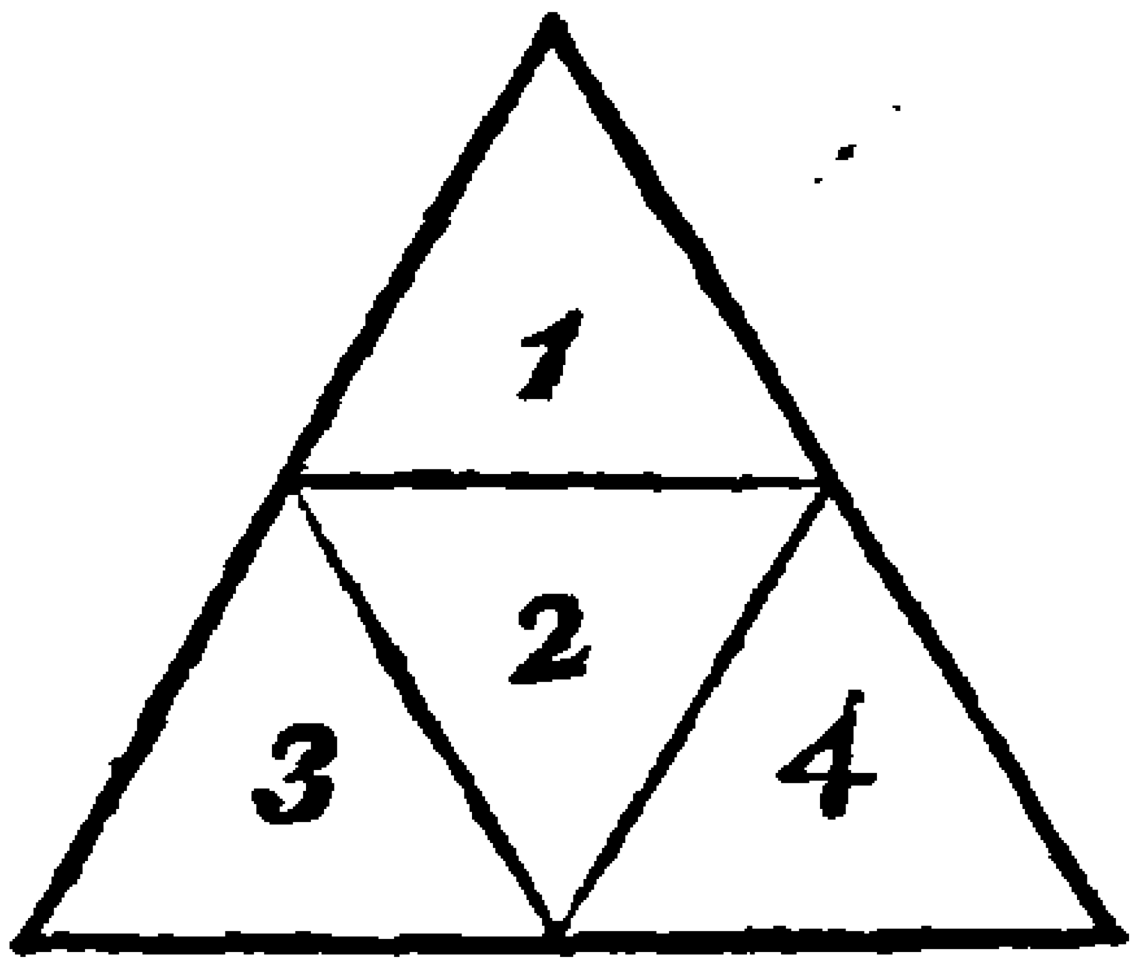
THE FIVE REGULAR BODIES; COMMONLY CALLED THE PLATONIC SOLIDS.

ALL thoſe Solid Bodies whoſe Sides are equal, and whoſe Superficies are ſimilar and equal, and which can be ſo incloſed within a *Sphere*, that each Angle ſhall touch the internal Surface of that *Sphere*, are called *Regular Bodies*; of which Bodies there are in Nature no more than *five*, *viz.* the *Tetraëdron*, *Hexaëdron*, *Octaëdron*, *Dodecaëdron*, and *Icoſaëdron*.

Of the Tetraëdron.

Def. A *Tetraëdron* is a Body contained under four equal and equilateral plane Triangles; conſequently, ſuch a Solid is a *Pyramid* ſtanding on an equilateral triangular Baſe; and its *Superficies* is equal to *four Times* the Area of the Baſe.

To conceive a more perfect Idea of the *Tetraëdron*, the Learner may draw a Figure like this upon Paſteboard,



or any other pliable Matter; then by cutting the Lines half through, turning up the Parts, and gluing them together, they will form a complete Tetraëdron; and this I would adviſe him to do, as he may by that Means more readily perceive the Reaſon of the following Operations.

Pro-

Problem I.

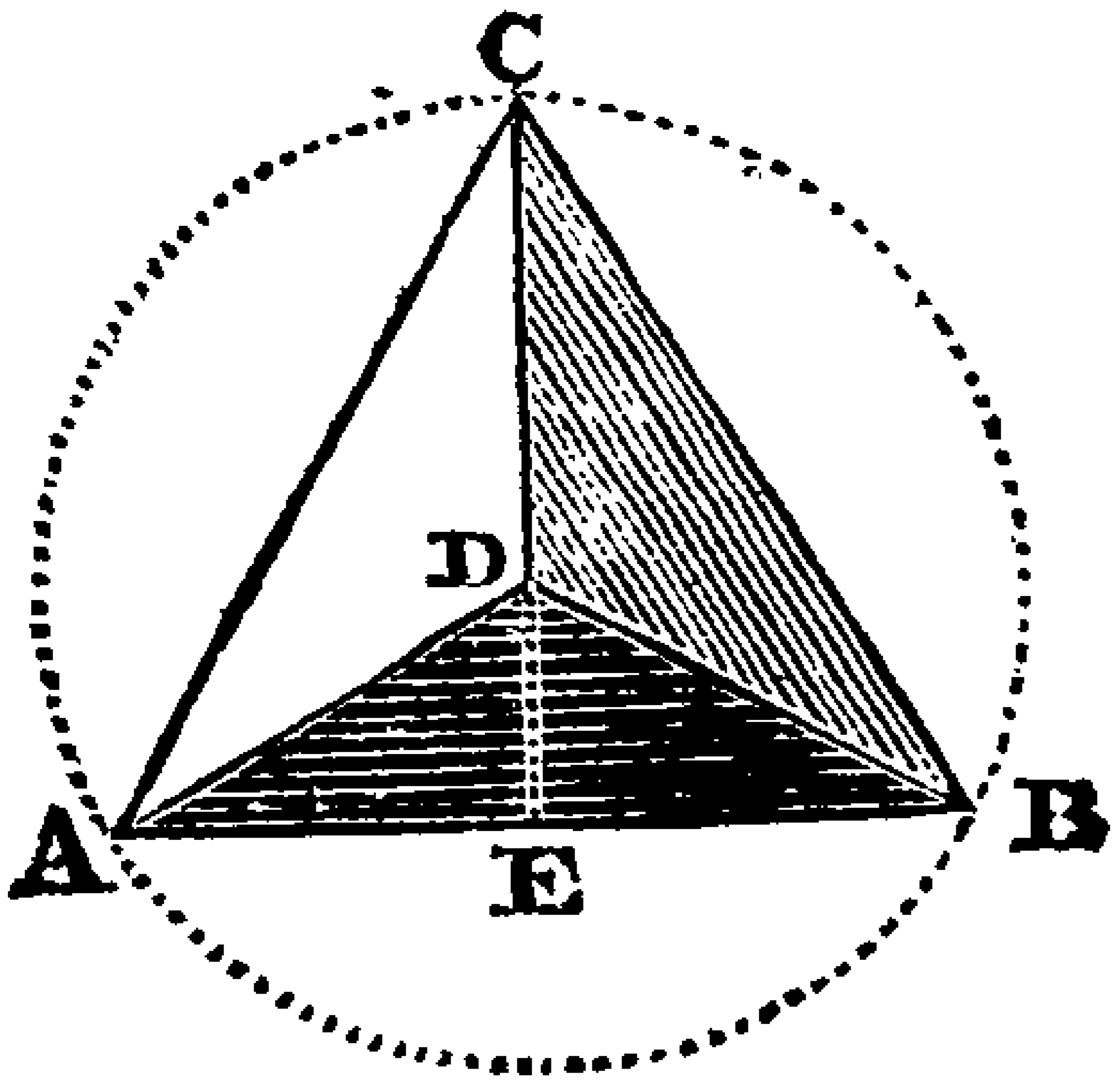
To find the Solidity of a *Tetraëdron.*

Rule.

Multiply the Area of one of the Triangles by $\frac{1}{3}$ of the Perpendicular Height of the Figure, and the Product will be the Solid Content.

Example.

Suppose A B C D E to represent a *Tetraëdron*, each of whose Sides, as A B, B C, and C A, are 6 Inches; the Height D E of one of the Triangles 5.196 Inches; and the Perpendicular Height of the Figure 4.899 Inches, what is its Solid Content?



Operation. 5.196, the Height of one *Triangle*, multiplied by 3, Half the Side of the Base, gives 15.588, the Area of one Triangle, which multiplied by 1.633, one-third of the Perpendicular Height of the Tetraëdron, gives 25.455204 Inches, the Solid Content required.

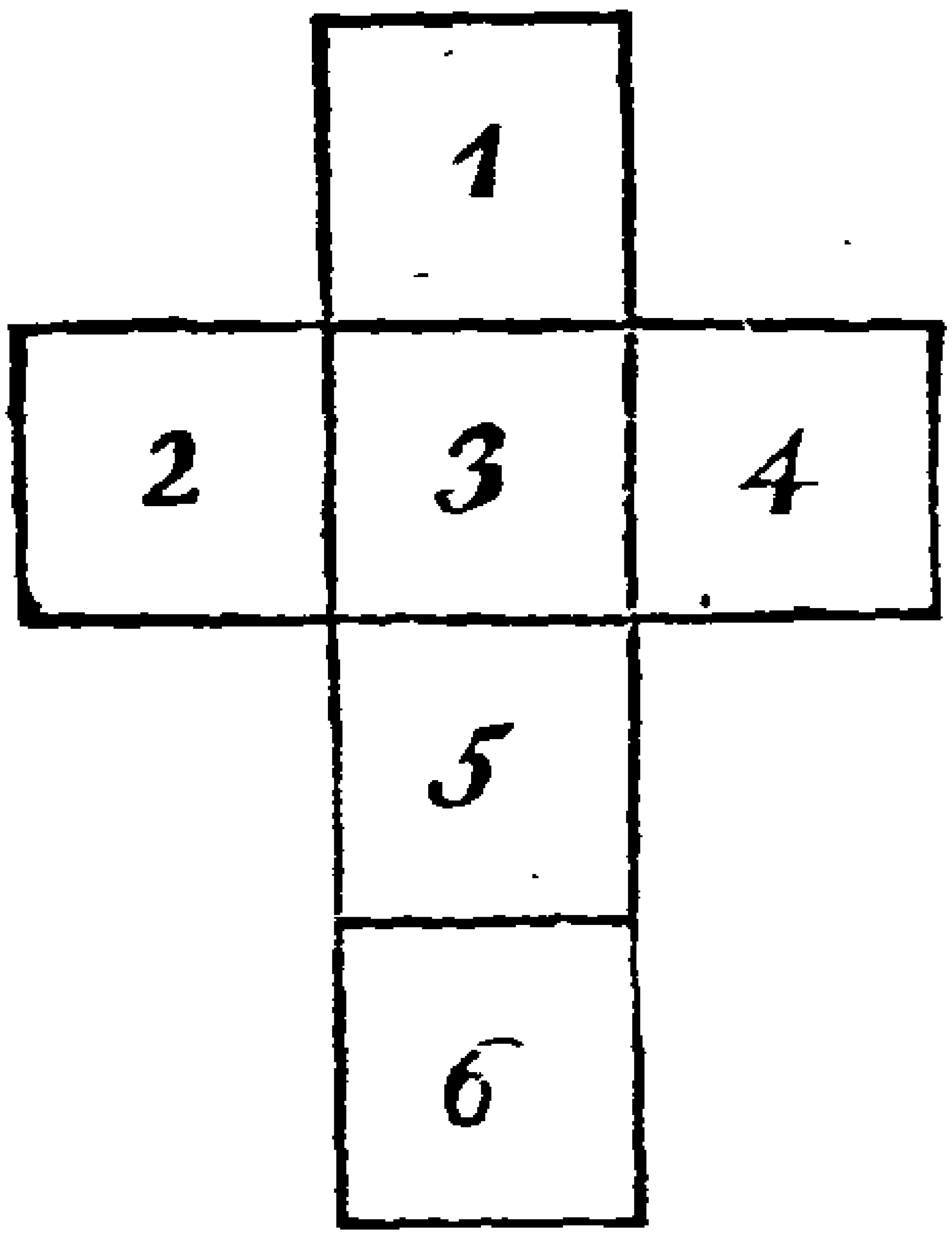
For the *Superficial* Content, multiply 15.588, the Area of one Triangle, by 4, the Number of Triangles, and it makes 62.352 Inches, the *Superficial* Content.

Of

Of the Hexaëdron.

Def. An *Hexaëdron*, or Cube, hath equal Length, Breadth, and Depth, like a *Die*, and is contained under *six* Square Planes; consequently the *Solidity* is equal to three Dimensions multiplied by each other, and the *Superficies* equal to *six Times* the Area of the Base, or one of its Sides.

A Figure drawn upon Pasteboard similar to this here inserted, having the Lines cut half through, folded up, and



glued together, will form the true Figure of an *Hexaëdron* or Cube.*

* An *Hexaëdron* being the same Figure as the *Cube*, the Measure of which having been considered before in *Stereometry*, makes it unnecessary to insert it here, unless to preserve the Propriety of representing the Five Platonic Solids in a regular Order.

Pro-

Problem 2.

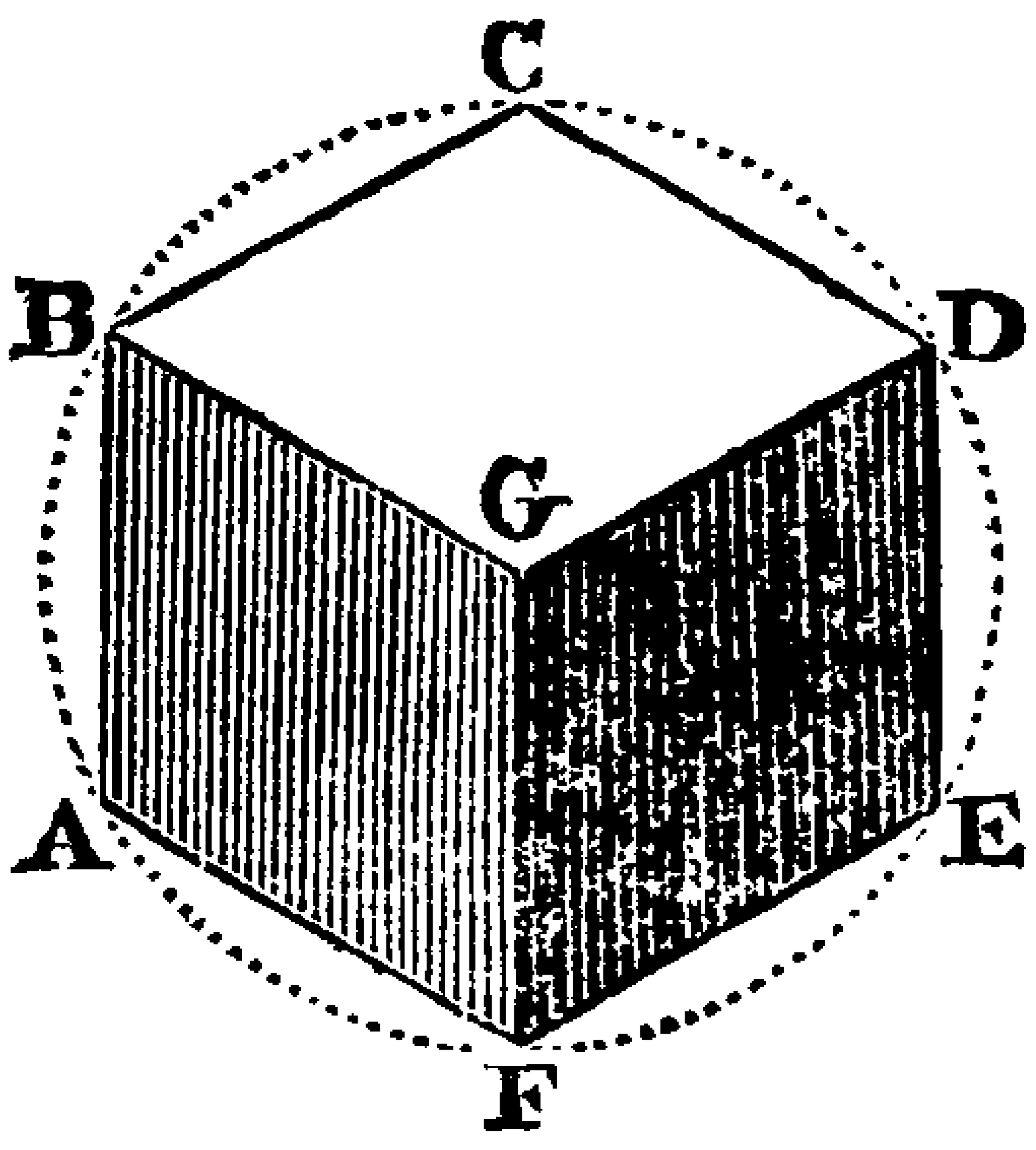
To find the Solidity of an *Hexaëdron.*

Rule.

The *Hexaëdron* being only a *Cube*, is meaſured by multiplying the Side into itſelf, and that Product again by the Side, and this laſt Product will be the Solidity required.

Example.

Suppoſe the Side A B = B C = C D, *&c.* of an *Hexaëdron* be 6 Inches, what is the Solid Content?



Operation. The Side 6 multiplied by 6, gives 36, which multiplied again by 6 gives 216 Inches, the *Solidity* required.

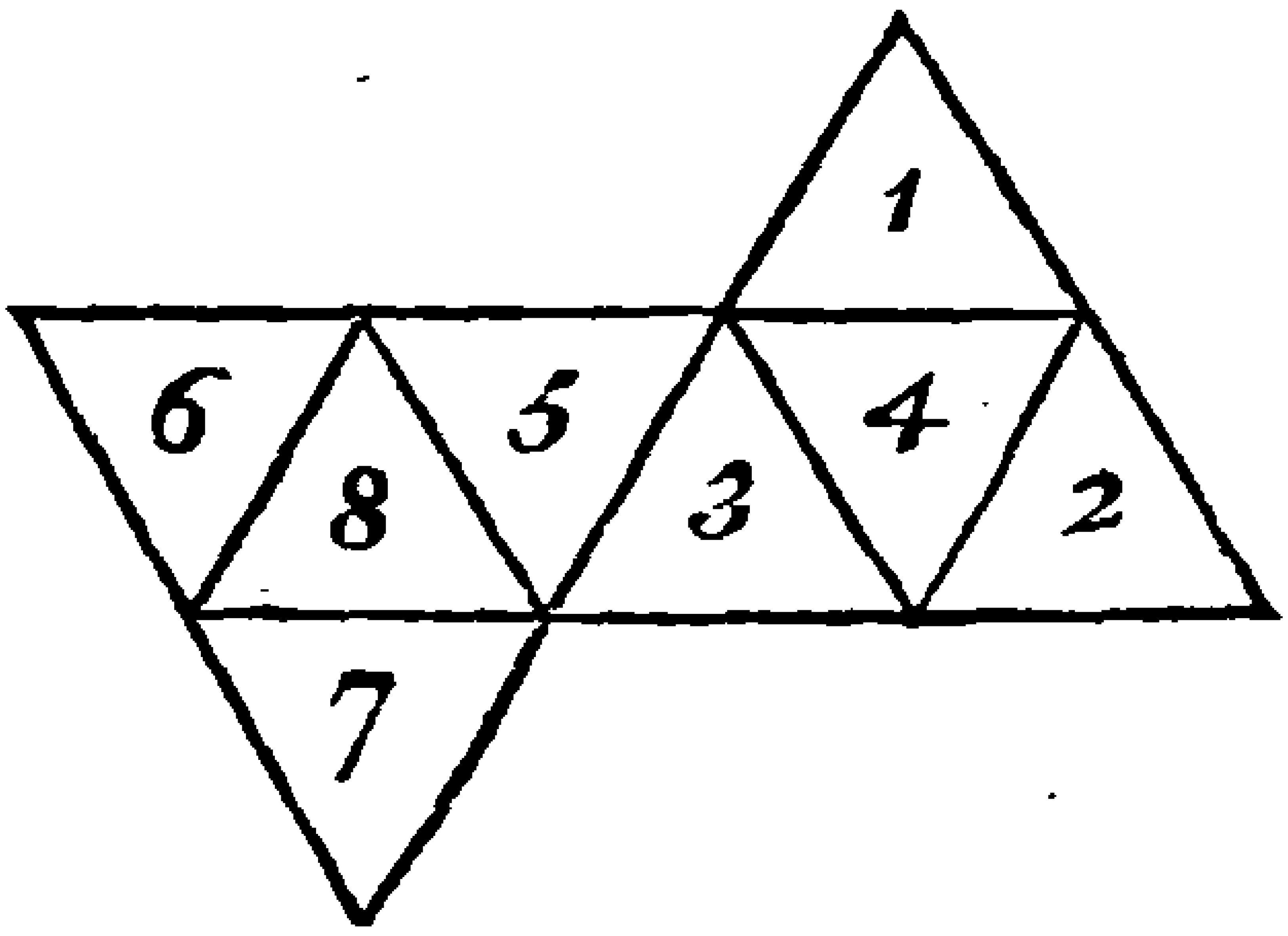
For the *Superficial* Content, multiply 36, the Area of one of the Sides, by 6, the Number of Sides, gives 216 Inches, the *Superficies* ſought.

Of

Of the Octaëdron.

Def. An *Octaëdron* is a Solid composed of *Eight* equal Pyramids, whose Tops all meet in a Point at the Center of the Solid; the Base of each being an equilateral Triangle, and each equal to the other; or, it is composed of two *Quadrangular Pyramids* joined together by their Bases. The *Superficies*, therefore, is equal to *eight* Times the Area of one Triangle, and the *Solidity* equal to the Solidity of the eight composing Pyramids, or to *two Quadrangular* ones.

A similar Figure to this here delineated, being drawn upon Pasteboard, cut half through in the Lines, folded



up, and glued together, will give the Learner an adequate Idea of an *Octaëdron*, as composed either of *eight Equilateral Triangular Pyramids*, or of *two Quadrangular ones.*

Problem 3.

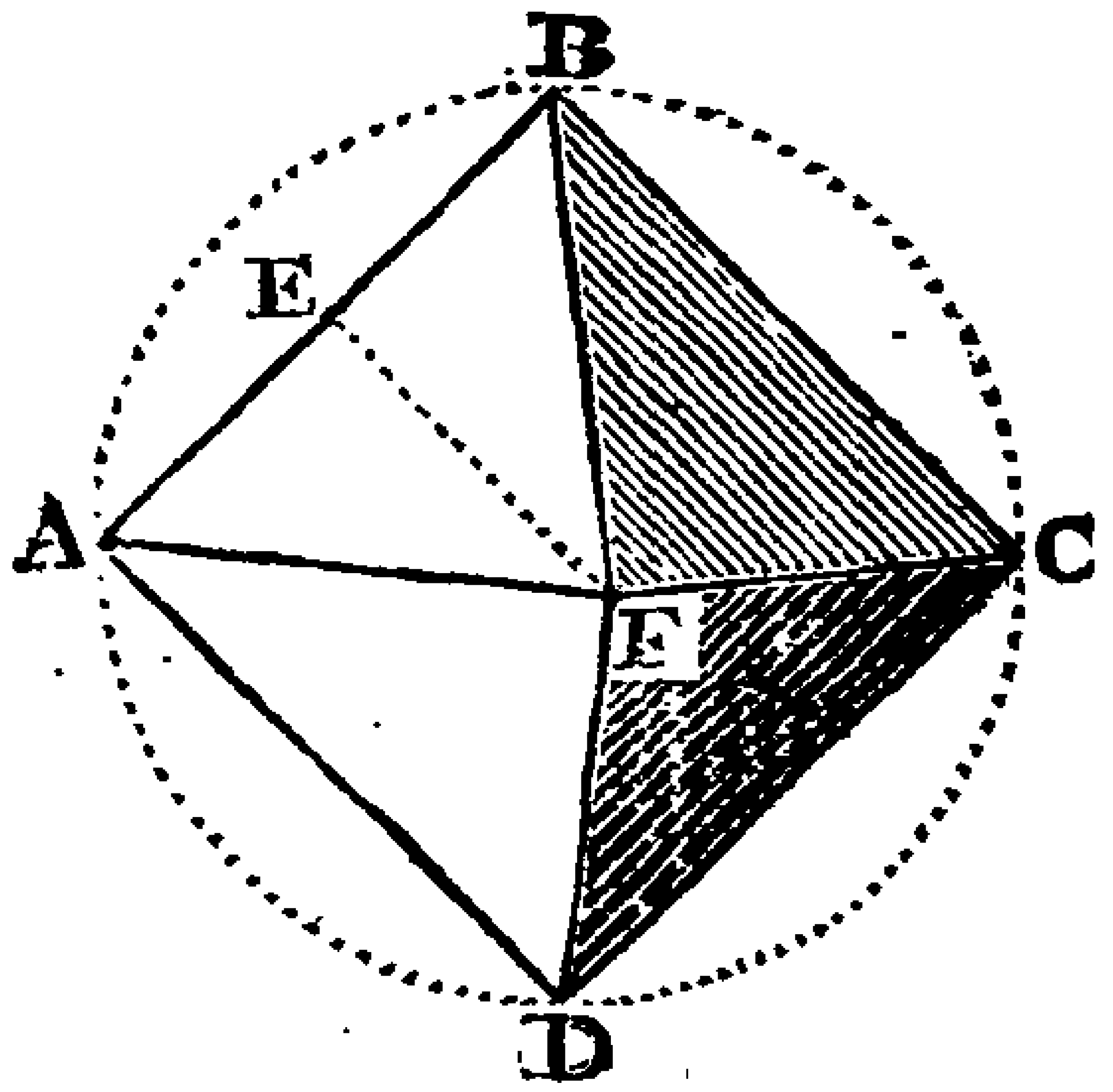
To find the *Solidity* of an *Octaëdron.*

Rule.

Multiply the Area of the Square Base in the Middle by $\frac{1}{3}$ Part of the Height of both Pyramids, and the Product will be the Solid Content required.

Example.

Suppose A B C D be an *Octaëdron,* whose Side A B = B C = C D, *&c.* is 6 Inches; the Height of one of its Sides, as E F, 5.196 Inches; and the Perpendicular Height of the two *Quadrangular Pyramids* B D 8.484 Inches; what is its Solid Content?



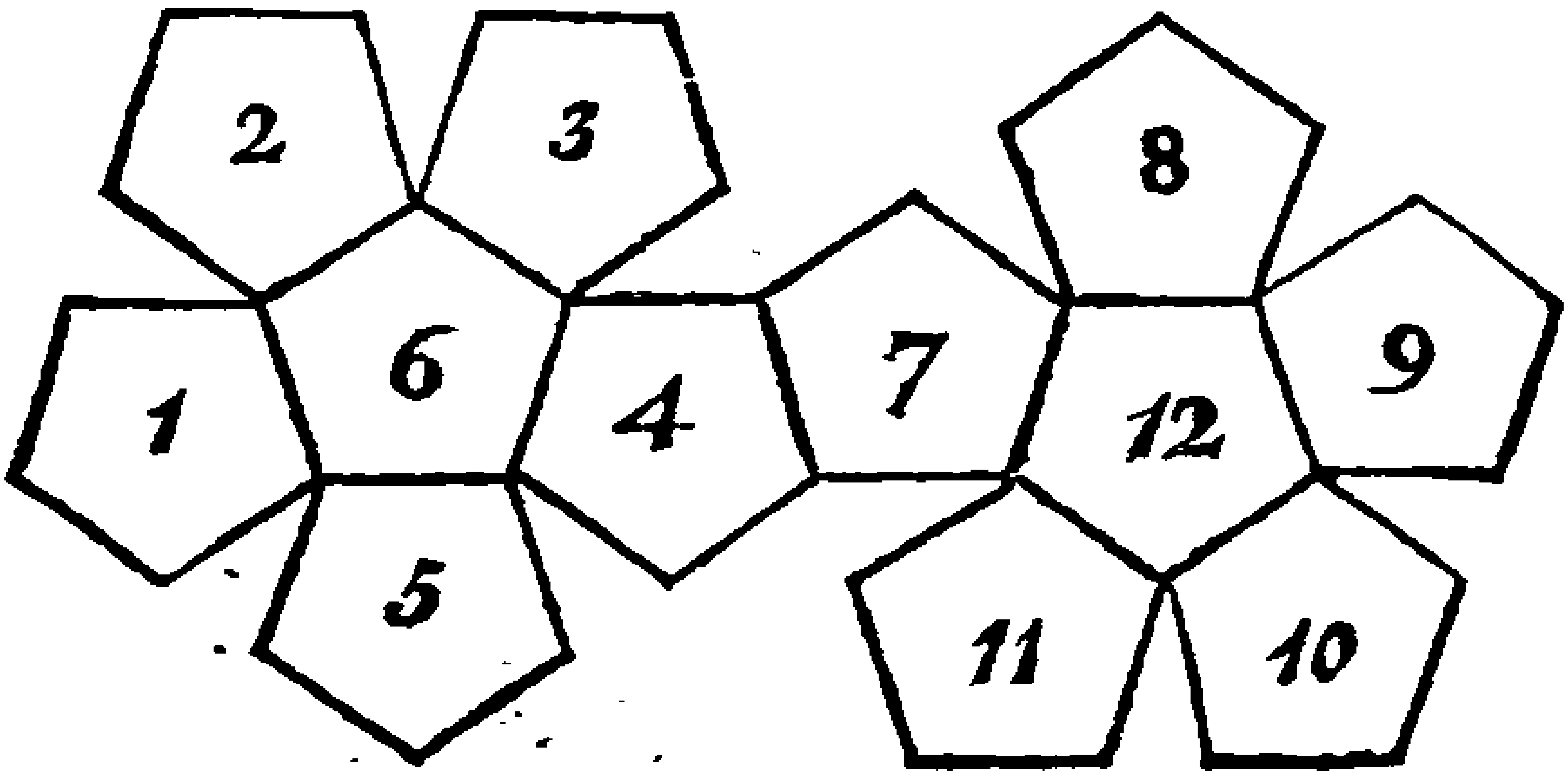
Operation. 6, the Side, multiplied by 6, is 36, the Area of the Square in the Middle, which multiplied by 2.828, *one-third of both Pyramids,* gives 101.808 Inches, the Solid Content sought.

For the *Superficial Content,* multiply 5.196, the Height of one Triangle, by 3, half the Side of the Base, and that Product multiply by 8, the Number of Triangles, gives 124.704 Inches, the *Superficial Content.*

Of the *Dodecaëdron*.

Def. A *Dodecaëdron* is a Solid compoſed of *twelve* equal Pyramids, whoſe Tops all meet in a Point at the Center of the Solid; the Baſe of each Pyramid being an equilateral Pentagon, and equal to each other. The *Superficies* of ſuch a Body is therefore equal to *twelve Times* the Area of one Pentagon; and the Solidity is equal to the Solidities of the *twelve* compoſing Pyramids.

If there be drawn upon Paſteboard a Figure like the following Projection; and the Lines be cut half through,



folded up and glued together, the ſeveral Pentagons will then form the true Figure of a *Dodecaëdron*.

Problem 4.

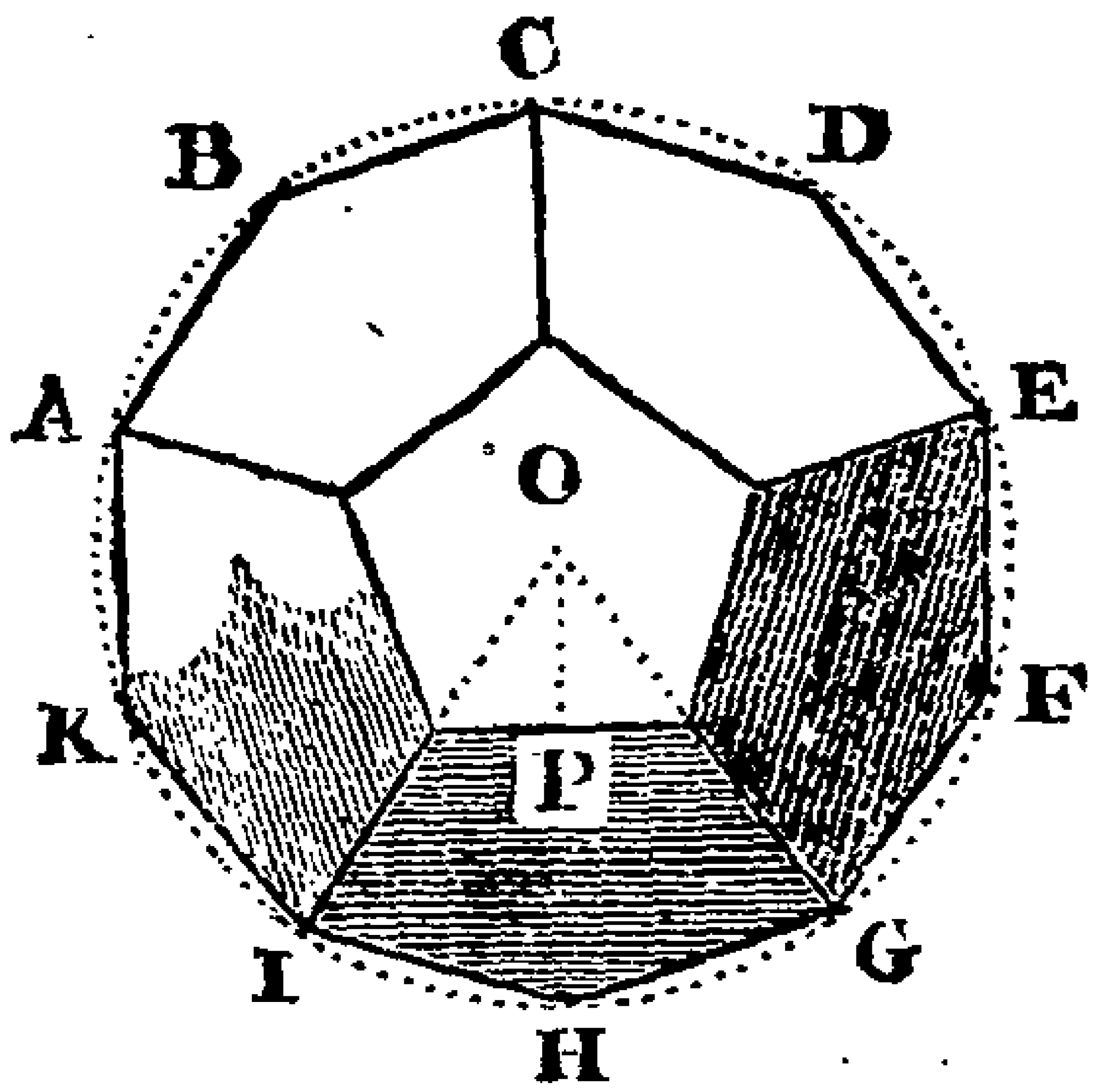
To find the Solidity of a *Dodecaëdron.*

Rule.

Find the Solid Content of one of the Pyramids, and multiply it by 12 (the Number of Pyramids contained in the Figure), the Product will be the Solid Content.

Example.

Let A B C D E, &c. represent a *Dodecaëdron*, each Side of which is equal to 6 Inches; the Height of one of the Pentagons, as O P, is 4.129 Inches; and the Altitude of the whole Figure 13.362 Inches; half of which is the Altitude of one of the Pyramids, *viz.* 6.681 Inches; what is its Solid Content?

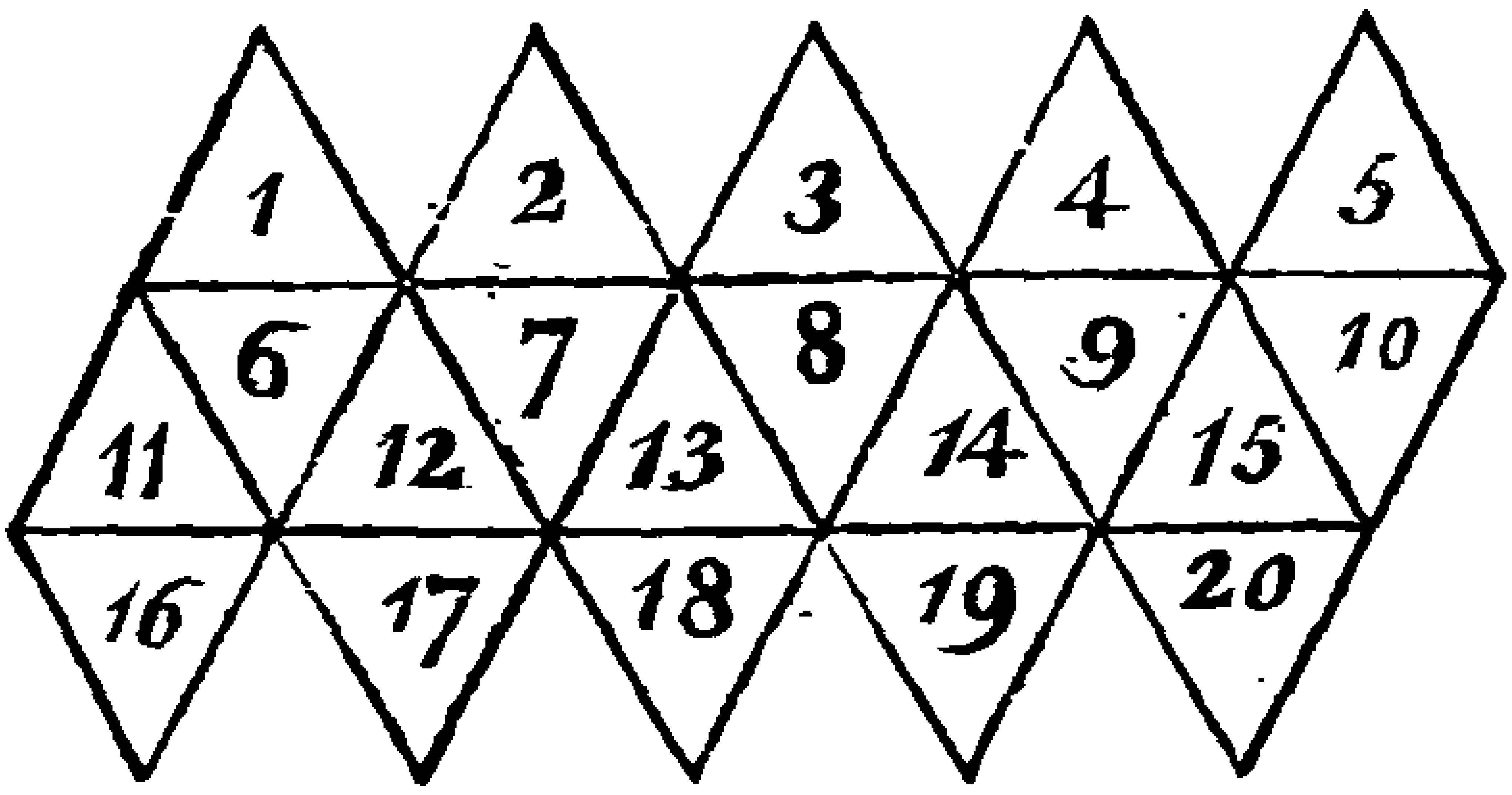


Operation. 4.129, Height of one Pentagon, multiplied by 15, half the Sum of its 5 Sides, gives 61.9350 for the Area of one of the Pentagons; which multiplied by 2.227, one-third of the Height of each Pyramid, gives 137.929245, the Content of one Pyramid; which multiplied again by 12, the Number of Pyramids, gives 1655.150940 Inches, the Solidity required.

For the *Superficial Content*, multiply 61.935, the Area of one Pentagon, by 12, the Number of Pentagons, and the Product gives 743.220 Inches, for the Superficial Content.

Of the *Icoſaëdron*.

Def. An *Icoſaëdron* is a Solid made up of twenty Pyramids, whoſe Tops all meet in a Point at the Center of the Body; the Baſe of each Pyramid being an Equilateral Triangle, and equal to each other. The Superficies, therefore, is equal to *twenty Times* the Area of one Triangle; and the Solidity equal to the Solidities of the *twenty* compoſing Pyramids.



We have added the annexed Figure, that the Learner, by drawing a ſimilar one upon Paſteboard, cutting the Lines half through, folding them up together, as directed before, for the other *Regular Solids*, may conceive a perfect Idea of the Figure and Dimenſions of an *Icoſaëdron*.

Pro-

Problem 5.

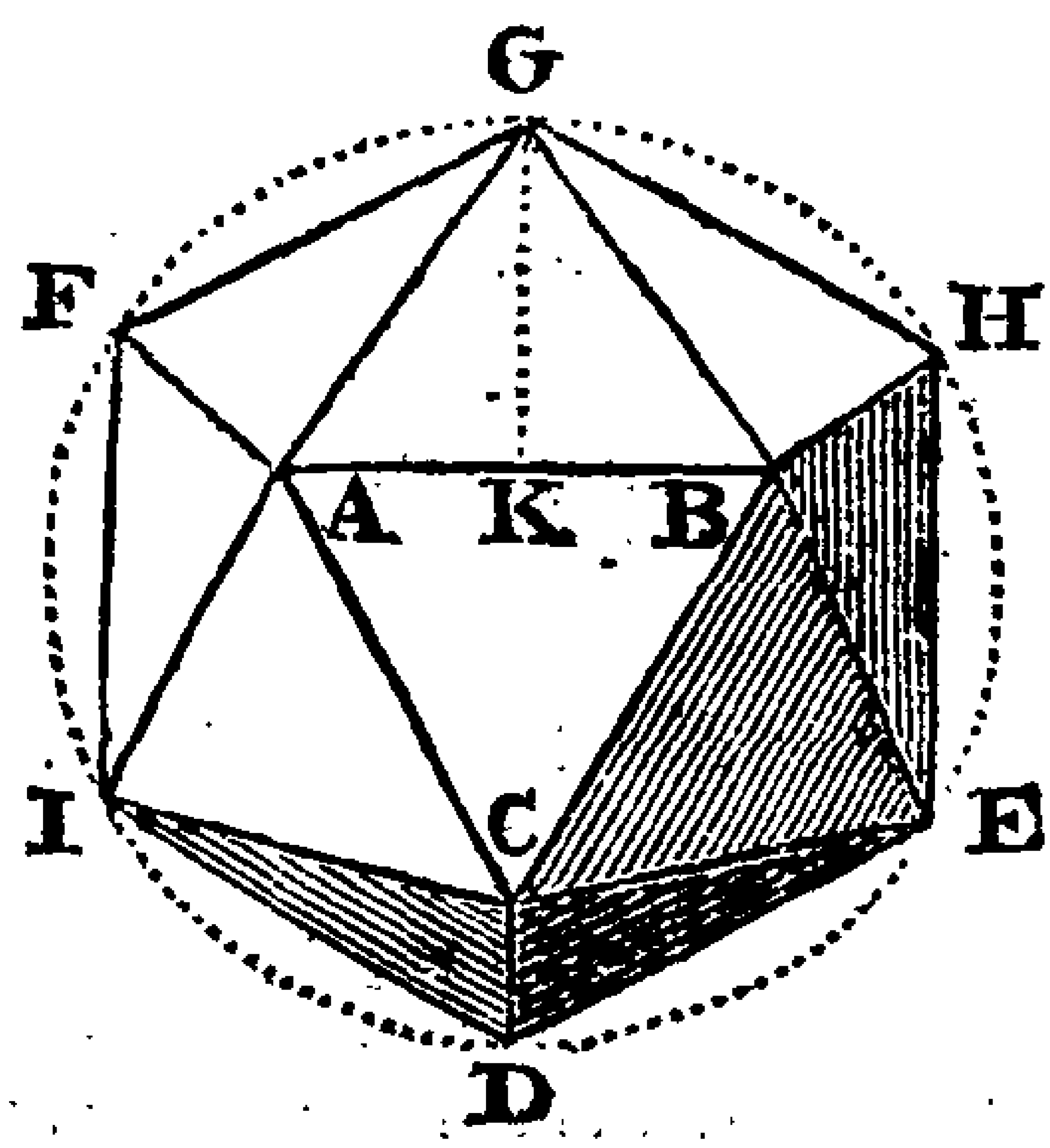
To find the Solidity of an *Icosaëdron*.

Rule.

Find the Content of one of the Triangular Pyramids, which multiply by 20, the Number of all the Pyramids, and the Product will be the Solid Content required.

Example.

Let ABCDE, &c. be an *Icosaëdron*, each Side of which is 6 Inches; the Perpendicular of one of the Triangles, as GK, is 5.196 Inches; the Height of the whole Figure is 9.069 Inches; half of which is 4.534 Inches, the *Altitude of one of the Pyramids*; what is its Solid Content?



Operation. Multiply 15.588, the Area of one Triangle, by 1.5115, the third Part of the Height of one Pyramid, the Product 23.561262 will be its Content, which multiplied by 20, the Number of all the Pyramids, gives 471.24612 Inches, the Solidity of the whole *Icosaëdron*.

For the *Superficial Content*, multiply 15.588, the Area of one Triangle, by 20, the Number of all the Triangles, and the Product 311.76 is the *Superficial* Content.

From the *Solidity* and *Superficies* thus found of the foregoing Bodies, the Solidity and Superficies of any other like Body may be easily obtained by having the Side only given. For, as all similar or like *Solids* (especially these regular ones) are to one another as the *Cube* of their like Sides; and their *Superficies* also being similar and alike, are therefore to each other as the *Squares* of their like Sides, we have this

Rule.

As the *Cube* of the Side of any of the foregoing Solids is to its Solid Content, so is the Cube of the Side of any other like Solid to its Solid Content.

And, as the *Square* of the Side is to its Superficial Content, so is the Square of the Side of the like Body to its Superficial Content.

Example.

What is the *Solidity* and *Superficies* of each of the Regular Solids, supposing each Side of them to be 1 Inch, or 1 Foot, &c?

For the Solid Contents.

Operation.

Cube of 6.	Solid Content.	Cube of 1.	Solid Content.
As 216	25.4552	1	.1178 — of *Tetraëdron.*
216	6.	1	1.0000 of the *Hexaëdron.*
216	101.808	1	.4714 + *Octaëdron.*
216	1655.1509	1	7.663 + *Dodecaëdron.*
216	471.2461	1	2.1816 — *Icosaëdron.*

For

For the Superficial Contents.

Operation.

Square of 6.	Superf. Cont.	Squ. of 1.	Superf. Cont.	
As 36	62.352	1	1.732	of the *Tetraëdron.*
36	216.	1	6.000	—— *Hexaëdron.*
36	124.704	1	3.464	—— *Octaëdron.*
36	743.22	1	20.645	—— *Dodecaëdron.*
36	311.76	1	8.66	—— *Icosaëdron.*

Note. By these last Numbers, the *Solidity* and *Superficies* of any of the Regular Solids may be found easier than by the former Operations. For, here you need only multiply the *Cube* of the given Side by the Number expressing the Solid Content in the Table, to know its *Solidity*; and the *Square* of the Side multiplied by the Number expressing the Superficial Content will give the *Superficies* of that Body.

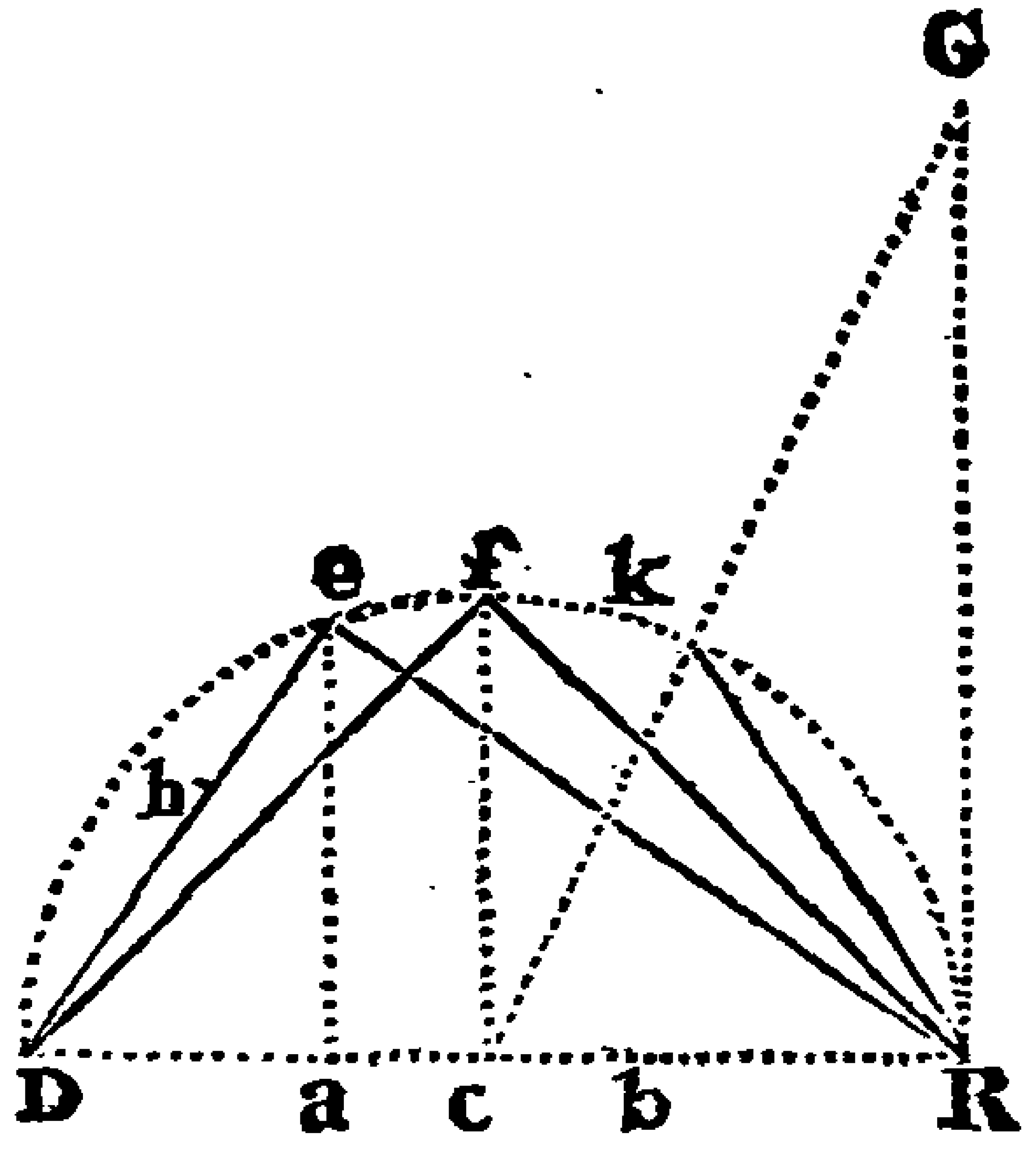
As it may be useful to determine the Length of the Sides of any of the Regular Bodies inscribed in a Sphere of any given Dimensions; we shall here annex an easy Geometrical Problem for that Purpose.

Problem 6.

To find the Length of the Sides of the five Regular Solids inſcribed in a Sphere of any given Dimenſions.

Example.

Suppoſe the Diameter of the given Sphere be 2 Inches, what are the Lengths of the Sides of each of the five Regular Bodies that can be circumſcribed by it?



Conſtruction. Let D R be the Diameter of the given Sphere of two Inches; and let $D a = a b = b R$ be $\frac{1}{3}$ (one third of that Diameter). Erect the Perpendiculars *a e* and *c f*, and draw the Chords D *e*, D *f*, *e* R, and *f* R. Then will

(1ſt.) R *e* be the Length of the Side of the *Tetraëdron* = 1.62 Inches.

(2d.) D *e*, the Side of the *Hexaëdron*, = 1.15 Inches.

(3d.) D *f* = *f* R, the Side of the *Octaëdron*, = 1.41 Inches.

(4th.)

(4th.) Cut the Chord D e in extreme and mean Proportion, by Problem 23, in b, and D b will be the Side of the *Dodecaëdron* = 0.71 Inches.

(5th.) Set up the Diameter D R perpendicularly at R, and from the Center c, to the Top at G, draw the Line c G, cutting the Arch at k, and draw the Chord k R, which will be the Side of the *Icoſaëdron* = 1.05 Inches.

If any of theſe five Bodies were required to be cut out of a Sphere of any other Diameter, the Rule will always hold,

As the Diameter of the Sphere 2 Inches, is to the Side of any one Solid inſcribed in it, as ſuppoſe the *Icoſaëdron* 1.05 Inches : : ſo is the Diameter of any other Sphere, ſuppoſe 12 Inches, to 6.3 Inches, the Side of the *Icoſaëdron* inſcribed in that Sphere. *

This Problem may be uſeful to thoſe who want to cut out any of the above Bodies in Wood or Stone to a determinate Size, either for *Dials*, or *Ornaments* for Gateways, *&c.*

* In this Manner the Sides of all the Regular Solids inſcribed in a Sphere of 12 Inches may be eaſily found to be as under:

The *Tetraëdron* 9.7; *Hexaëdron* 6.9; *Octaëdron* 8.4; *Dodecaëdron* 4.2; *Icoſaëdron* 6.3 Inches.

Note. There is one Thing very remarkable reſpecting theſe Five Bodies, which is, that if an abſolute *Plenum* takes Place in the Univerſe (a Doctrine held by ſome Philoſophers), then the conſtituting Particles of Matter muſt be in the Shape of ſome one of theſe Solids; for there are no other Bodies, let their Figurability be what they may, but will, when combined together, leave ſome Vacuity or Interſtice between them.

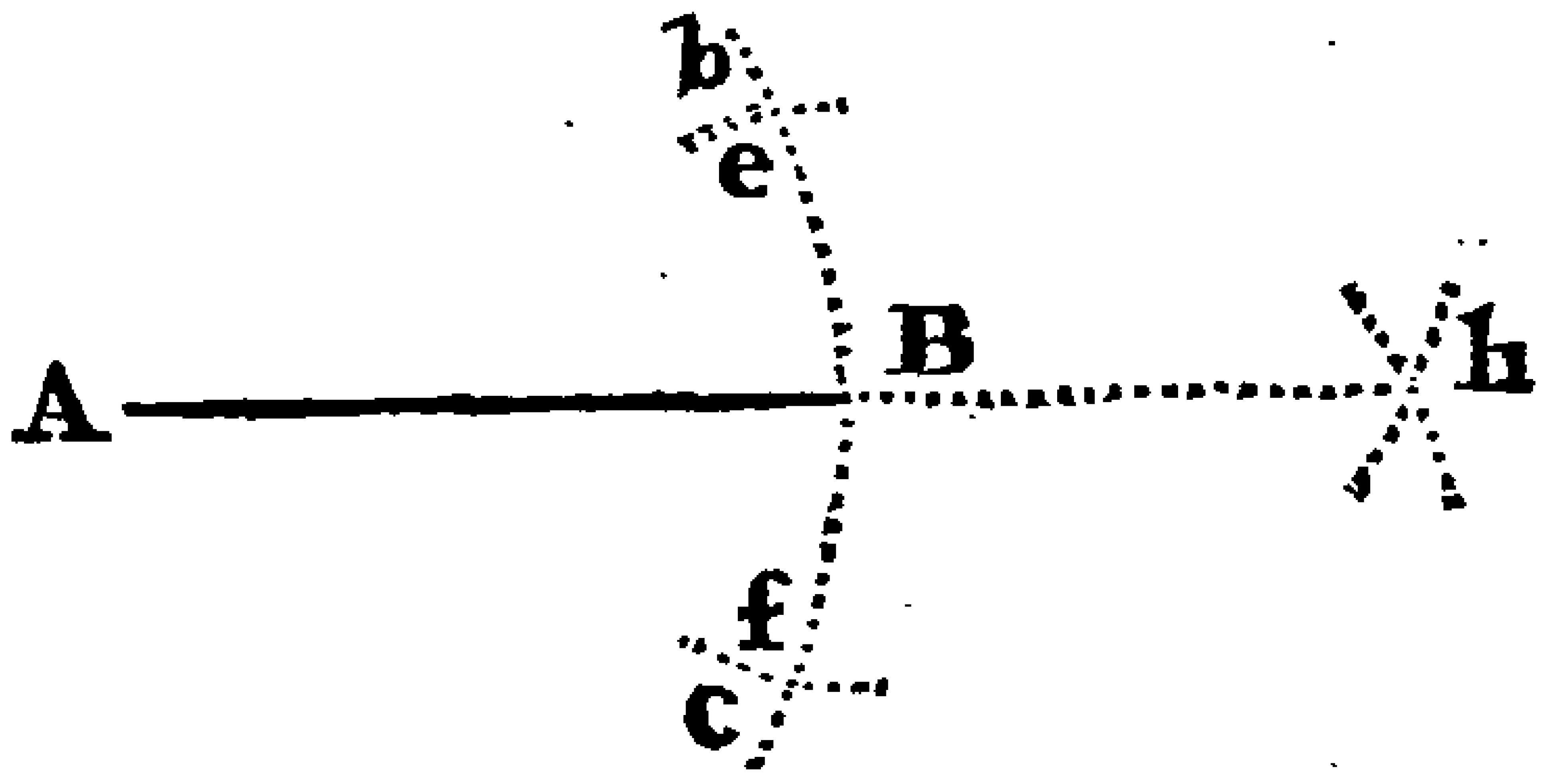
ADDI-

ADDITIONAL PROBLEMS.

Problem I.

TO continue a Right Line to a greater Length than can be drawn by a Ruler at one Operation.

Suppofe A B be the Line given, which cannot be made longer at one Operation, by Reafon of the Ruler being of the fame Length.

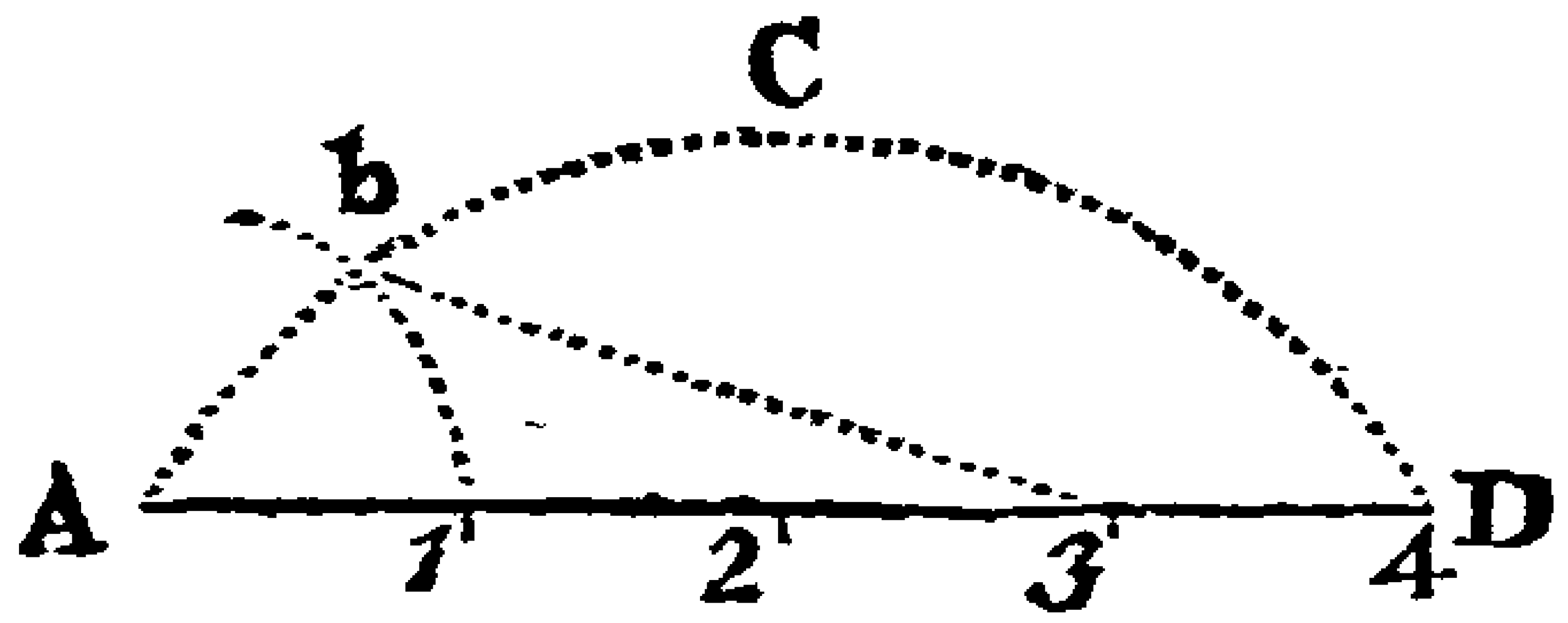


Operation. With the Compaffes opened to the Length of the given Line A B, fet one Foot in A, and with the other defcribe the Arch *b c*; upon which, from the End of the given Line at B, fet off two Points, as *e* and *f*. On each of which Points alternately fet one Foot of the Compaffes (opened to any Widenefs) and defcribe the Arches interfecting each other at *h*; to which, from the End of the given Line, lay a Ruler, and continue the faid Line at Pleafure. By this Means a Line of any determinate Length may be drawn with a very fhort Ruler.

Problem 2.

To find the *Length of any Arch* of a Circle.

Let A C D be the Arch, whose Length is required.



Operation. Divide the Chord A D into 4 equal Parts, and set off one Part from A to *b*; then draw a Line from *b* to the End of the 3d Division on A D, and it will be nearly equal to *half* the Arch; which doubled, will give the Length of the *whole Arch* A C D required.

To find the same more exactly in Numbers.

Rule.

Multiply the Radius of the Circle by the Number of Degrees in the given Arch, and that Product multiply again by .0174533 (a Decimal), and this last Product will be the Length of the Arch required.

Suppose the Diameter of a Circle be 22.6 Inches, and the Arc, or Part of the Circumference given, be 52 Degrees 15 Minutes, what is its Length?

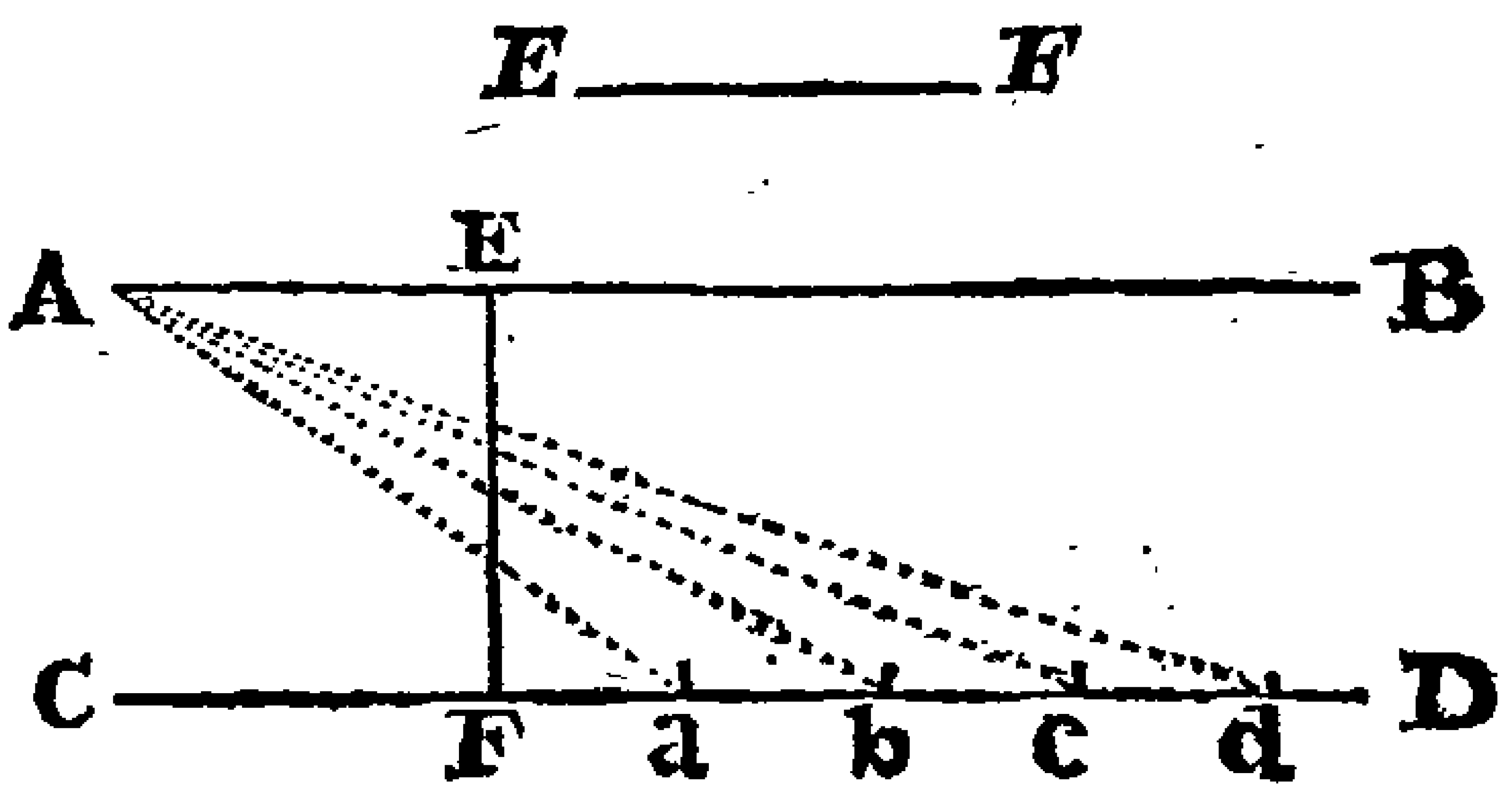
Operation. The Decimal of 15 Minutes is .25, which added to 52 Degrees is 52.25. Then 52.25 × 11.3 the Radius = 590.425, which × .01745 gives 10.30291625 Inches, the *true* Length of the Arch required.

Pro-

Problem 3.

To divide a given Right Line into an *infinite Number* of Parts.

Let the Line given be E F to be divided into a Number of Parts, exceeding any finite Number.

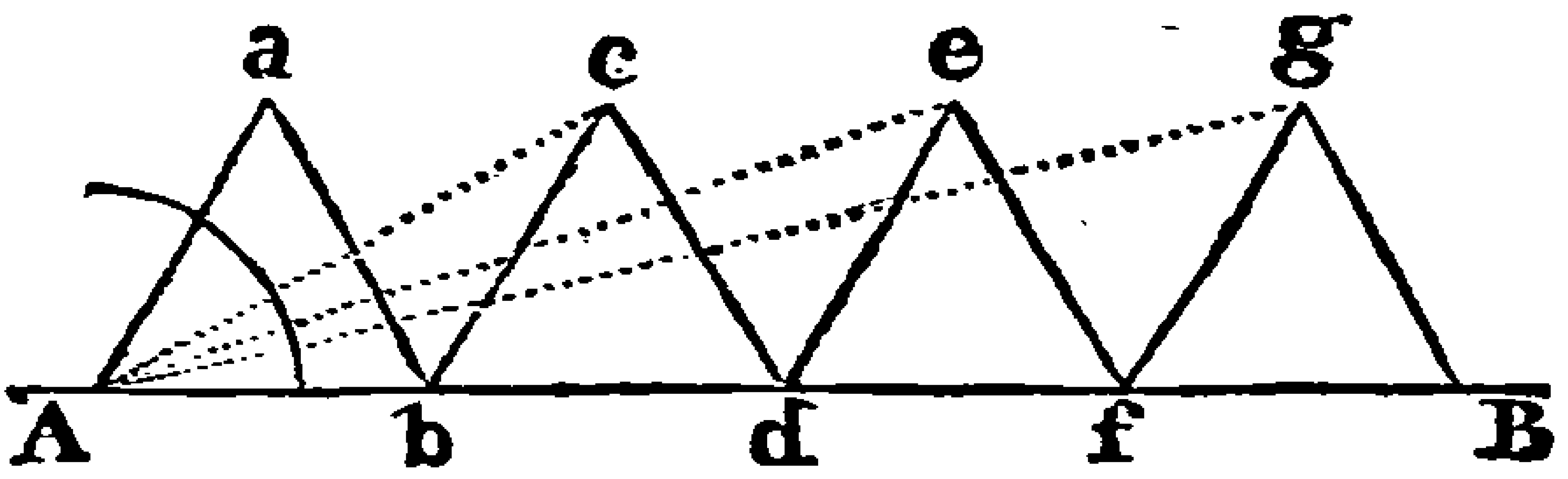


Operation. First, set the Line E F upright between the 2 parallel Lines A B and C D, and suppose them infinitely extended to the Right Hand; then it is evident, that in the Line C D infinitely extended there may be taken an infinite Number of Points, *a*, *b*, *c*, *d*, &c. Now if to each of these Points there be drawn Right Lines from the Point A taken in the Line A B, to the Left of the Line E F, each of these Lines A *a*, A *b*, A *c*, &c. will cut off a small Portion of the Line E F; but because the Points *a*, *b*, *c*, &c. are infinite in Number, so likewise are the Lines A *a*, A *b*, A *c*, &c. and consequently the Parts, or small Portions, they will cut off from the Line E F will be infinite in Number too. Whence it is manifest that the Line E F, however small, may be divided into an *infinite Number* of Parts.

Note. The *smallest Particle of Matter*, as well as the largest, is capable of an *infinite Division*.

Problem 4.

To ſhew that an *Angle*, as well as a Line, may be continually diminiſhed, and yet never be reduced to Nothing.



Operation. Let A B be a Right Line produced to an infinite Length beyond B. On this Line let there be placed an infinite Number of Equilateral Triangles, as A *a b*, *b c d*, *d e f*, &c. cloſe to each other. Then from the Point A, draw the Lines A *c*, A *e*, A *g*, &c. to the Tops of the 2d, 3d, 4th, &c. Triangles. Whence it is plain, that every Line drawn from A, to the Top of every ſucceeding Triangle, will make a leſs Angle with the Line A B, than the Line immediately before it. But no Right Line drawn from the Point A to the Top of any Triangle ſet upon the Line A B, how far off ſoever, could ever coincide with the Line A B; therefore the Angle at A will be continually diminiſhing, but can never be exhauſted, or come to nothing.

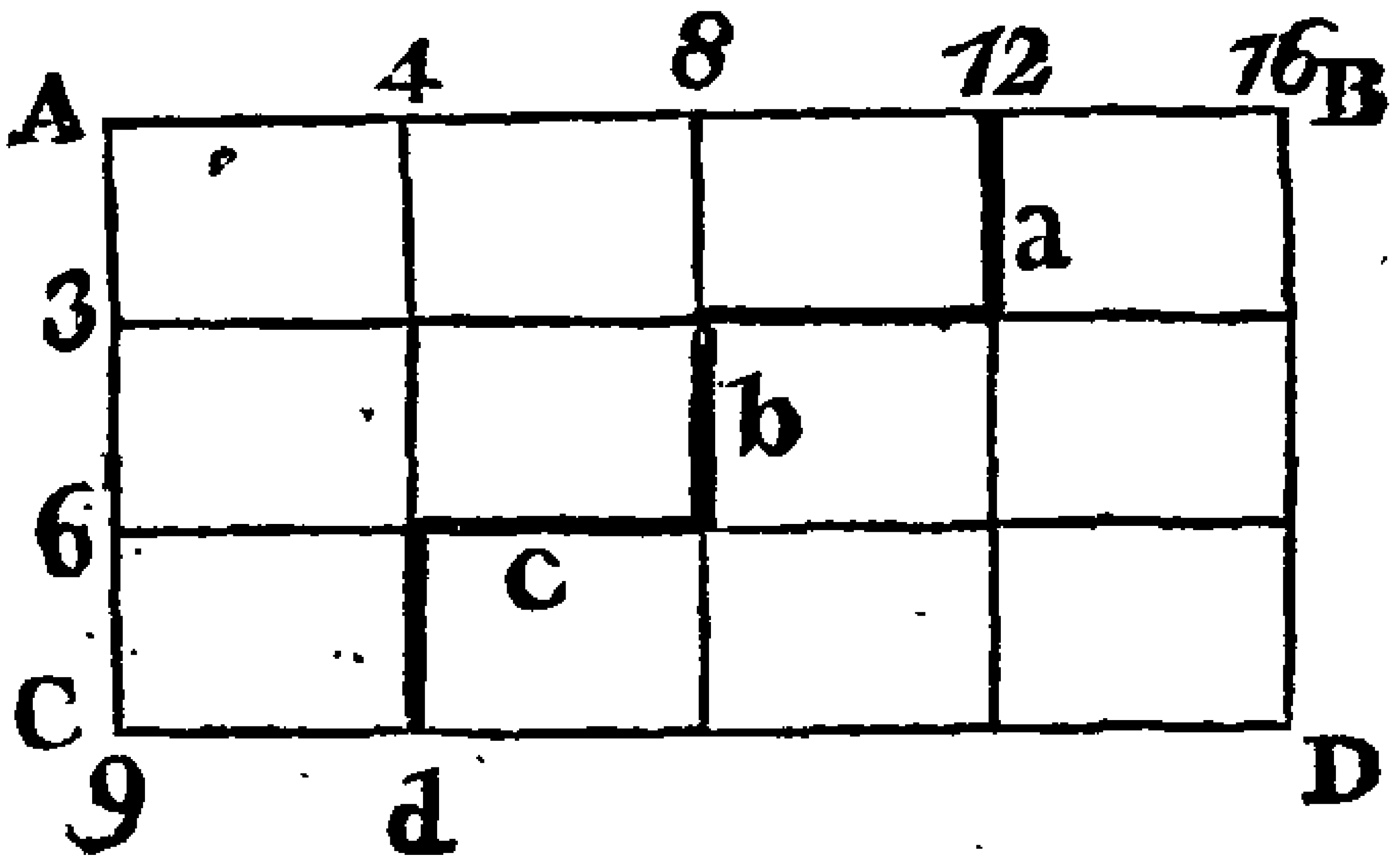
Note. The Line *a b* of the firſt Triangle will never be quite cut off by any Line drawn from A to the Top of any Triangle; a Part of it towards the Bottom will ſtill remain, which proves here, as in the laſt Problem, *that Matter is diviſible ad infinitum.*

Pro-

Problem 5.

To reduce a *Parallelogram to a Square* equivalent in Area to it.

A Man having a Pannel of his Cupboard 12 Inches ſquare broken out, and having by him a Board 9 Inches broad, and 16 Inches long, which is equal in Area to it, wants to know how this Board muſt be cut through into 2 Pieces, ſo as exactly to fit the ſaid Hole of 12 Inches ſquare.

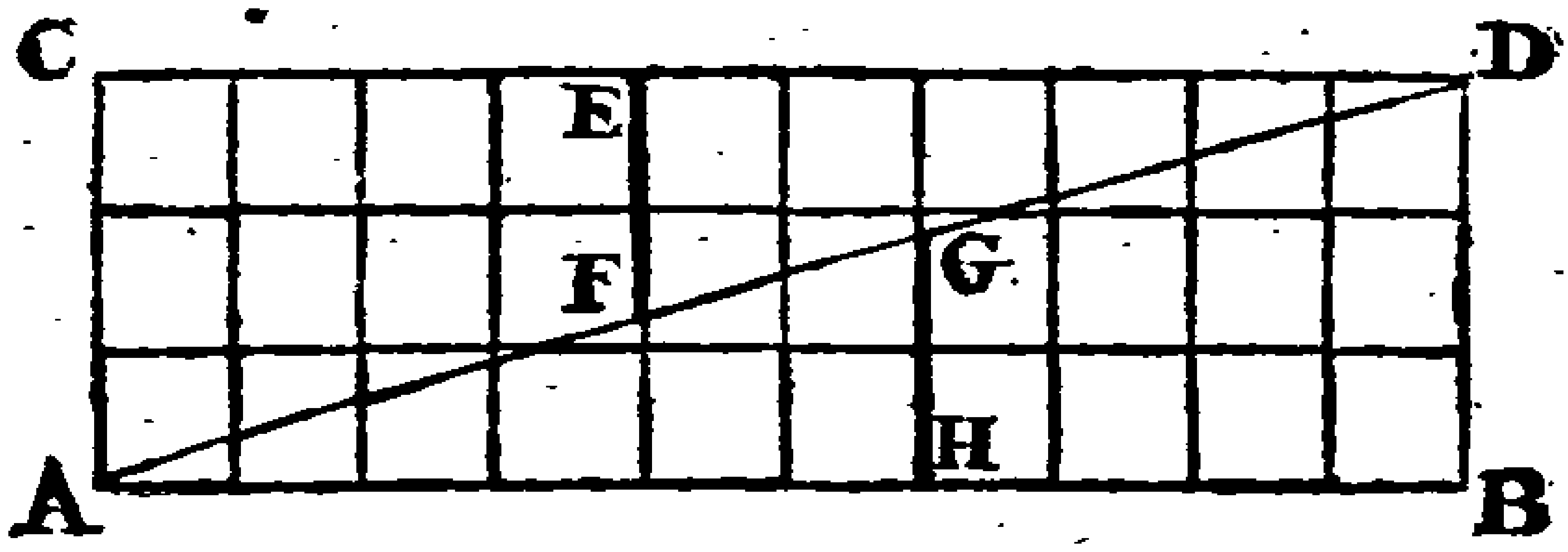


Operation. Firſt, divide the Breadth A C into 3 equal Parts, and the Length into 4 equal Parts, and draw Lines acroſs as in the Figure. Then, if the Board be cut through the Lines *a*, *b*, *c*, *d*, &c. and the Point B brought under the Place marked 12, the other Parts of it will fall into ſuch Places as ſhall form one complete Square of 12 Inches exactly.

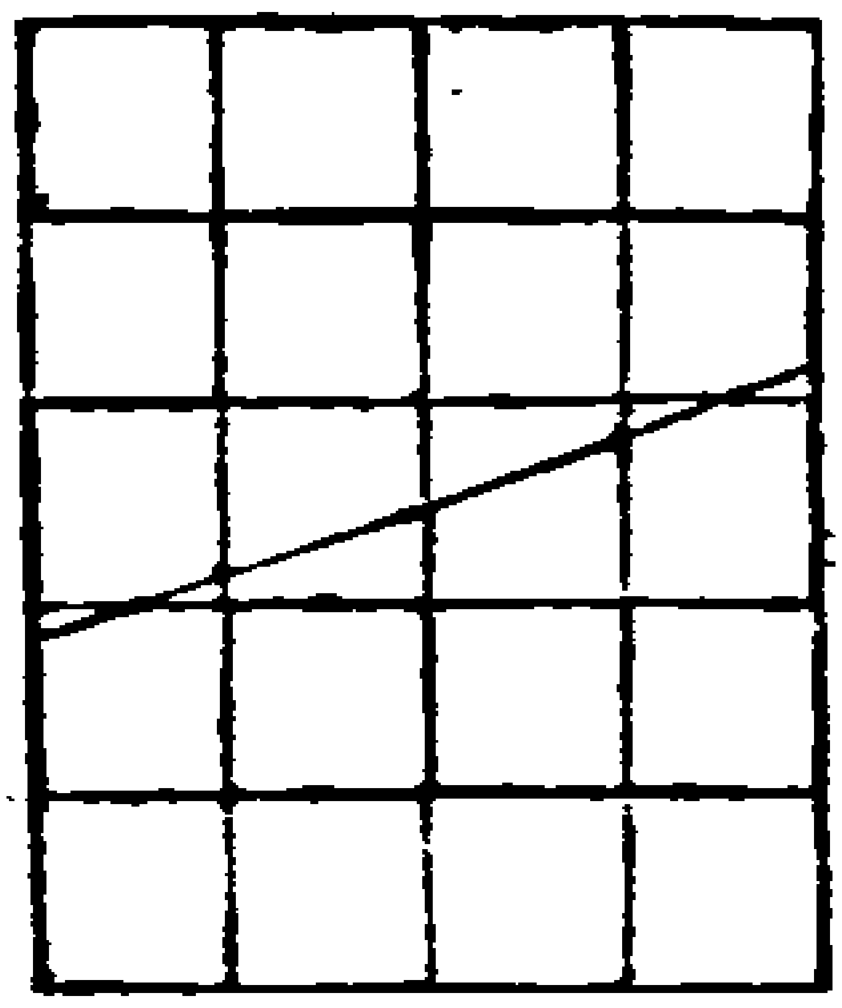
Problem 6.

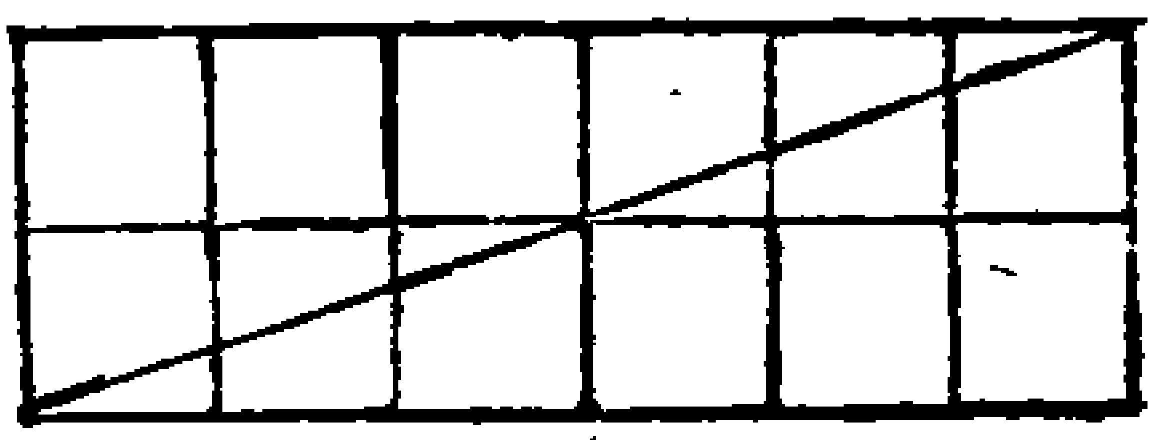
To increase the Surface of a *Geometrical Parallelogram.*

Let A B C D be the *Parallelogram* to be increased; and suppose its Breadth A C is 3 Inches, and its Length A B 10 Inches.



Operation. First, divide the shorter Side A C into 3 equal Parts, and the longer Side A B into ten equal Parts, and draw the Lines across as is done in the Figure, so will the whole Surface be divided into 30 equal Parts, or 30 square Inches. Then from A to D draw the Diagonal Line A D, which will cut the Figure into two equal Triangles. Again, cut those Triangles into 2 Parts, through the Lines E F and G H, *i. e.* through the 4th Line from the bigger End, by which Means there will be produced 2 *Triangles* and 2 *Trapeziums*, which joined together as in the following Figures, will make 32 Squares, instead of the 30 drawn in the first Figure, so that here is apparently an *Increase of 2 Square Inches* more.





Note. If you draw in each Square of the first Figure the reverse Side of a *Guinea* or *Shilling*, there will be only 30 Pieces, but immediately joining them, as in the 2d and 3d Figures, you will have 32; that is, 2 Pieces more than before.—*Good Gain.*

Pro-

Problem 7.

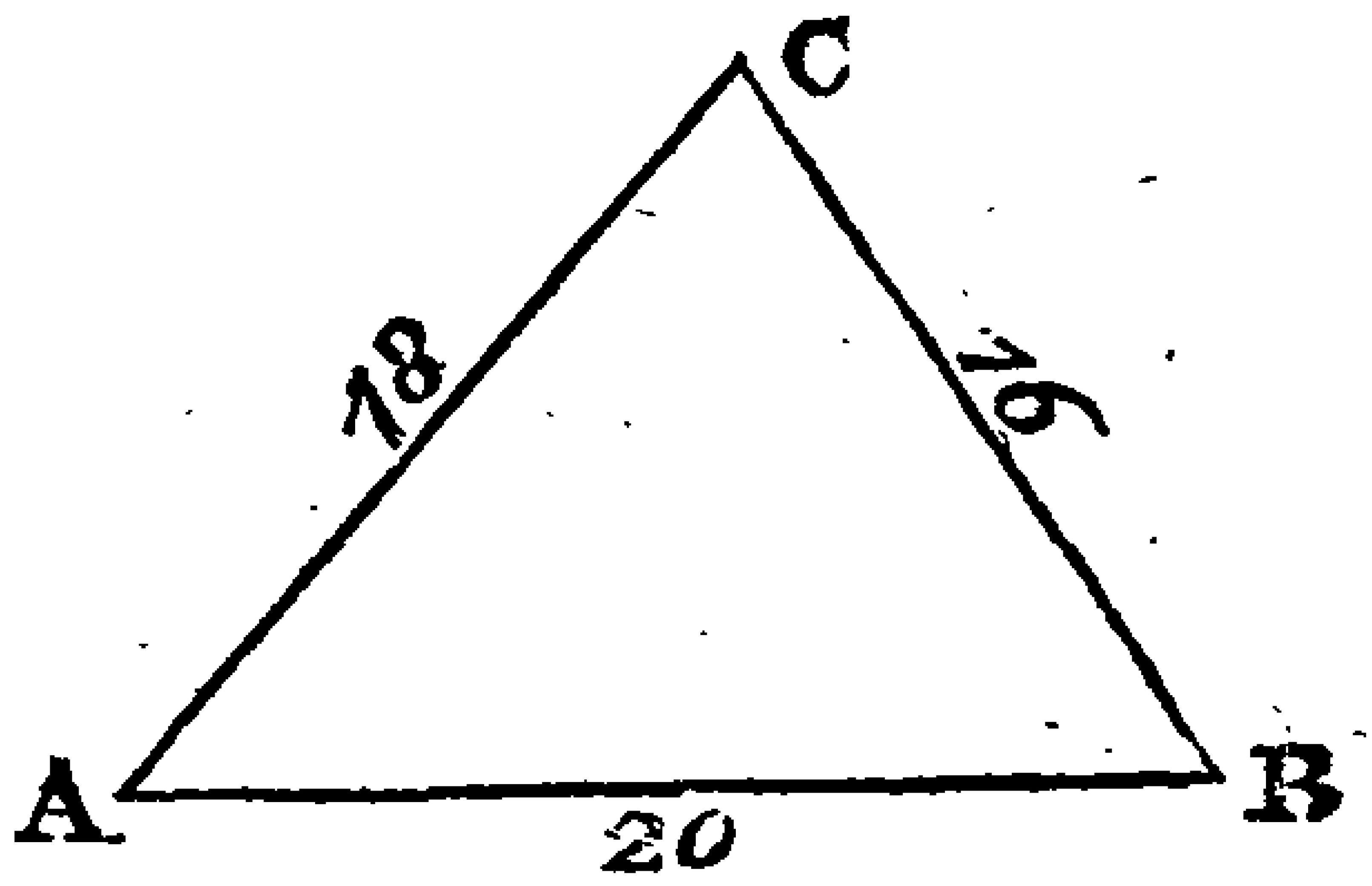
To find the Area of an *oblique plain Triangle* without falling a Perpendicular.

Rule.

From Half the Sum of the three Sides, ſubtract each particular Side; then multiply the half Sum and the three Differences continually; the Square Root of the laſt Product will be the Area of the Triangle.

Example.

Suppoſe A B C be a Triangle, whoſe three Sides are as follow, *viz.* A B 20 Inches, A C 18 Inches, and B C 16 Inches, what is its Area?



Operation. The Sum of the three Sides is 54; the Half is 27. Then $27 - 20 = 7$; $27 - 18 = 9$; $27 - 16 = 11$; and $27 \times 7 = 189 \times 9 = 1701 \times 11 = 18711$, whoſe Square Root is $= 136.78$ Inches, the Area required.

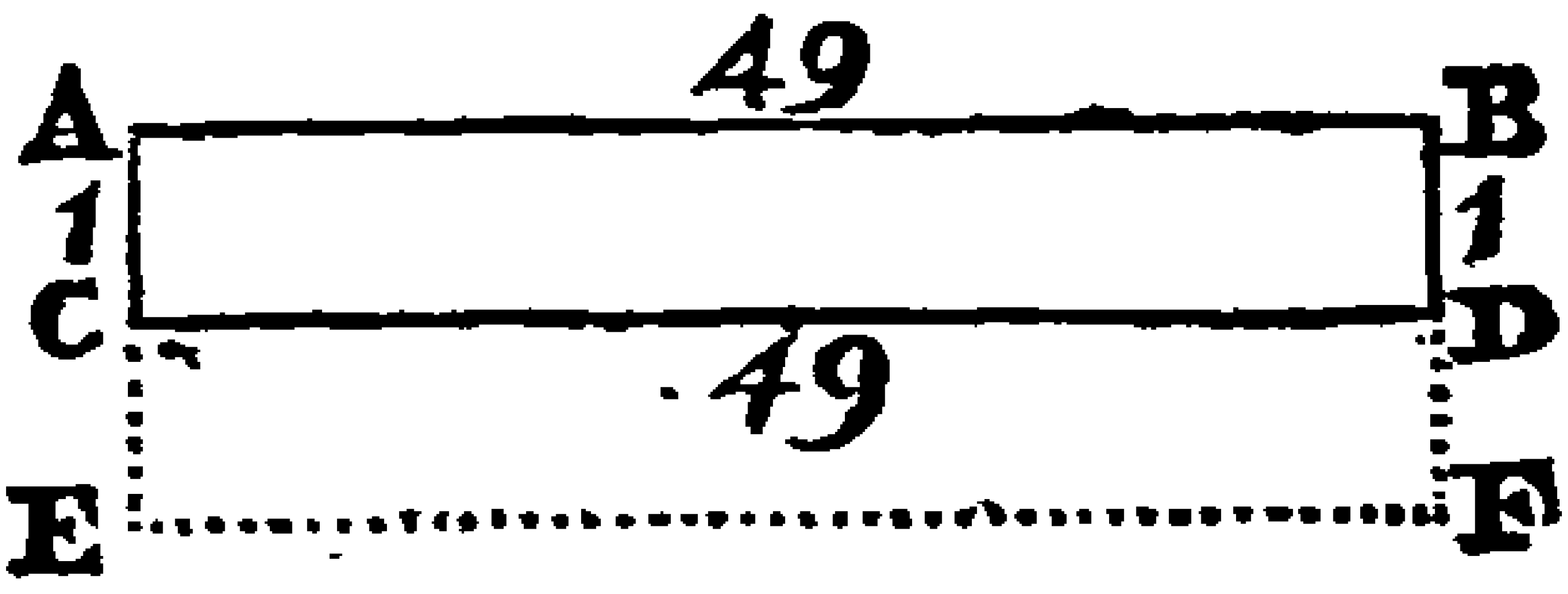
By this Problem the Content of a Piece of Ground may be found without the Surveyor's going into it.

Pro-

Problem 8.

If 100 Hurdles will fold 100 Sheep, how many will 102 Hurdles fold? *

Suppose the Parallelogram A B C D to represent the Situation of the Hurdles when they fold 100 Sheep, where the Sides A B and C D are 49 Hurdles each, and the Ends A C and B D only 1 Hurdle each.



Then it is very evident, that if to each of the Ends A C and B D, there be added 1 Hurdle more, the Parallelogramic Space will be just as large again. Whence it follows, that if the former Space A B C D will fold 100 Sheep, the additional Parallelogram C D E F will fold 100 more, consequently the Whole, with the Addition of two Hurdles, will fold 200 Sheep.

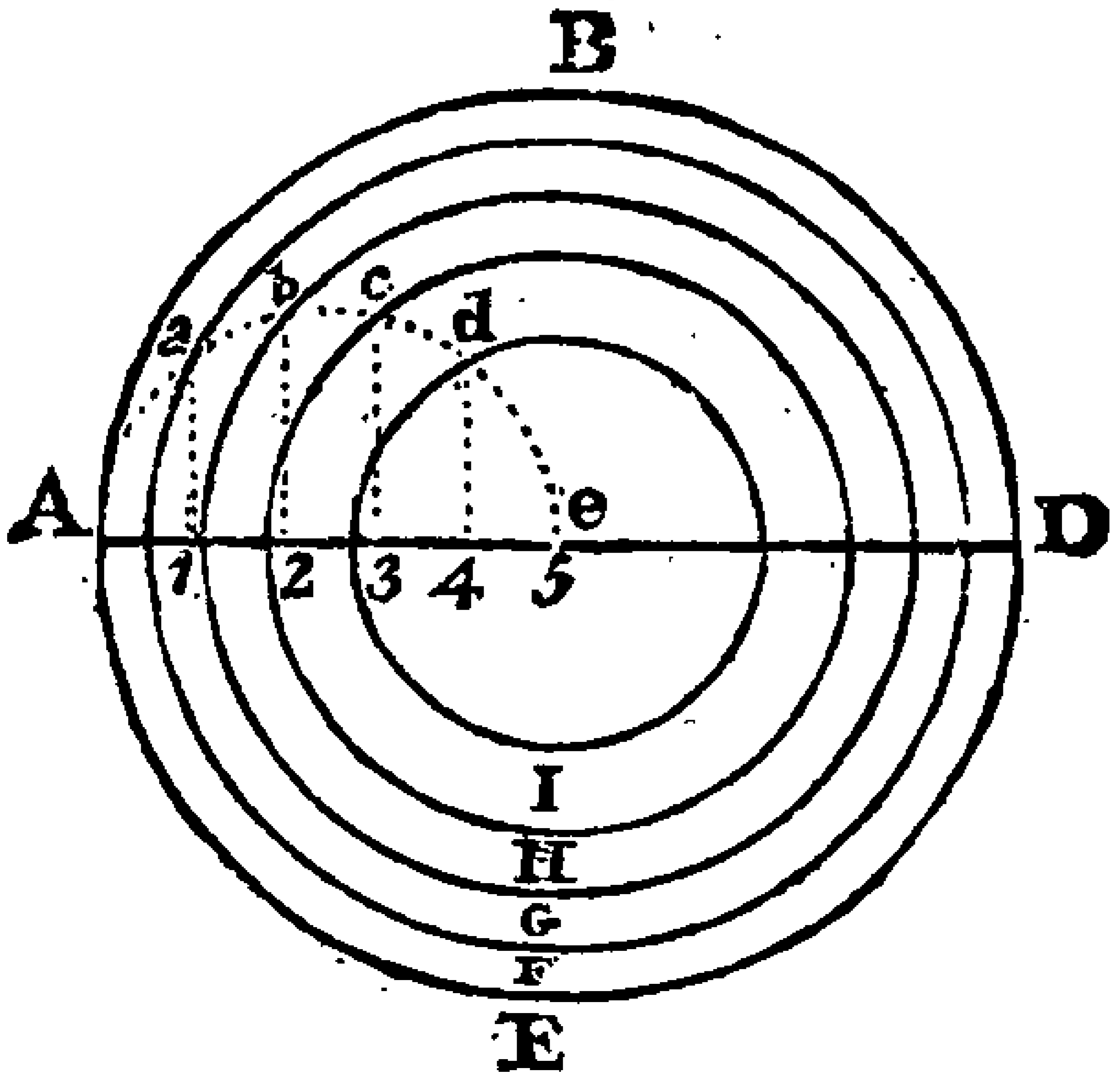
* This is generally called the *Shepherd*'s Problem.

Pro-

Problem 9.

To divide the Area of a Circle into any Number of equal Parts by concentric Circles.

Suppoſe A B D E be a Circle whoſe Area is required to be divided into 5 equal Parts by the concentric Circles F G H I.



Operation. Firſt, divide the Semi-diameter A *e* into 5 equal Parts, as A 1, 2, 3, 4, 5, and on the middle Point at *e*, as a Center, with the Radius or Opening *e* A, deſcribe the Semi-circle A, *a*, *b*, *c*, *d*, *e*. Then, from the Points of equal Diviſion at 1, 2, 3, 4, &c. raiſe Perpendiculars till they meet the Semi-circle in the Points *a*, *b*, *c*, *d*, through which Points draw the concentric Circles E, F, G, H, I, and it is done.

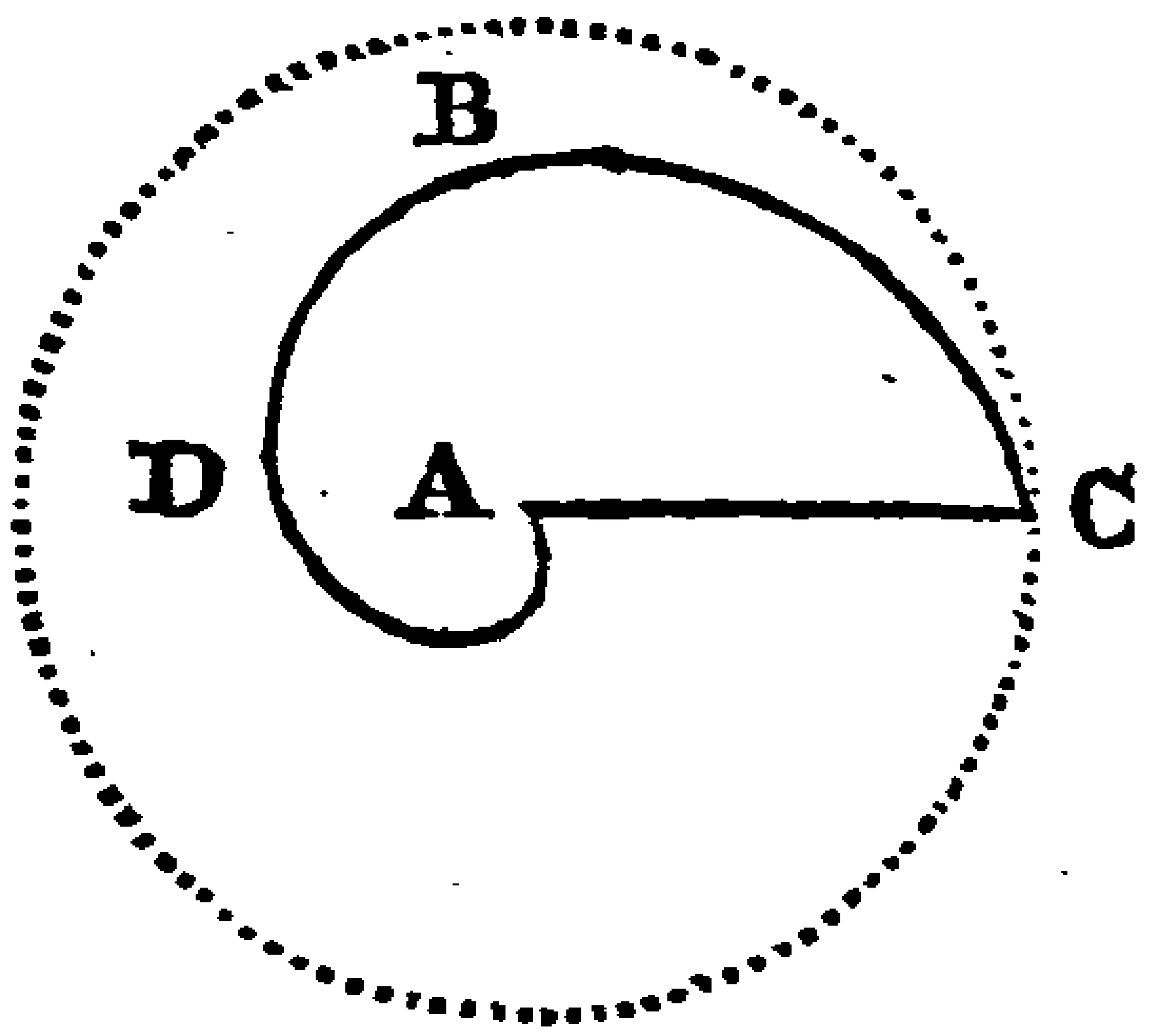
Note. Suppoſe 5 Smiths ſhould agree to purchaſe a *Grinding Stone* among them, each paying an equal Share of the Price, and that each Man ſhould have the Uſe of the Stone to wear off a fifth Part till it come to the laſt Man, who was to wear it out. The *firſt* Man ſhould wear the Stone from E to F; the *ſecond* from F to G; the *third* from G to H; the *fourth* from H to I; and the *fifth* from I to the Center or Axis at *e*.

Pro-

Problem 10.

To find the Area of any Space of *Archimedes'* Spiral.

Let the Space given be A D B C A, to find its Area.



Rule.

Make the Diſtance A C the Radius of a Circle circumſcribing the Spiral; then, find the Area of the whole Circle, and divide it by 3; the Quotient will give the Spiral Area A D B C A required. *

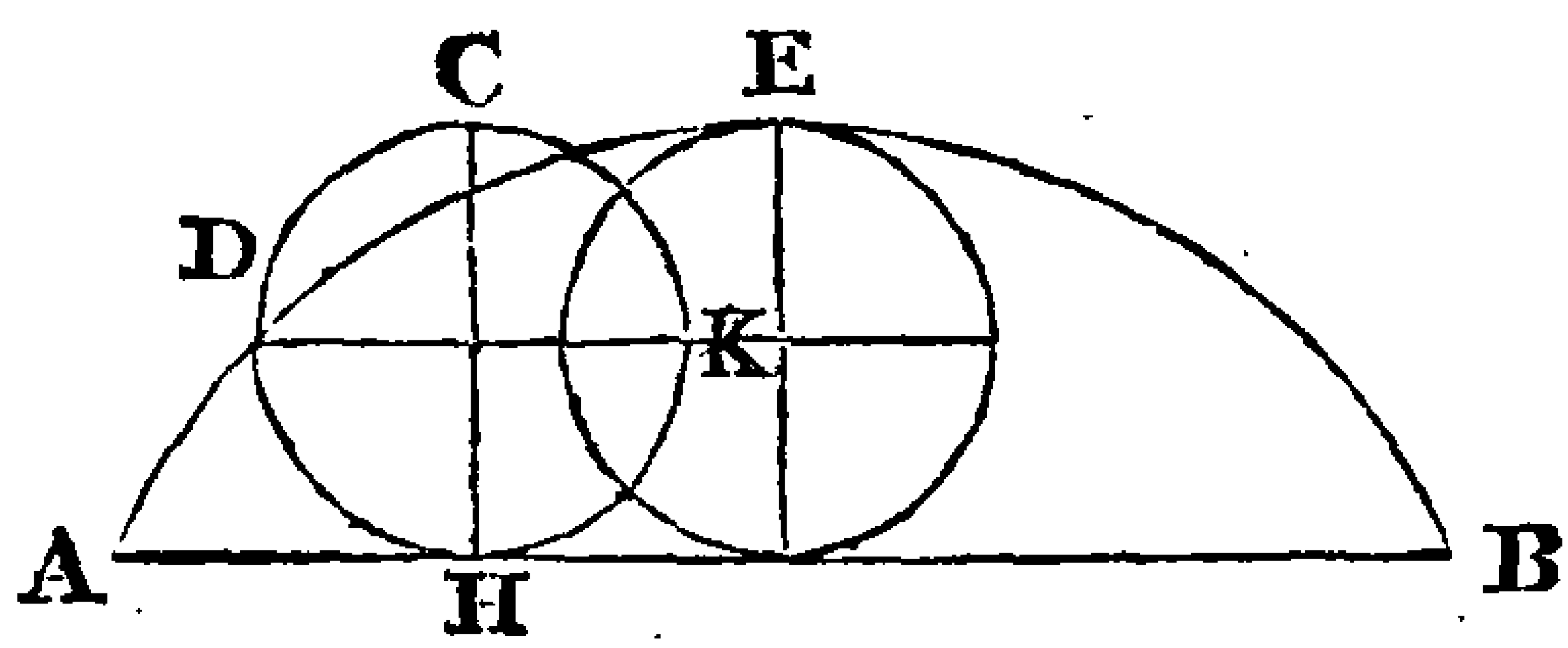
* The Area of the Spiral Space of *Archimedes* is always equal to $\frac{1}{3}$ (one-third) Part of the circumſcribing Circle.

Pro-

Problem 11.

To find the Area of a *Cycloid*.

Let the *Cycloidical Space* given be A B E, to find its Area.



If a Circle or a Wheel roll along upon the right Line A B, till it perform one Revolution, *i. e.* till it meaſure out a right Line equal in Length to its Circumference, then that Point in the Circle, as D, which firſt touched the right Line A B, will deſcribe, when it arrives at B, the Curve A E B, called a *Cycloid.*

Rule to find the Area.

Find the *Area* of the *generating Circle* H D C K, and multiply it by 3, the Product will give the *Cycloidical* Space required.

To find the *Length of the Arch* A E B.

Multiply the *Diameter* of the generating Circle H D C K by 4, and the Product will give the Length of the Arch required.

The Area of a *Cycloid* is equal to three Times the Area of the *generating Circle*; and the Length of the Arch is equal to 4 Times the *Diameter* of the ſame Circle.

Pro-

Problem 12.

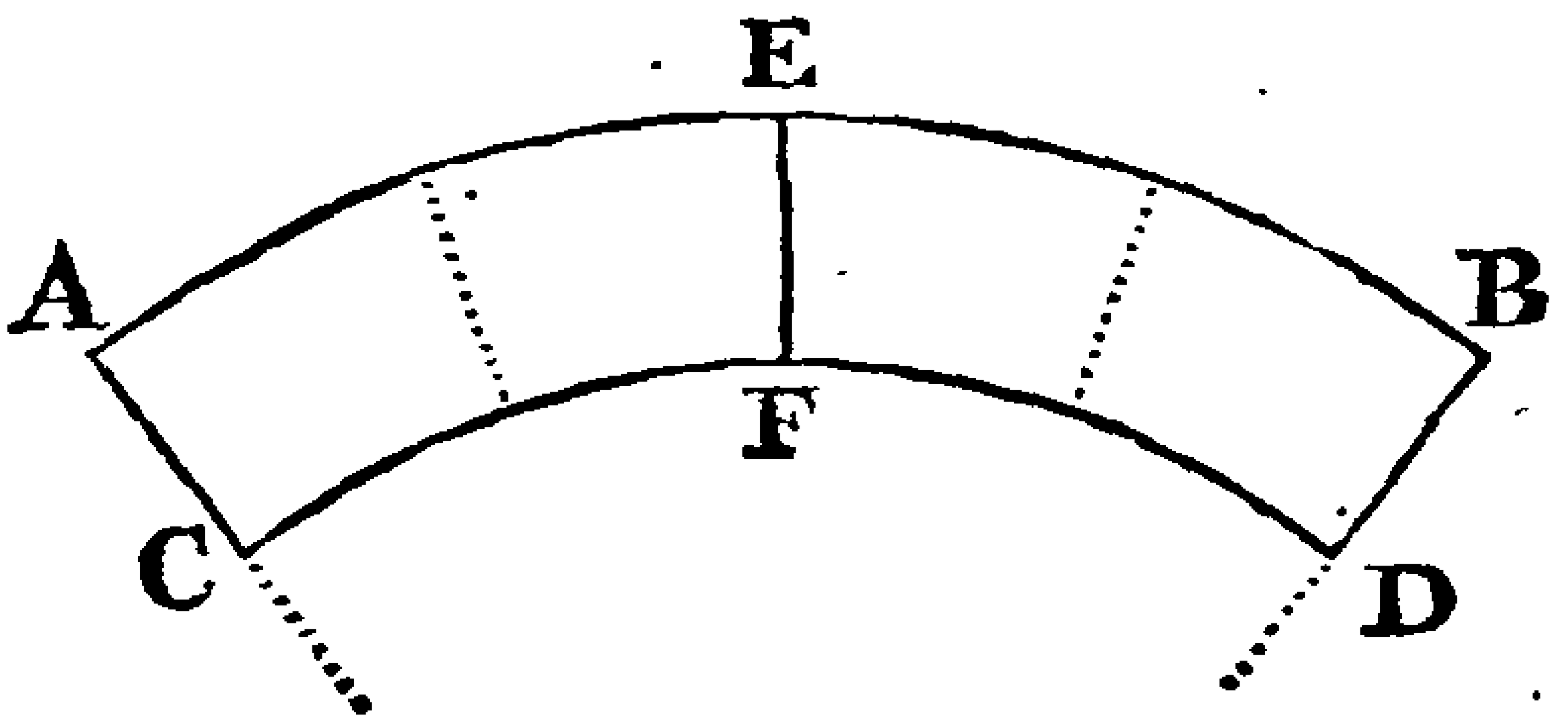
To find the Area of a *Segment*, or Part of a *Sector of a Circle.*

Rule.

Multiply half the Sum of the two Arches by the Distance between them, (or by one of the Ends) and the Product will give the Area.

Example.

Suppose the Length of the Arch A B be 84 Inches, the Arch C D 72.5 Inches, and the Distance between them E F 3.5 Inches; what is the Area of the Segmental Space A B D C?



Operation.

The Sum of the two Arches A B and C D is 156.5; the half is = 78.25; which multiplied by E F or A C = 3.5, gives 273.875 Inches, the Area required.

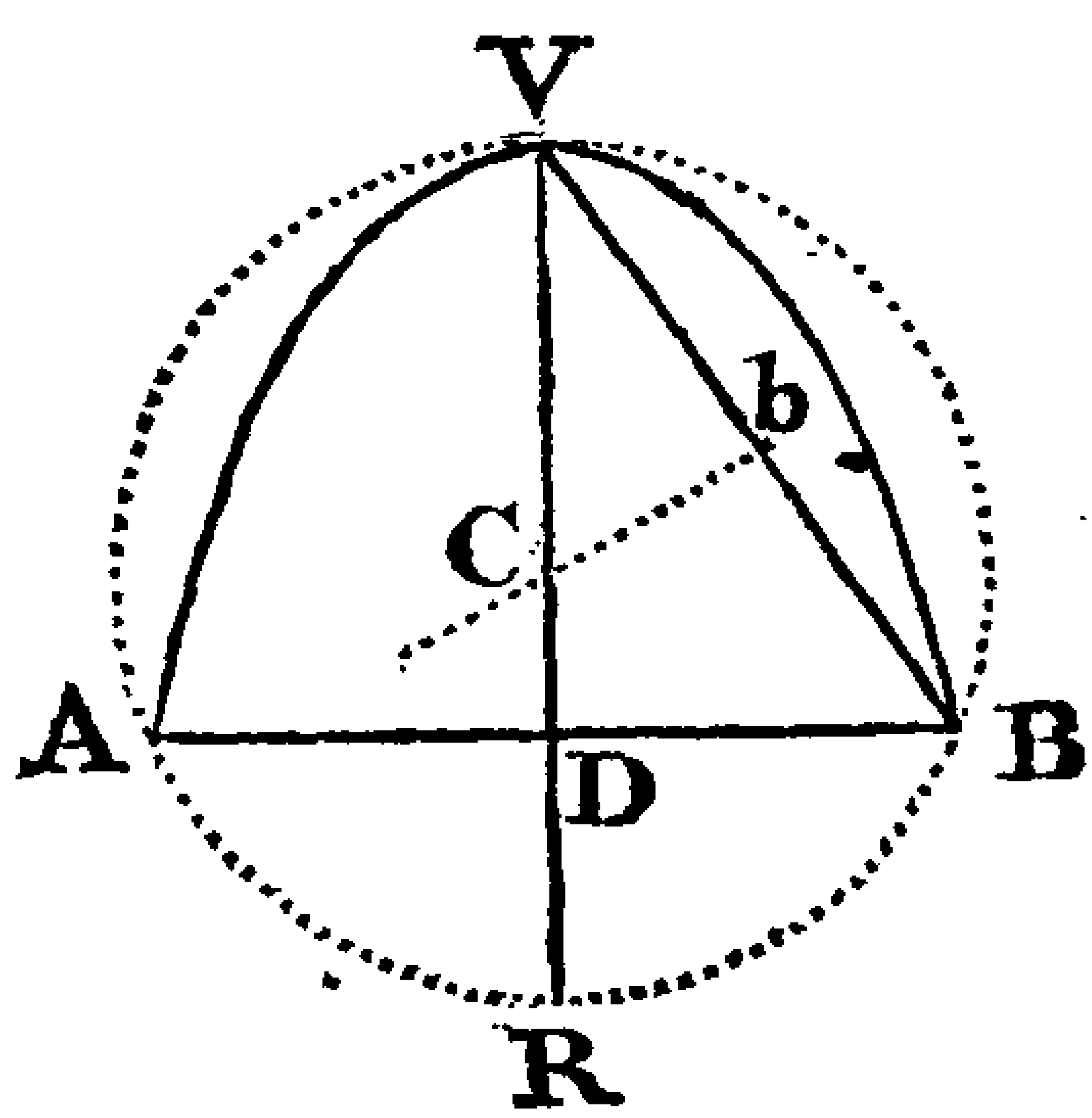
Pro-

Problem 13.

To deſcribe a *Parabola*, by having only the Baſe or greateſt Ordinate A B, and the Height or Axis V D given.

Conſtruction.

First find the *Latus Rectum* thus:

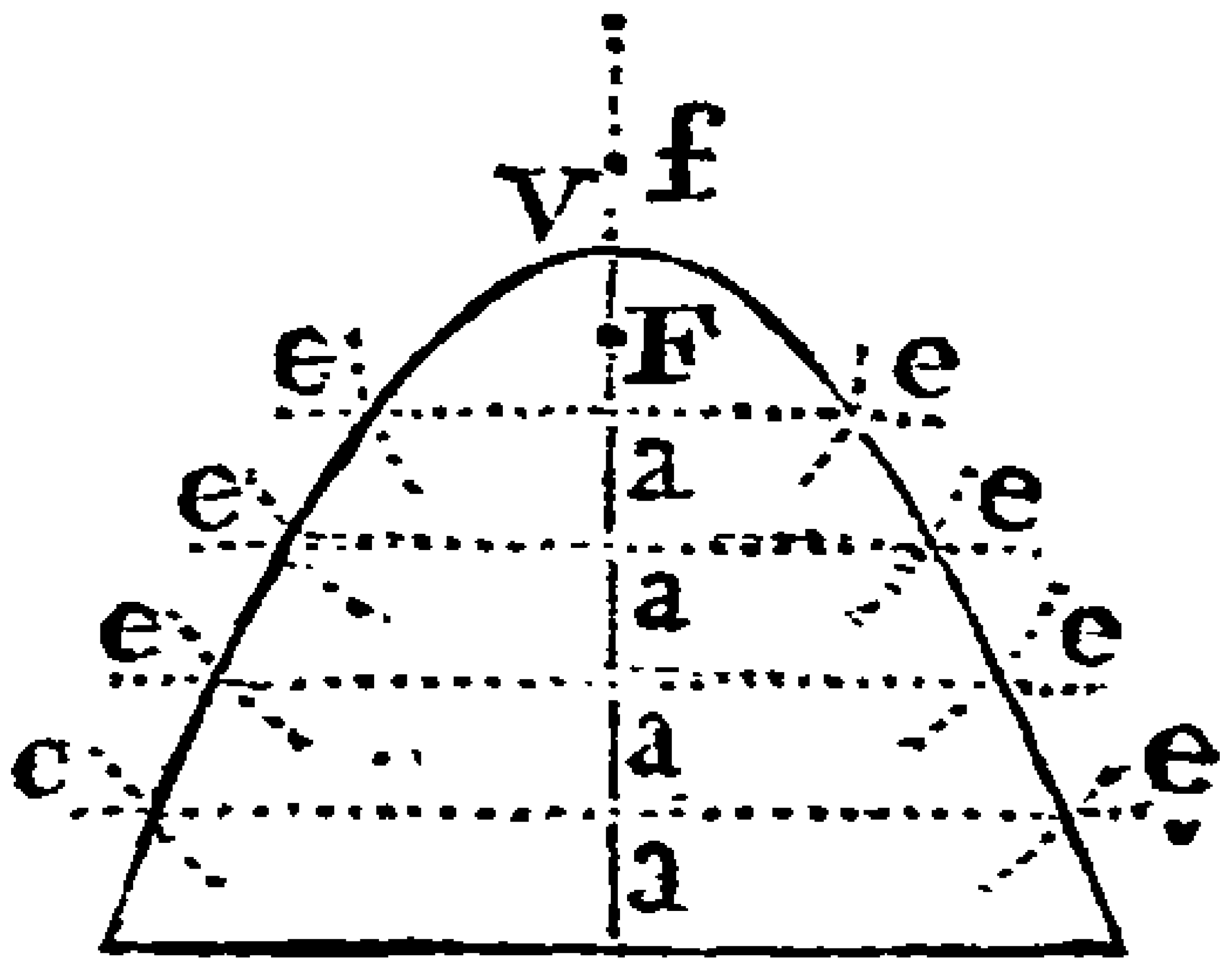


Draw a Line from the Vertex V to the End of the Ordinate or Baſe at B. Divide that Line into two equal Parts, as at *b*; upon *b* erect a Perpendicular, and where it cuts the Axis V D, as at C, ſet one Foot of the Compaſſes, and with the other opened to the Vertex V, deſcribe the Circle as in the Figure. So will the Diſtance between the Baſe or Ordinate A B, and that Point where the Axis continued cuts the Circle, as D R, be the *Latus Rectum* ſought. One Quarter of which is always the true Diſtance of the *Focus* from the Vertex or Top of the *Parabola*. This being obtained, we may proceed to delineate the *Parabola* as follows.

To

To delineate the *Parabola*.

Take $\frac{1}{4}$ of the *Latus Rectum* DR in the foregoing Scheme, and ſet it from the Vertex V in the next Figure both Ways, upwards and downwards, to *f* and F, (equal to the Diſtance of the *Focus* from V.)



Next, let a Number of Points, as *a*, *a*, *a*, &c. be taken in the Axis, and through each draw perpendicular Lines, as *e e*, *e e*, *e e*, &c. and all parallel to one another. Then with a Pair of Compaſſes take the Diſtance *a f*, and with one Foot in the *Focus* F ſtrike Daſhes acroſs each Parallel reſpectively, in *e* and *e*, &c. then with an even Hand draw a Curve through thoſe Points, and it will form the *Parabola* required.

Pro-

Problem 14.

To find the Length of the *Transverse* and *Conjugate Axis* of an *Hyperbola*, having the Base and Height given, with the Height also above the Place where the *Hyperbola* measures across exactly equal half the Breadth of that Base.

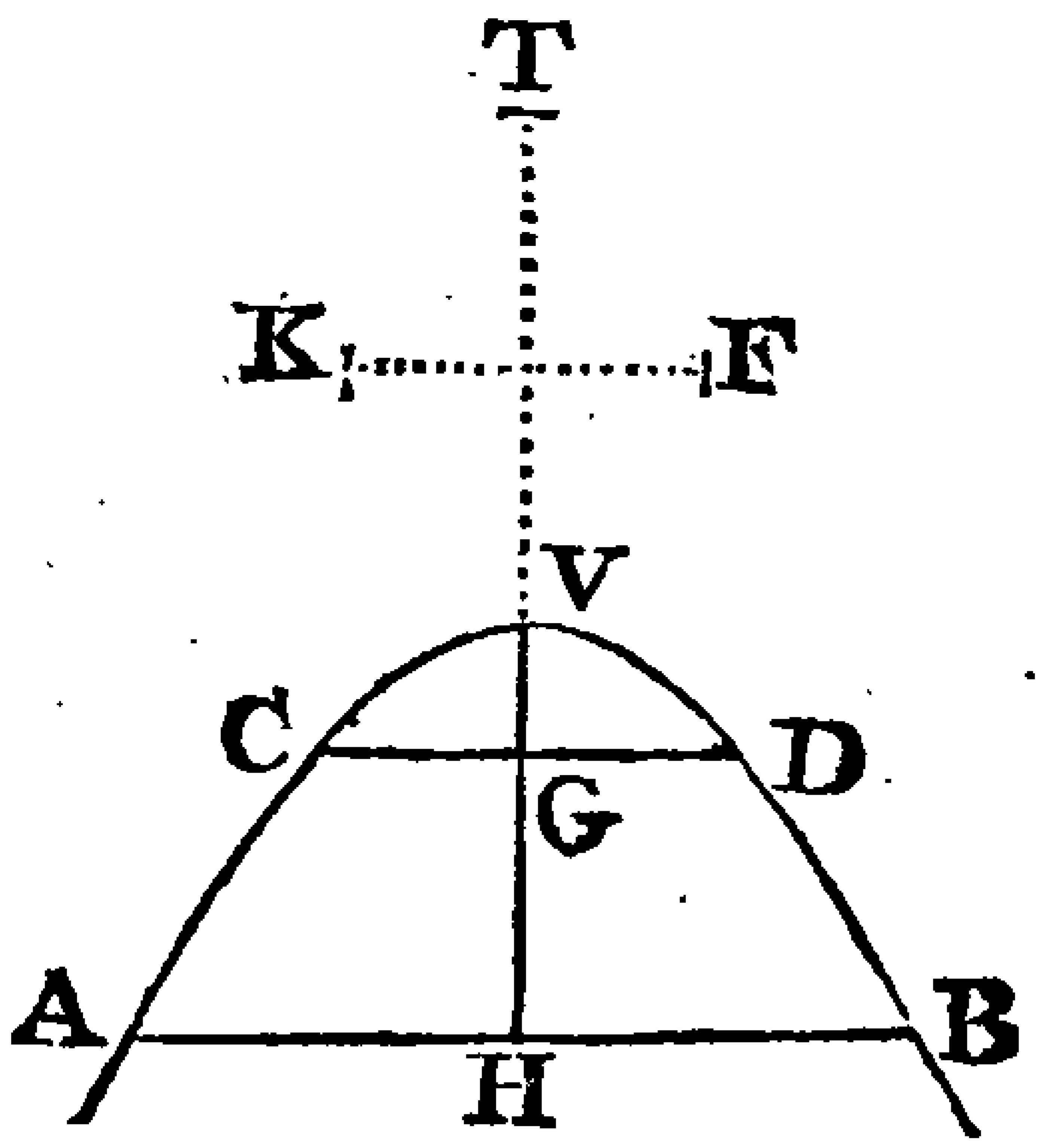
(1st.) To find the *Transverse Axis*.

Rule.

From the Square of the whole Height, take 4 Times the Square of the lesser Height; divide the Remainder by 4 Times the lesser Height, lessened by the whole Height, and the Quotient will give the Length of the *Transverse Axis* required.

Example.

Suppose an *Hyperbola*, whose Base A B is 96 Inches, and Height V H 40; and suppose the Height V G, above the Breadth of 48 Inches, equal half the Base, to be 12.111 Inches; what is the Length of the *Transverse Axis* T V, above the Vertex V of the Hyperbola?



Operation.

The Square of the whole Height 1600, lessened by 4 Times the Square of the lesser Height 586.7052, &c. leaves for a Remainder 1013.2147, &c. which divided by 4 Times the lesser Height 48.444, lessened by the whole Height 40, which is = 8.444, gives in the Quotient 120, the Length of the *Transverse Axis* sought.

(2dly.) To find the *Conjugate Axis*.

Rule.

Multiply the *Height* of the *Hyperbola* by the *Sum of the Tranſverſe and Height*: Make the *Square Root* of that Product a *Diviſor* to the Product of the Tranſverſe and half the Breadth of the Baſe, ſo will the Quotient ariſing be the *Conjugate Axis* required.

Example.

Suppoſe, as in the laſt Example, the Tranſverſe Axis T V to be 120 Inches; the Height of the Hyperbola V H 40 Inches; and the Baſe A B 96, conſequently its half, 48 Inches; what is the Length of the *Conjugate Axis* K F?

Operation.

The Height 40, multiplied by the Sum of the Tranſverſe and Height, is = 1600, the Square Root of which is = 80: By which Root divide the Product of the Tranſverſe and half the Baſe multiplied together, = 5760, and the Quotient = 72 gives the Length of the Conjugate Axis ſought.

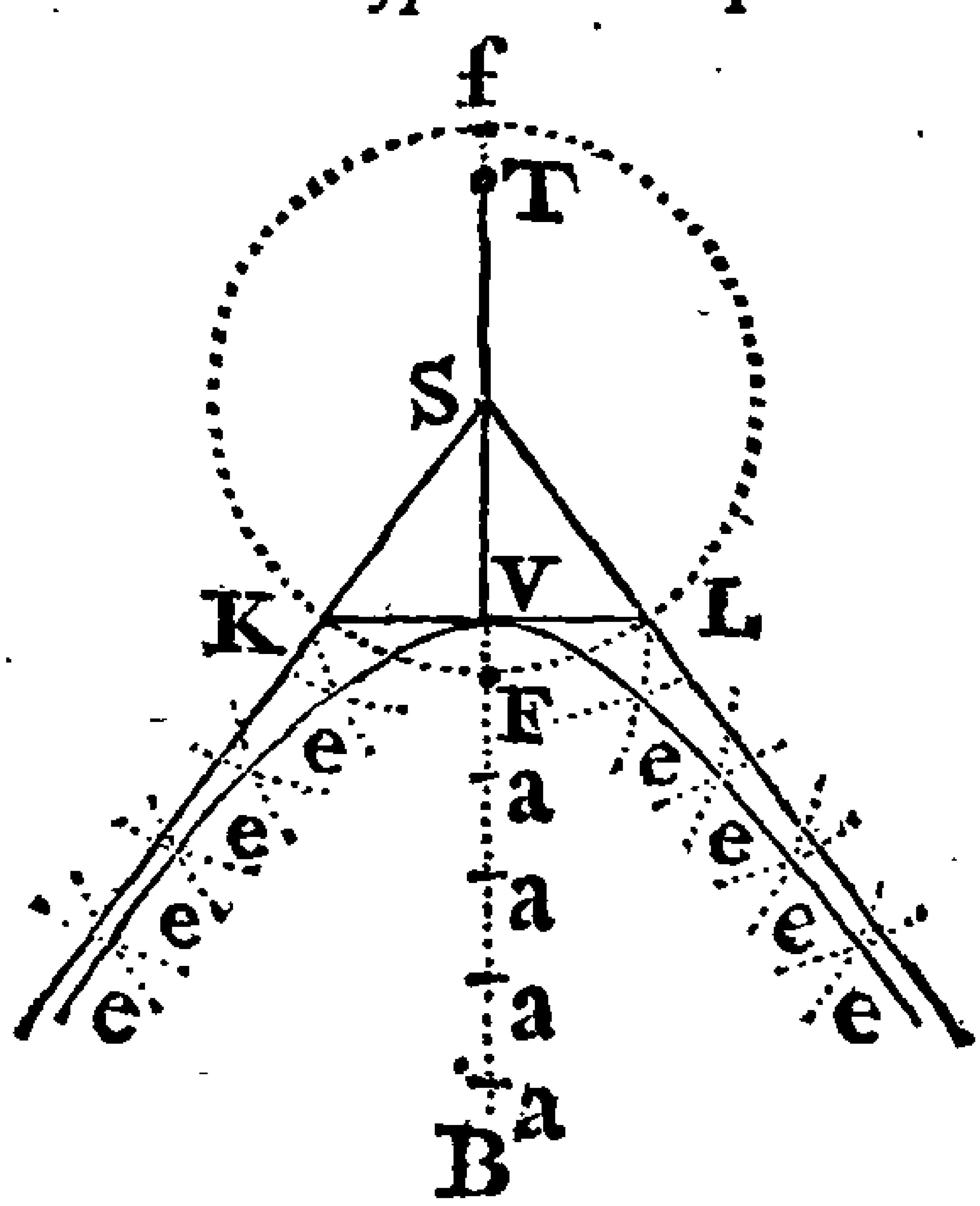
Pro-

Problem 15.

To delineate or deſcribe an *Hyperbola* by having the *Tranſverſe Diameter* T V, and the Conjugate Diameter K L given.

Conſtruction.

Draw the Line T V for the *Tranſverſe Diameter* or Axis, and continue it upwards and downwards at Pleaſure. Take the Conjugate Diameter or Axis K L, and place the Middle of it on the End of the Tranſverſe at V, ſo that thoſe Lines may ſtand at right Angles to each other. Divide T V into 2 equal Parts at S; and from S, with the Compaſſes opened to K or L, deſcribe the Circle K *f* L F, cutting T V continued in F and *f*, which are the Focii of the Hyperbola. In T V continued downwards take any Number of Points, as *a*, *a*, *a*, &c. and from F and *f*, as Centers, with the Diſtances T *a* and V *a* in the Compaſſes, deſcribe Arches cutting each other in *e*, *e*, *e*, &c. Then, through the ſeveral Points *e*, *e*, *e*, draw the Curve *e* V *e*, and it will be the *Hyperbola* required.



Note. If two right Lines be drawn from the Point S, by the Ends K and L of the Conjugate Diameter, they will be the *Aſymtotes* of the *Hyperbola*, whoſe Property it is to approach continually nearer the Curve, yet never to meet it.

Problem 16.

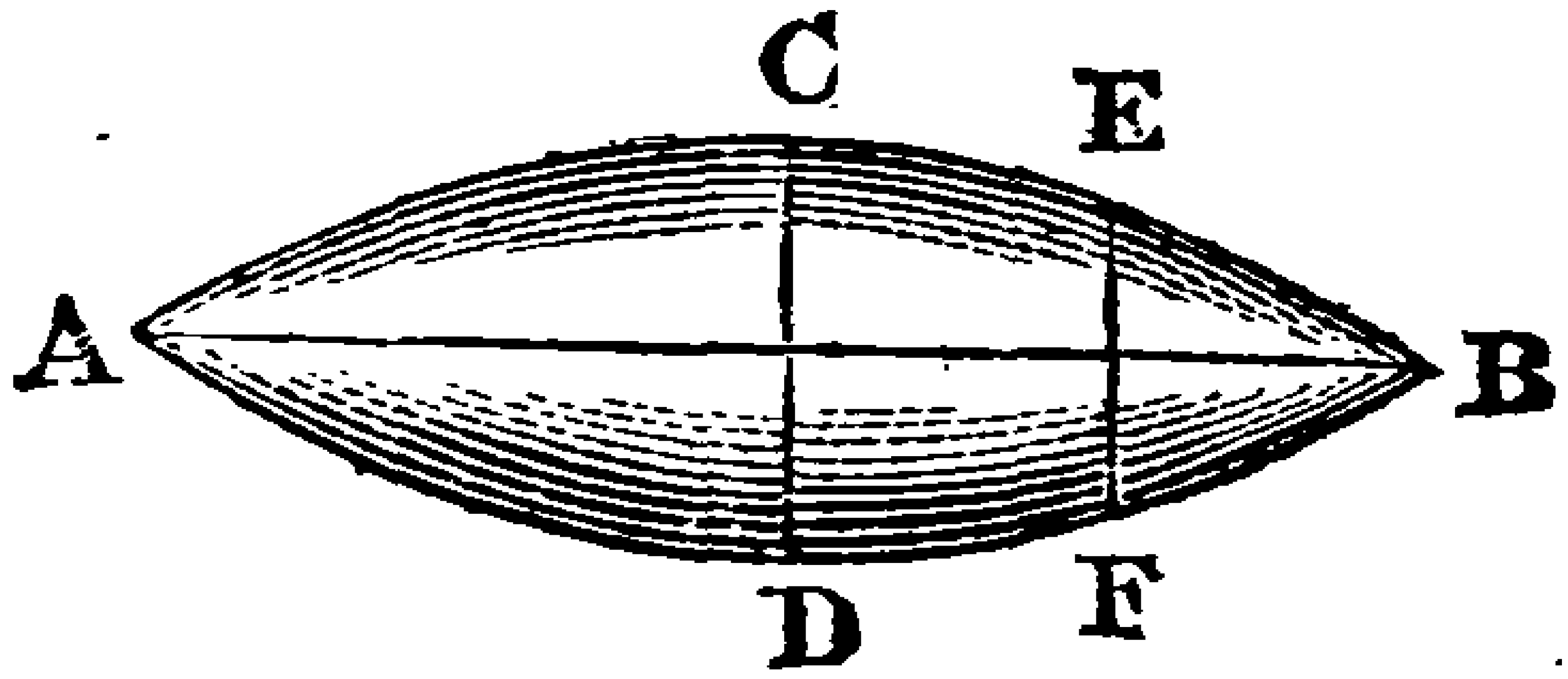
To find the Solidity of a *Circular*, *Elliptical*, *Parabolical*, or *Hyperbolical Spindle*.

General Rule.

To the *Square* of the Diameter in the Middle of the Spindle, add the *Square* of *double* the Diameter at $\frac{1}{4}$, or one fourth of the Length, (*i. e.* exactly between the Middle and one of its Ends) multiply the Sum by the Length, and the Product again by .1309, and it will give the Solidity very nearly.

Example.

What is the Solidity of a Spindle (of any of the above Forms) whose Length A B is 20 Inches, the greatest Diameter C D 6, and the Diameter E F at $\frac{1}{4}$ of the Length 4.74 Inches?



Operation.

The Diameter C D $\square$ = 36, added to double the Diameter E F $\square$ = 89.8704, makes 125.8704; which × by 20, the Length A B, is = 2517.408; this × again by .1309, gives 329.5287072 Inches, for the Solid Content required.

Problem 17.

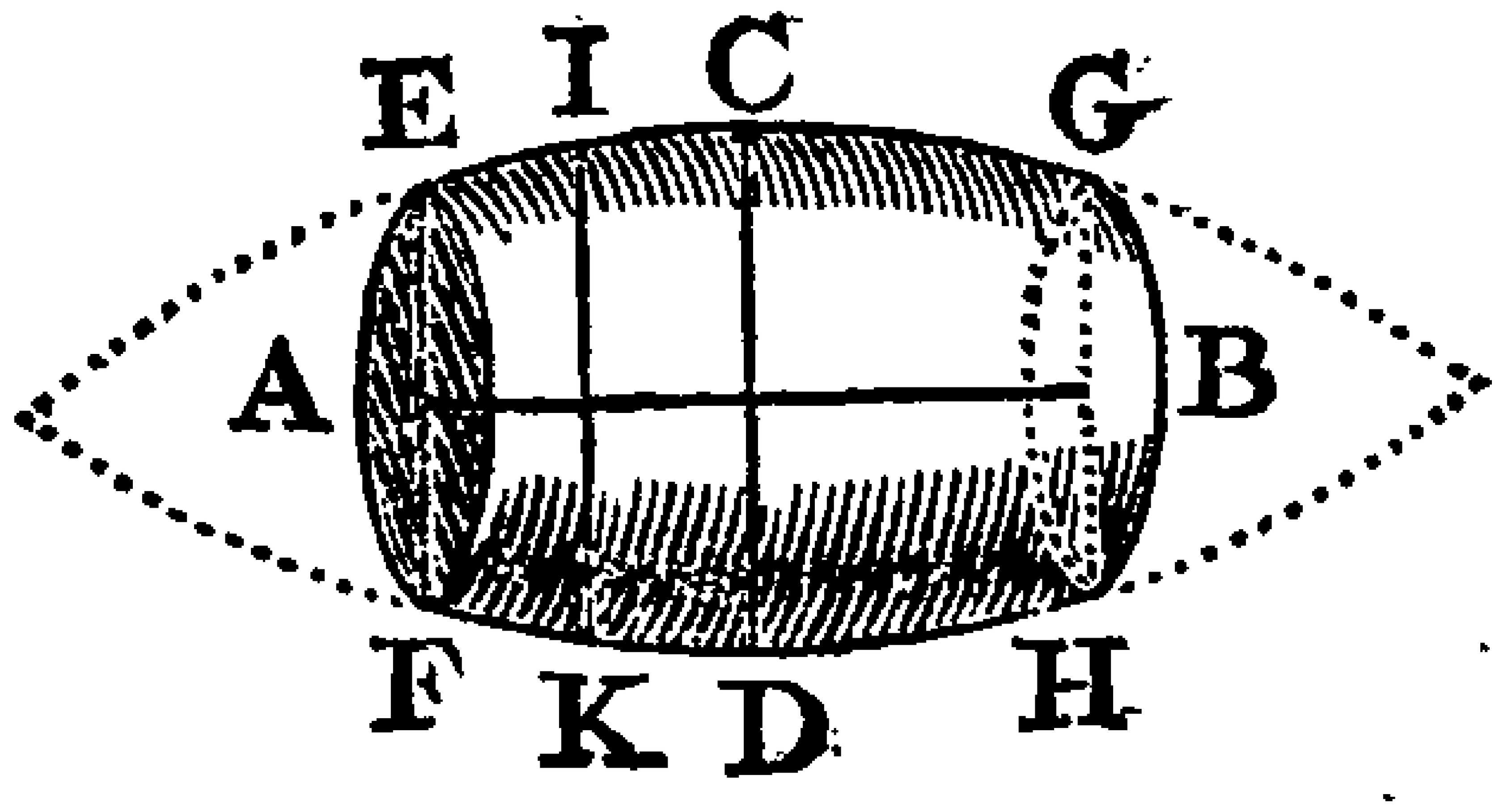
To find the Solidity of a *Frustum* or *Segment* of an *Elliptical*, *Parabolical*, or *Hyperbolical Spindle*.

General Rule.

Add together the *Squares* of the *greatest* and *least* Diameters, and the Square of *double* the Diameter in the Middle between the two; multiply the Sum by the Length, and the Product again by .1309 for the Solidity.

Example.

What is the Solidity of the Middle *Frustum* of any Kind of *Spindle*, whose Length A B is 20 Inches; the middle, or greater Diameter C D 16 Inches; the Diameter at each End, E F and G H, 12 Inches; and the Diameter at $\frac{1}{4}$ of the Length, *(i. e.)* between the Middle and the End, I K, 14.5 Inches?



Operation.

The □ of C D = 256 + □ E F = 144 + twice I K □ = 841 is = 1241. This × by 20, the Length, A B is = 24.820, which multiplied again by .1309, gives in the Product 3248.9380 Inches, for the Solid Content.

Problem 18.

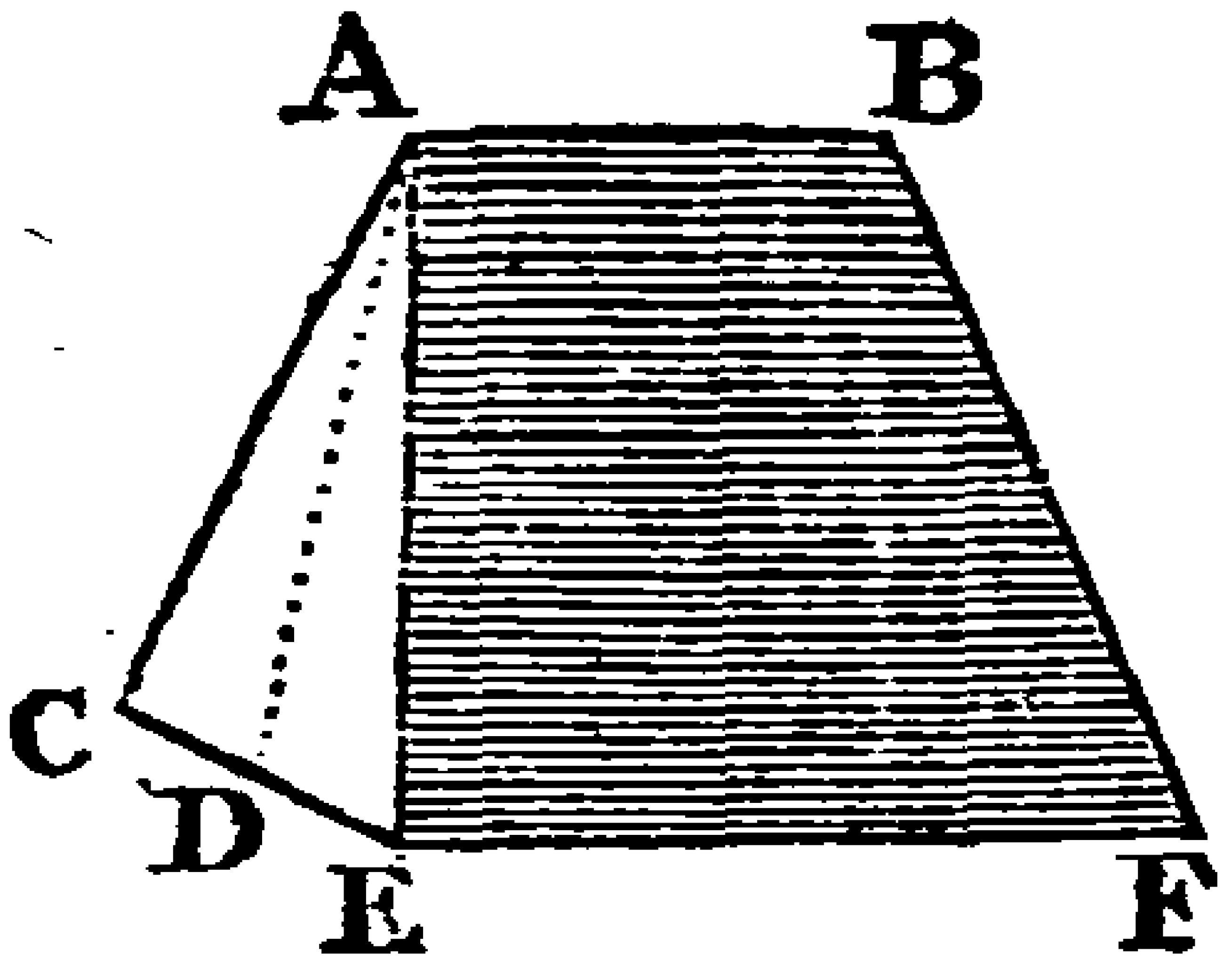
To find the Solidity of a *Wedge.*

Rule.

To *twice* the Length of the *Back* add *once* the Length of the Edge; multiply this Sum by the Product of the Height multiplied into the Breadth of the Wedge, and $\frac{1}{6}$ (one ſixth) of this laſt Product will be the Solidity.

Example.

Suppoſe a *Wedge* whoſe Height A D is 8 Inches; its Edge A B, 3 Inches; the Length of the Back E F, 4 Inches; and the Breadth of the Back C E, 2 Inches; what is its Solid Content?



Operation.

Twice the Length of the Back E F $=$ 8 $+$ *once* the Length of the Edge A B $=$ 3 is $=$ 11; this $\times$ by 16 (the Height A D $=$ 8 multiplied by the Breadth C E $=$ 2) gives 176; which $\div$ by 6 gives 29.333, &c. Inches, the Solid Content.

Note. When the *Back* and *Edge* are of the ſame Length, the Wedge is equal to *half a Priſm* of the ſame Baſe and Altitude; and its Content may be found by multiplying the *Area of the Back by half the Height* of the Wedge.

Pro-

Problem 19.

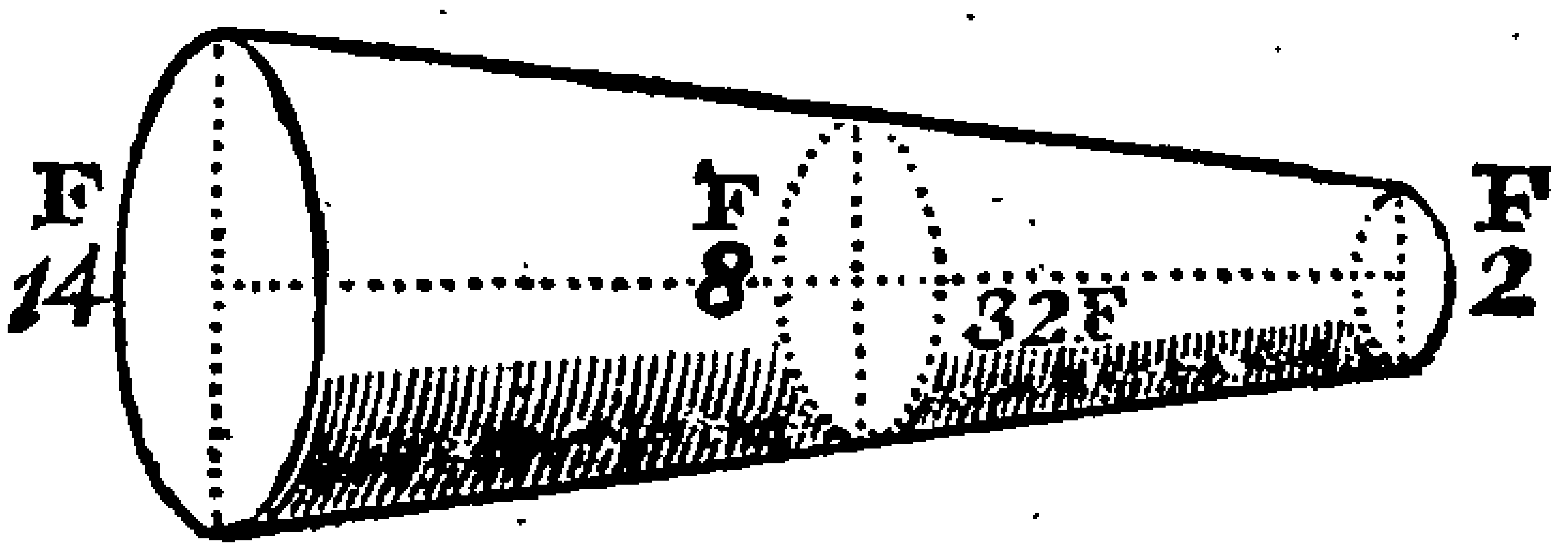
To cut a Tree through in ſuch Manner, that the two Parts meaſured ſeparately, ſhall produce a greater Solidity, than that of the whole Tree.

Rule.

Cut it through exactly in the Middle, or at half the Length, and the two Parts will meaſure (conſiderably) more than the Whole did before.

Example.

Suppoſe a Tree to girth 14 Feet at the greater End, 2 Feet at the leſs, and 8 Feet in the Middle, and that the Length is 32 Feet.



Operation.

By the common Method, the whole Tree meaſures only 128 Feet. When cut through the Middle, the greater Part meaſures 121 Feet, and the leſs Part 25 Feet, which together make 146 Feet, and exceeds the whole by 18 Feet.

Problem 20.

To cut a Tree ſo that the Part next the greater End may meaſure the moſt poſſible, and ſometimes conſiderably more than the whole Tree.

Rule.

Cut it through where the Girth is equal to $\frac{1}{3}$ (one third) of the greateſt Girth, and the greater End will then meaſure the greateſt poſſible.

To find where the Tree muſt be cut through from the leſs End, to make the Girth there equal to $\frac{1}{3}$ of the Girth of the greater End.

Rule.

From the Girth of the greater End, ſubtract 3 Times the Girth of the leſs End, and divide the Remainder by the Difference between the greater and leſs Girths; this Quotient multiply by $\frac{1}{3}$ of the Length of the Tree, and the Product will give the Length of the Piece to be cut off from the leſs End, to leave a Remainder greater than the whole Tree.

Operation.

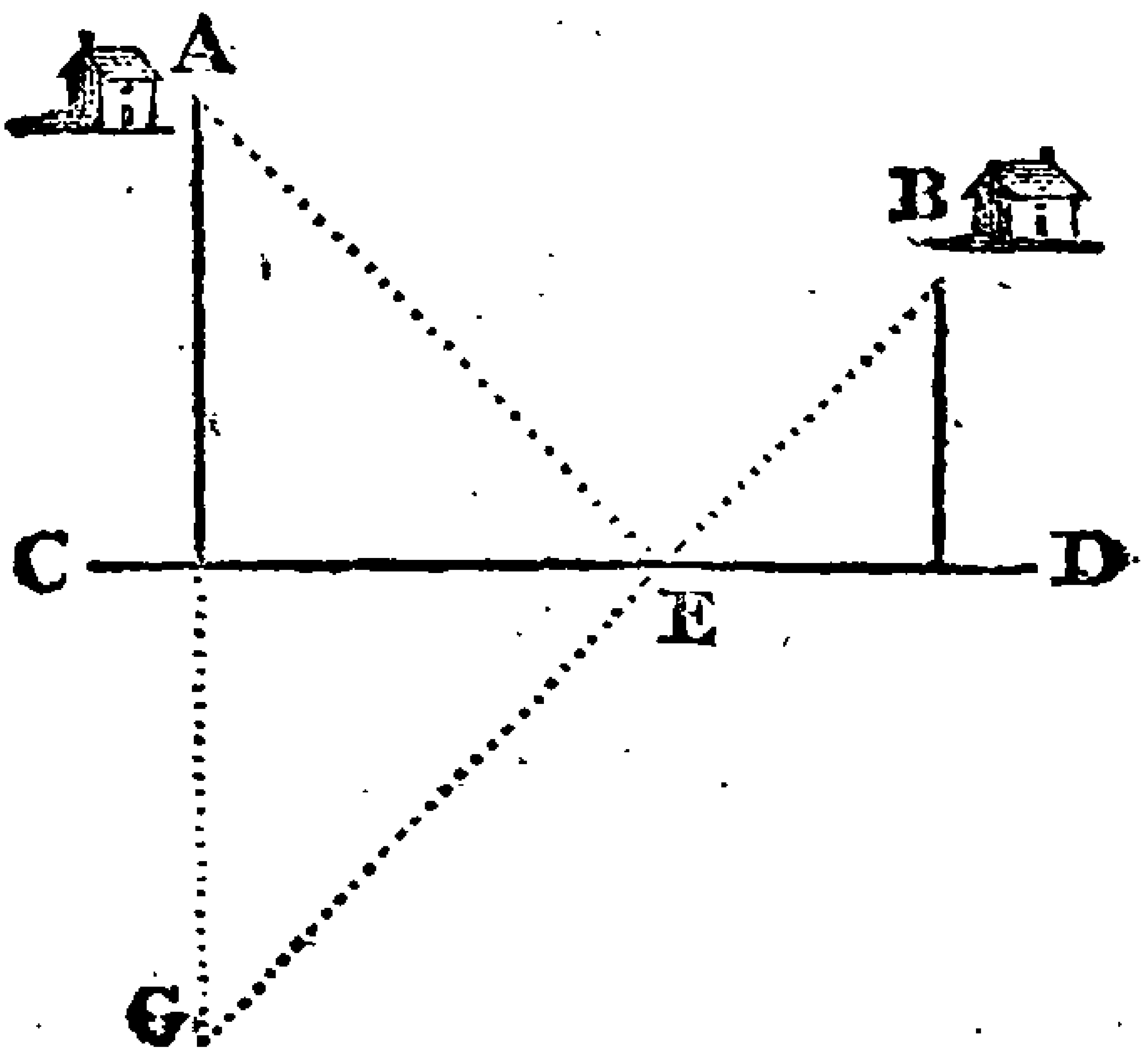
Taking here the ſame Example as in the laſt Problem, we ſhall have 7.1 Feet the Length to be cut off; 24.9 Feet, the Length of the remaining Part; and 4.666 Feet, the Girt at the Place cut off. Now the Content of the whole Tree as before is only 128 Feet; but the Content of the greater Part here is 135.5 Feet, which exceeds 128 by 7.5 Feet, and is the greateſt poſſible.

If the greateſt Girth doth not exceed the leſs 3 Times, the Tree cannot be cut as required by the Problem; for when the leaſt Girth is equal exactly to $\frac{1}{3}$ of the greater, the Tree then meaſures to the moſt poſſible.

Pro-

Problem 21.

A Perſon, for a conſiderable Wager, is to travel in a certain Time from the Town at A, to another at B, but he is obliged to call at a Place which is ſomewhere on the (Road or) Line C D: Now his Time being limited, he is deſirous to know (by Geometry) the Situation of the Place upon the Line C D, from whence the Diſtance to A and B ſhall be the leaſt poſſible?

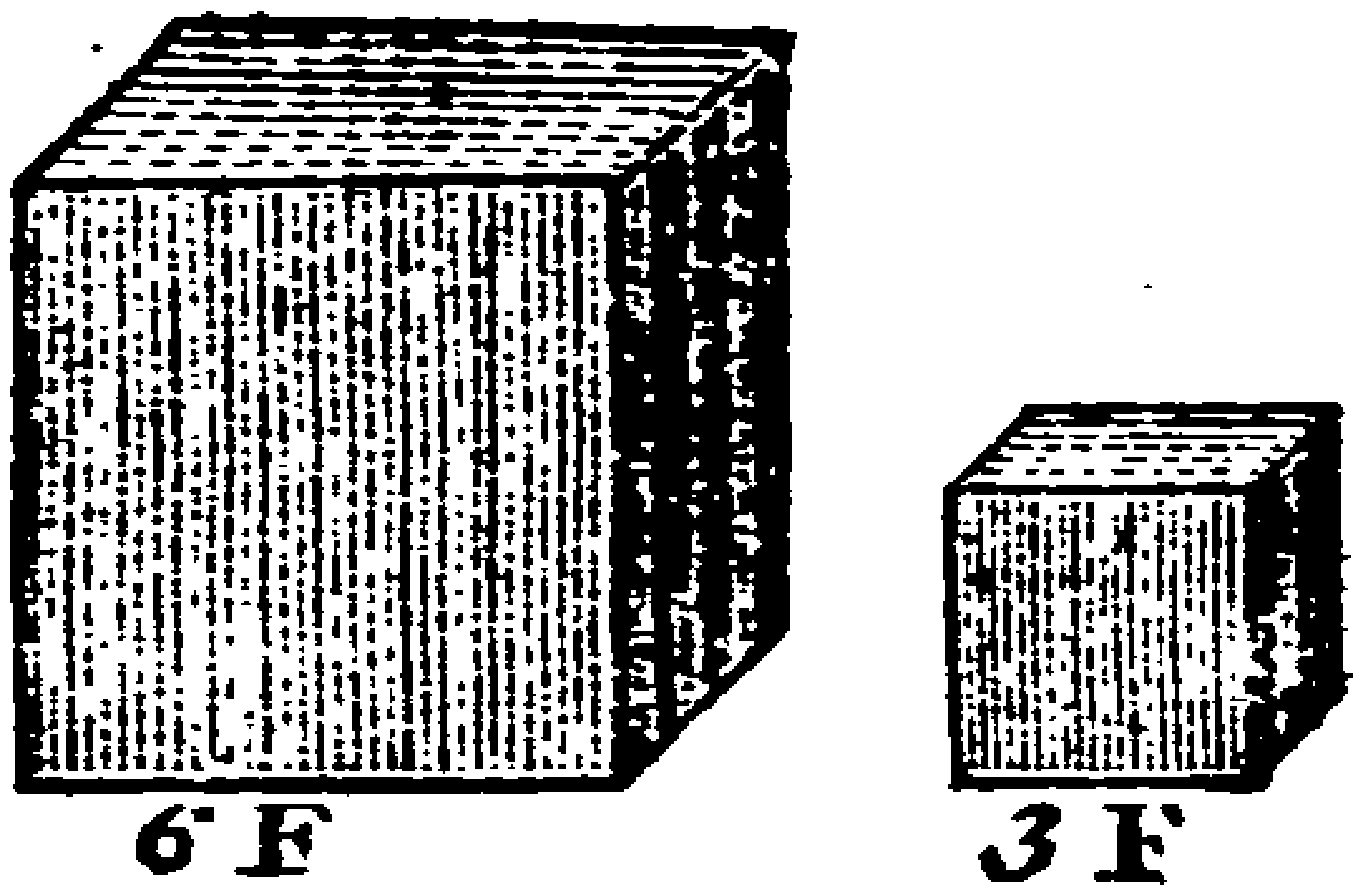


Operation.

Firſt, produce the Line A C downwards towards G, and make C G = C A. Next, draw the Line G B through the Line C D at E; draw alſo the Line from A to E; then will G E = A E; and the Angle C E A = C E G. But the Sum of G E and E B is the leaſt poſſible, when they are in the ſame Direction, or make one right Line. Conſequently the Point E is the Place on the Line C D, at which the Traveller muſt muſt call to make the Journey the leaſt poſſible.

Problem 22.

A Farmer borrowed a *Stack of Hay* of his Neighbour, which meaſured 6 Feet every Way, * and paid him back again by 2 equal Cubical Pieces, each of whoſe Sides were 3 Feet. Query, whether the Lender was fully paid?



Operation.

The Content of the larger Cube is 216 Feet. The Content of the ſmaller only 27 Feet. The 2 Payments, therefore, amounted to no more than 54 Feet, which is juſt $\frac{1}{4}$ (or fourth Part) of what was borrowed; conſequently the Farmer is ſtill indebted $\frac{3}{4}$ more to his Neighbour.

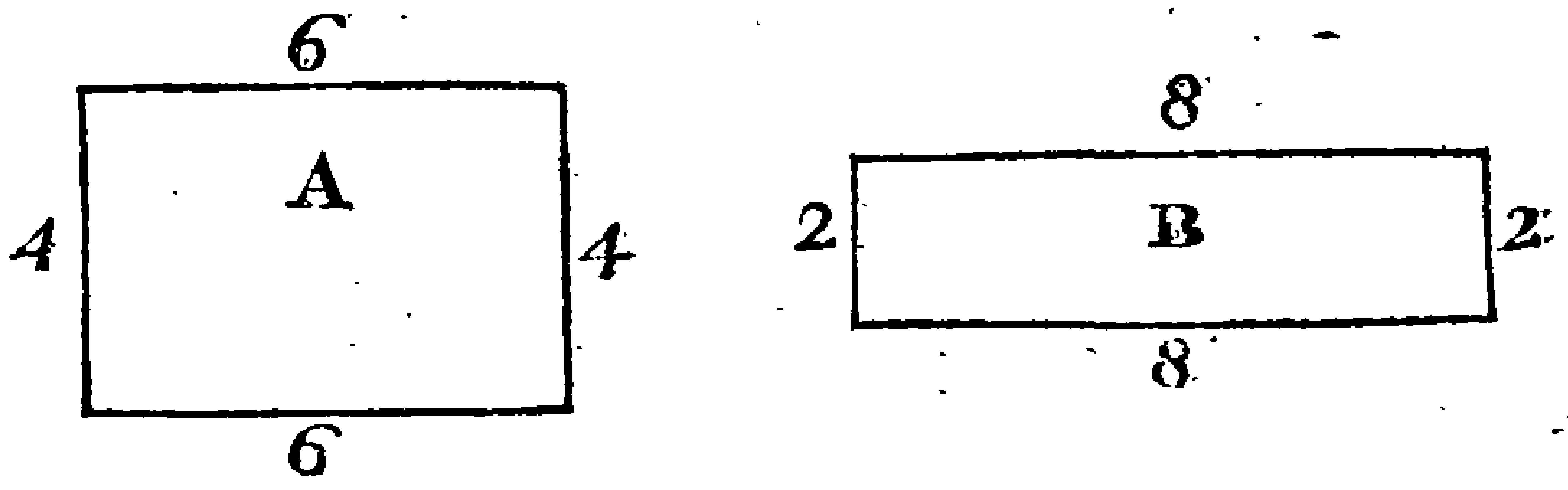
* That is, 6 Feet in Length, 6 in Breadth, and 6 in Depth.

Pro-

Problem 23.

To find the Difference of the Areas of *Isoperimetrical* Figures; that is, of Figures whose Number of Sides and Measure about them is equal, the one to the other.

It is observable that two Figures of the same Number of Sides, and the same Perimeters, may have their Areas very different from each other. For, suppose, in the two following *Parallelograms*, the Length of the former, marked A, be 6 Feet, the Breadth 4 Feet, and the Perimeter 20



Feet: And suppose the Length of the latter, marked B, be 8 Feet, the Breadth 2 Feet, and Perimeter 20 Feet, the same with the other; then the Area of the one will be 24 Feet, and the Area of the other but 16 Feet, which is equal only to $\frac{2}{3}$ of the former.*

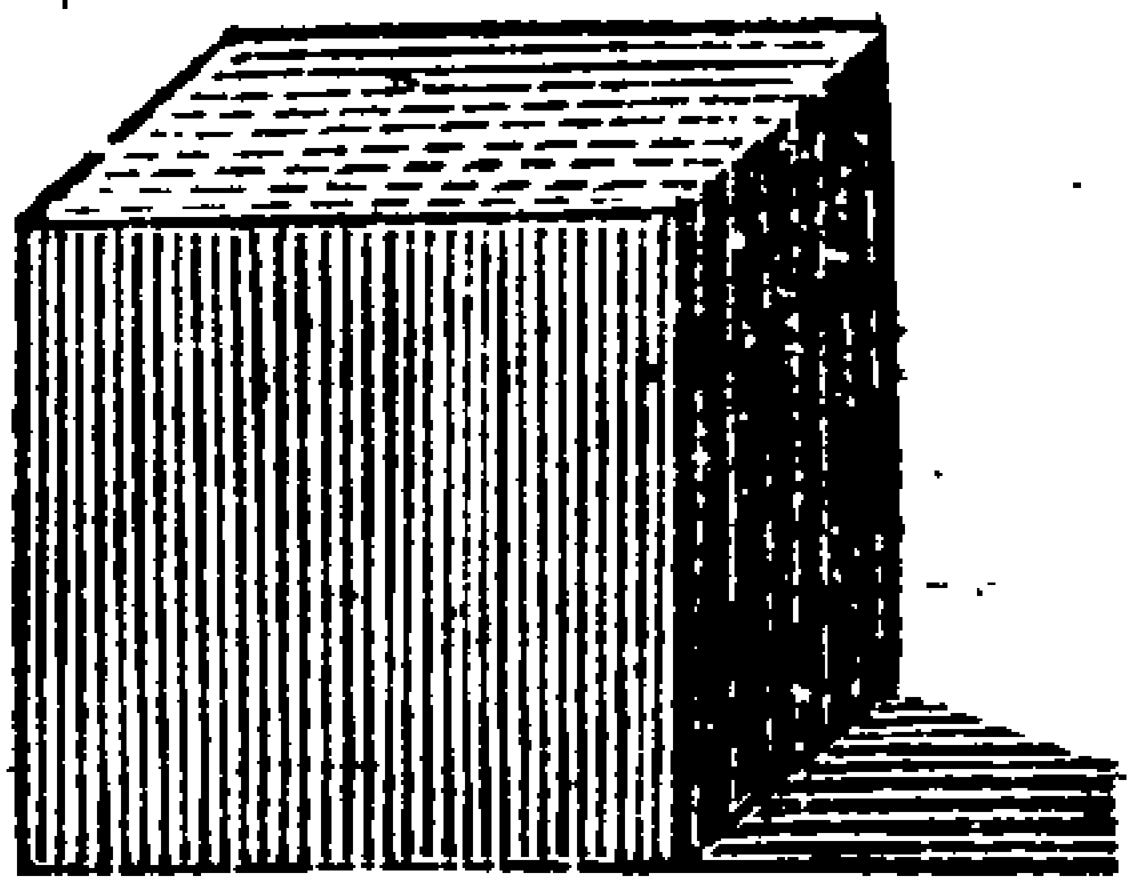
Hence we discover that of all *Isoperimetrical* Figures having the same Number of Sides, the nearer their Sides and Angles approach to an Equality, the greater is their Area or Surface, and that the Perimeter only being given is not sufficient to determine the Content.

* Many People form their Opinion of the Size of a Piece of Ground, a Camp, or a City, only from the Circumference or Perimeter: when they are told, therefore, that *Megalopolis* contains in Circumference 50 Stadia,

Stadia, and *Lacedæmon* no more than 48, and yet that this laſt City is twice as large as the former, they know not how to believe it; and if any one, deſigning to increaſe their Surprize, ſhould affirm, that it is poſſible that a Ground, Camp, or City, which contains only 40 *Stadia* in Perimeter, may be *twice* as large as another that contains 100 *Stadia*, they are ſtruck with the greateſt Amazement. The Cauſe of this Surprize ariſes from their Neglect of Geometrical Learning in their Youth; and I was rather inclined to take ſome Notice of theſe Matters, becauſe not the Vulgar alone, but ſome even of thoſe who are employed in the Adminiſtration of States, or placed at the Head of Armies, are ſometimes aſtoniſhed, and not able to conceive, that *Lacedæmon* is a much greater City than *Megalopolis*, though it be conſiderably leſs in Perimeter.

Problem 24.

To find the Side of a *Cubic Block of Gold*, which being coined into Guineas, would pay off the National Debt.



Operation. At the Tower of London 44 Guineas and $\frac{1}{2}$ are coined out of 1lb. Troy, or 5760 Grains of Gold; the Standard Weight therefore of 1 Guinea is 129 $\frac{4382}{10000}$ Grains.

The Weight of a *Cubic* Foot of such Gold is 7524921 $\frac{7}{10}$ Grains, or 1306lb. 4oz. 10dwt. 9 $\frac{7}{10}$ grs. Troy Weight. Out of this Quantity 58135 $\frac{1}{4}$ Guineas may be coined, which is equal in Value to 61042l. 0s. 3d. Sterling.

The National Debt at this Time is about 250,000,000l. Sterling, which turned into Guineas make 238095238.09, whence we have this Proportion.

Guineas.		Cubic Feet.		Guineas.	
If 58135.25	require	1,	what will	238095238.09	require?

Answer 4095.539 Cubic Feet.

Consequently, a Lump of Gold equal in Bulk to 4095.539 Cubic Feet coined into Guineas, will pay the National Debt; and the Side of such a Cube will be found to be 16 Feet nearly.

Problem 25.

What *Annuity* will be sufficient to pay off the *National Debt* of 250 Millions in 30 Years, at 4 *per Cent.* Compound Intereſt?

Operation.

The Annuity of 1*l.* for 30 Years, at 4 per Ct. is	0.0578301
Which multiplied by the Debt — —	250000000
Gives the Annuity ſought — — —	14.457525
From which deduct the Intereſt yearly paid, at 4 per Cent. — — —	10.000000
There remains the additional Sum to be raiſed	4.457525

Conſequently, the *National Debt*, allowing it, this preſent Year 1783, to be 250 Millions, and the Intereſt yearly paid at 4 per Cent. to be 10 Millions; then will an *Additional Sinking Fund* of 4.457525*l.* per Ann. clear off the whole Debt in 30 Years.

☞ If former Miniſters had been prudent enough to have diſcharged this Debt as oft as it had amounted to 40 or 50 Millions, the Annuity to have been raiſed would have been but ſmall; *i. e.* between 2 and 3 Millions yearly at moſt. Had this Meaſure been adopted, we ſhould have been at this Time, not only free from the heavy and ruinous Load of Taxes we groan under, but, with our preſent Reſources, ſhould have been a mighty People;—eaſy at Home, and formidable Abroad!

Pro-

Problem 26.

Of *Magic Squares*.

Magic Squares are Numbers in progreſſive Order in a *Natural Square* ſo diſpoſed in the Cells of a Geometrical Square, that the Sums, (if the Numbers given are in Arithmetical Progreſſion,—but their *Products*, if the Numbers are in Geometrical Progreſſion) of each Row taken either *Perpendicularly*, *Horizontally*, or *Diagonally*, are equal.

Thus the nine Digits placed in their Order in the *Natural Square* may be diſpoſed in the following Manner in the two *Magic Squares* to make the Sum of 15 each Way.

Natural Square.

1	2	3
4	5	6
7	8	9

Magic Squares.

2	7	6
9	5	1
4	3	8

4	9	2
3	5	7
8	1	6

In like Manner the *Natural Square* of 16 Cells may be diſpoſed 4 different Ways in *Magic Squares*, making 34 each Way.

Natural Square.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Magic Squares.

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

16	3	2	13
9	6	7	12
5	10	11	8
4	15	14	1

4	15	14	1
9	6	7	12
5	10	11	8
16	3	2	13

13	3	2	16
12	6	7	9
8	10	11	5
1	15	14	4

If the Numbers in the *Natural Square* are in a Geometrical Proportion, the Products of each Row will be equal to each other, taken either Perpendicularly, Horizontally, or Diagonally in the *Magic Square.*

Natural Square.

2	4	8
16	32	64
128	256	512

Magic Square.

16	512	4
8	32	128
256	2	64

The Products of the ſeveral Lines in the Magic Square make each Way 32768.

Magic Squares ſeem to have been ſo called from their being uſed in the Conſtruction of *Amulets* or Charms, as Preſervatives againſt Miſchief, Witchcraft, or Diſeaſes.

Pro-

Problem 27.

TO *Square the Circle.*

That is, to find the Side of a Square whose Area ſhall be perfectly equal to that of a Circle. *

This is a Problem that has employed the Geometricians of all Ages; much Care and Time has been expended in the Performance, yet they have not been happy enough to accompliſh it. The Difficulty ariſes from not being able to aſcertain exactly the Proportion between the *Diameter* and the *Circumference*, upon which the true Area of the Circle depends. Some Mathematicians have carried on their Calculations till the Proportion between them has been aſſigned much nearer than *one* ſingle Grain of Sand, compared to a Sphere as large as the Orbit of Saturn. Since then we can only proceed by *Approximation*, without ever arriving perhaps at a final Concluſion; we will here offer to the Learner's Attention a very ſhort as well as an eaſy Method, by which the Proportion between the *Diameter* and *Circumference* of the Circle may be found as near as *Two Hundred Thouſand Parts are to One*, which will be a Solution exact enough for all the common Purpoſes of Life.

* The *Emperor Charles* V. offered a Reward of One Hundred Thouſand Crowns to the Perſon who ſhould perfectly ſolve this celebrated Problem; and the States of *Holland* have propoſed a large Reward for the ſame Purpoſe.

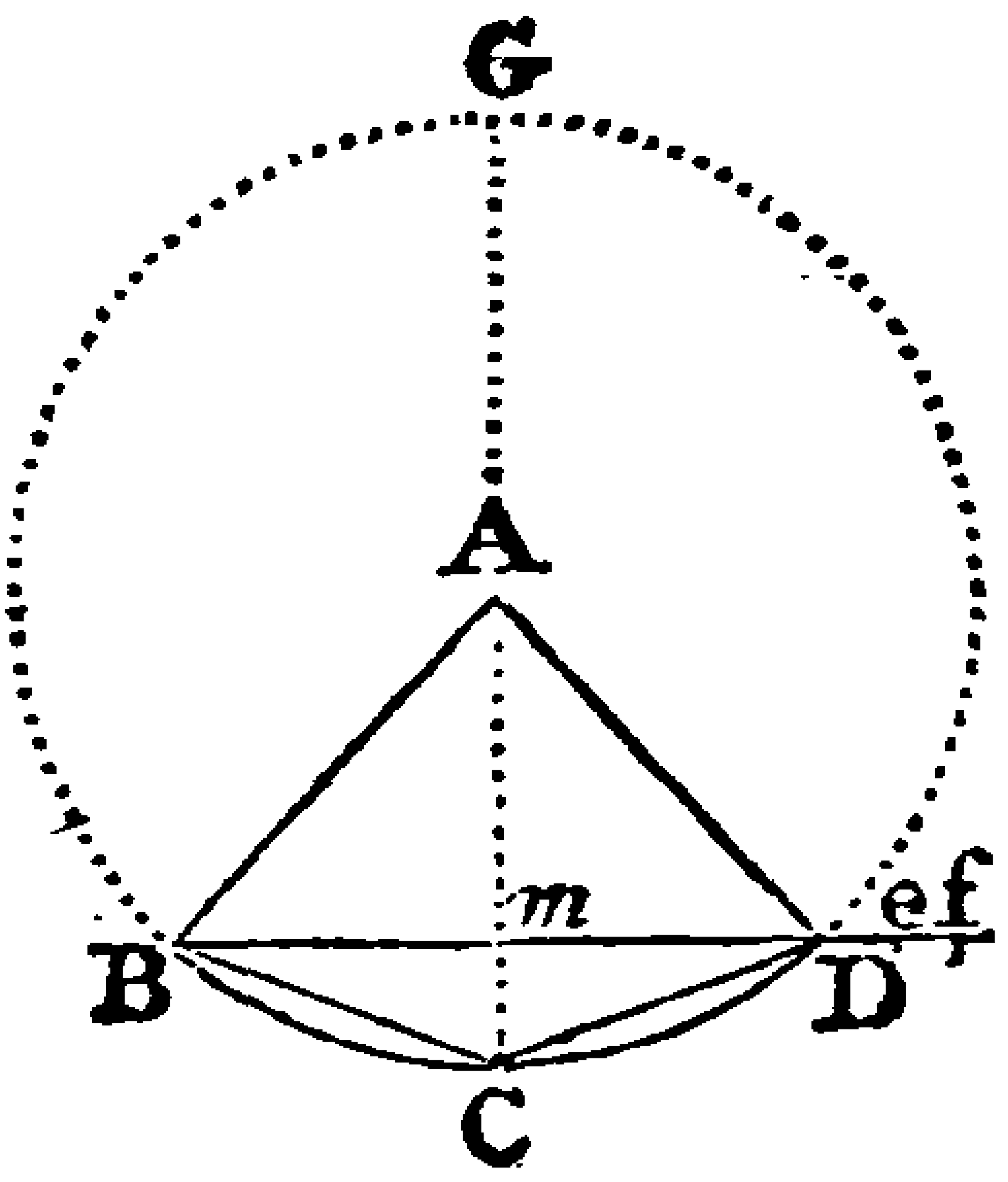
Exam-

Example.

Suppose the Arch of a given Circle be B C D; the Center A; the Diameter G A C; and A B or A D, the Radius, or Semidiameter; to find the Length of the said Arch B C D, and thereby the Length of the whole Circumference G B C D.

(1st.) *Geometrically.*

Divide the given Arch B C D into two equal Parts at the Point C, and draw the Chords B C, C D, and B D. Extend the Chord B D to *e*, so that the Line B *e* may be equal to the Length of the two Chords B C and C D. Prolong the Line B *e* to *f*, so that the Line *e f* may be equal to the third Part of the Line D *e*; and then the Line B *f* shall be *nearly* equal to the Length of the Curve B C D. *Nearly* I say, because the Line B *f* is a *very little* less than the Arch B C D; but when the Arch does not exceed 30 Degrees, the Difference is so small, that, of a Hundred Thousand Parts that may be given to the Radius A B or A D, the Difference will not amount to one of those Parts.



(2d.) *Arithmetically.*

Every one who has made any Progreſs in Geometry, knows, that the Chord of 60 Degrees of a Circle, is preciſely equal to the Radius or Semidiameter of that Circle. If the Arch therefore of 60 Degrees be divided into two equal Parts by the Radius A C in the foregoing Figure, we ſhall have a right angled Triangle formed, in which are given the Radius A B or A D = 50000; half the Chord D B = D *m* = 25000, to find the Seg. Am. (by 47, 1 B. of Euclid) which ſubtracted from the Radius A C, leaves the Length of the Line *m* C. Then, in the Triangle D *m* C right angled at *m*, there is given C *m* and D *m* to find C D the Chord of 30 Degrees = 25882. This done, divide the Chord C D or C B into two equal Parts, and proceed in the ſame Manner as before to find the Chord of 15 Degrees, which will be = 130530. This doubled makes 26106, from which take the Chord of 30 Degrees = 25882, and there will remain 224 = the Line D *e*; the third Part of which is 74, for the Line *e f*. And that Line *e f*, being added to the Line B *e*, the joint Sum will be 26180, for the whole Line B *f*, equal to the Length of the Arch B C D, which Arch (as being one-twelfth Part of the Circle) will, when multiplied by 12, give 314160, for the Circumference of the whole Circle.

Hence we diſcover, that when the Diameter of a Circle conſiſts of 100000 Parts, the Periphery or Circumference is about 314160 of ſuch Parts. And conſequently, the Diameter of a Circle is to its Circumference in a Ratio, nearly, as 1 to 3.1416.

To find the Area of the Circle.

The *Circumference* being thus found, the *Area* of the Circle may be eaſily obtained. For all Circles may be ſuppoſed to be made up of a vaſt Number of ſmall plain Triangles.

Triangles, whose Heights, or acute Angles, all meet in the Center, and whose little Bases united form the Circumference. Whence, if we multiply the *Height*, which in this Case may be taken equal to the *Radius* = .5 *, by half the Sum of the Bases; that is, by *half the Circumference*, which is = 1.5708, the Product = .7854 will be the Area required; that is, the Area of a Circle whose Diameter is 1.

To find the Area of any other Circle, &c.

And by this *Area*, may the *Area* of any other Circle be found, by knowing only its *Diameter* without the Trouble of finding its Circumference. For as all like Superficies are to each other, as the Squares of their like Sides; so are all Circles to each other as the *Squares of their Diameters*; whence it will always hold—

$$\text{As}\ \left.\begin{array}{c}\text{Diameter}\\ 1\ \square\\ \text{squared}\end{array}\right\} : \begin{array}{c}\text{Area}\\ .7854\end{array} :: \left.\begin{array}{c}\square\\ \text{Diam. given}\\ \text{squared}\end{array}\right\} : \text{Area sought.}$$

To find the Side of a *Square* equal to the Area of the Circle. You need only extract the Square Root of the circular Area, and it is done; for that Root will be the Side of the Square required; and which in this Case will be found to be .886217.

* The several Triangles being conceived to be infinitely small, their Perpendicular Height, and Sides, which are so many Radii, may be taken one for the other without any sensible Error.

Note.

Note. The Circle is the most capacious of all Figures; for if the Sides of a *Triangle*, *Square*, *Pentagon*, *Hexagon*, or any other regular Polygon, be equal to the *Circumference* of a Circle, it will not contain so great an Area as the Circle does. For, suppose the Circumference of a Circle be 75.4 Inches, its *Area* will be 452.4 Inches. Now, the *fourth Part* of the Circumference is 18.85, which squared gives only 355.3225 Inches, consequently is 97.3775 Inches less than the true Area of the Circle. Hence, we see the Ground and Foundation of the Error of measuring *round Timber*, by taking $\frac{1}{4}$ of the Girth for the Side of the Square equal; which false Way ought, I think, no longer to take Place, but be finally banished from the Recommendation and *Practice* of all thinking Men, who have any Connection with such Kind of Mensurations.

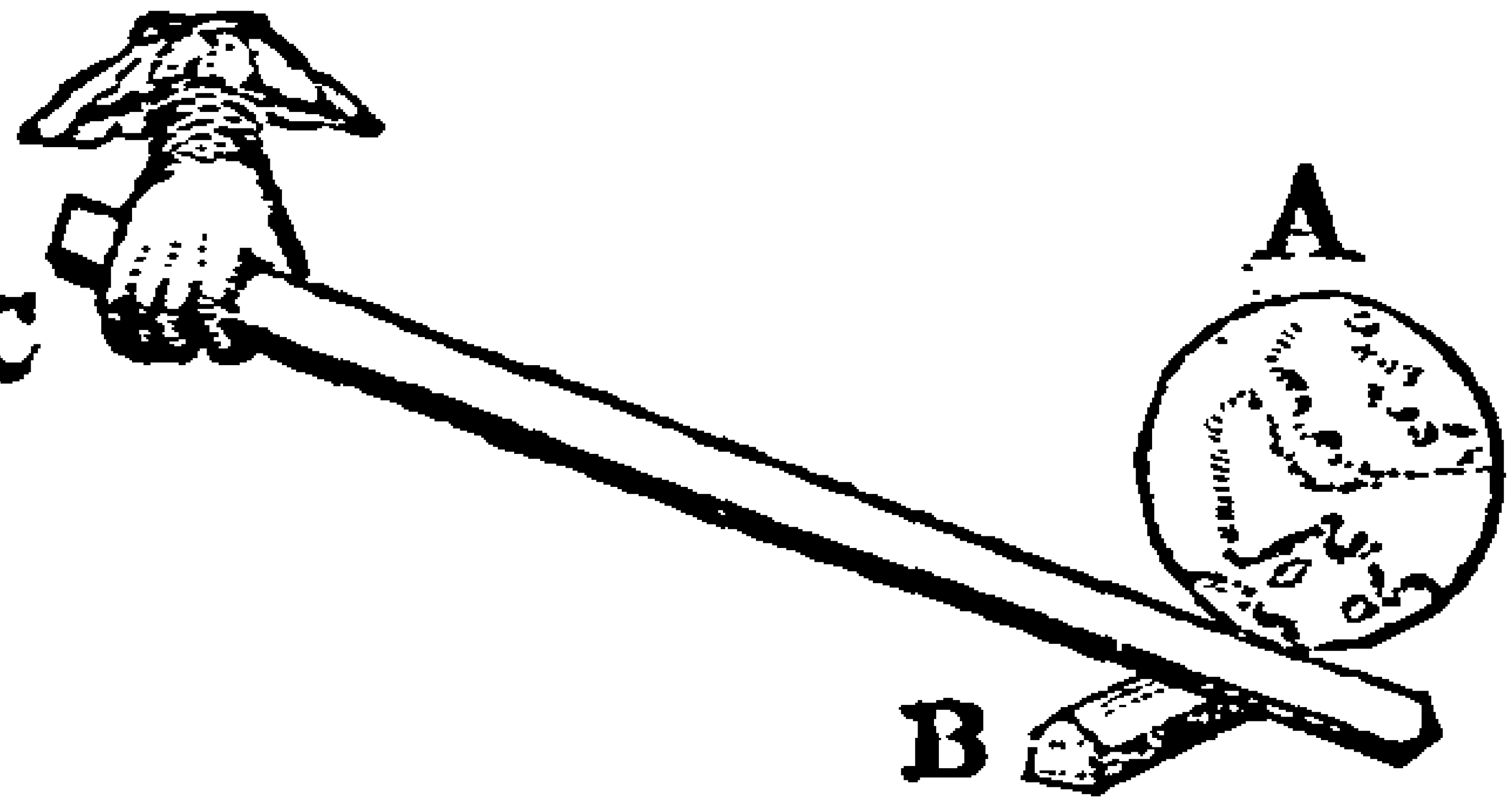
Again, if the Circumference of a Circle be 1, the Side of a Square of equal Area to that Circle is .2821; whereas by the false Method of the *Girth* it is but .25.

Pro-

Problem 28.

To raise the Earth according to the Proposal of the great *Geometrician Archimedes* of *Syracuse*.

The celebrated *Archimedes* affirmed, that he could move the Earth, if he had a Place at a Distance from it to stand upon to manage his Machinery.* This he proposed to do, perhaps, with a long Lever in the following Manner.



Thus, suppose A to represent the Earth; B the Place, Prop, or Center of Motion; and C the Place where the moving Power is to be applied.

Then, suppose a Man to pull or press the End of the long Arm with the Force of 200 Pound Weight, and that the Earth contains, in round Numbers, 4,000,000,000,000,000,000,000, or 4000 Trillions of Cubic Feet, each at a mean Rate weighing 100 Pound; and that the Prop or Center of Motion of the Lever is 6000 Miles from the Earth's Center: In this Case, the Length of the Lever from the Prop to the moving Power or Weight ought to be 12,000,000,000,000,000,000,000,000, or 12 Quadrillions of Miles; and so many Miles must the Power move in order to raise the Earth but one Mile. Whence, it is easy to compute, that if *Archimedes*, or the Power applied, could move as swift as a Cannon-Ball, it would take 27,000,000,000,000, or 27 Billions of Years, to raise the Earth one Inch.

* πός μοι στω, καὶ τὸν κοσμὸν κινήσω: Give me a Place to stand on, and I will move the Earth.

Note.

Note. If any other Machine, ſuch as a Combination of Wheels, Pullies, or Screws, was propoſed to move the *Earth*, the Time it would require, and the Space gone through by the Hand that turned the Machine, would be the ſame as above. Hence we learn, that however boundleſs our Imagination and Theory may be, the actual Operations of Man are confined within narrower Bounds, and more adapted to our real *Wants*, than to our Deſires. With this Reflection, we cloſe the preſent Work, wiſhing the Reader may receive that Benefit and Advantage from it, which was deſigned by the Author.

Note. The following Problem ſhould have been added at Page 150.

To meaſure the *Middle Zone* of a *Sphere* or *Globe*.

Rule.

To the Squares of the Radius, or Semidiameter of each End, add $\frac{1}{3}$ (one third) of the Square of their Diſtance, or the Height of the Zone. This Sum multiplied by the ſaid Height or Diſtance, and that Product again by 1.5708, will give the Solid Content.

FINIS.